The moon, theory and tables.

collection of articles.

various authors, 1824-96

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ON THE

PRESENT STATE

OF THE

SCIENCE OF THE TIDES,

WITH DIRECTIONS FOR

TIDE OBSERVATIONS.

BY THE REV. WM. WHEWELL,
Fellow and Tutor in Trinity College, Cambridge, England.

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[*Reprinted by request of Prof. A. D. Bache.*]

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ON THE SCIENCE OF THE TIDES, &c.

TO THE COMMITTEE ON PUBLICATIONS OF THE JOURNAL OF THE FRANKLIN INSTITUTE.

Gentlemen,—It is no doubt well known to you, and to those of your readers who follow the progress of general science, that the Rev. Mr. Whewell, of Trinity College, Cambridge, is engaged in endeavouring to advance the important, and hitherto, comparatively, neglected, science of the tides, the first results of these investigations being the memoir on, and map of, cotidal lines, contained in the Transactions of the Royal Society of London, for 1833. Through the kindness of a mutual friend, I have received the articles,* also from the pen of Mr. Whewell, on the subject just referred to, which accompany this note, and which I should feel obliged by your inserting in that part of the Journal of the Institute where they will be most likely to meet the eye of any one who may be disposed to contribute good tide observations to the stock which Mr. Whewell is now accumulating, for the further elucidation of his subject. By this publication I shall probably best comply with the wishes expressed by Mr. Whewell. In addition I have only to offer to communicate to that gentleman any observations which may be addressed to me, and in the way which the author may desire.

Very respectfully yours,

Philada. Aug. 26, 1834. A. D. Bache.

On the present state of the Science of the Tides. By the Rev. W. Whewell, *Fellow and Tutor of Trinity College, Cambridge.*

The state of our information concerning the tides is at present exciting considerable attention among English mathematicians; and it will probably not be long before we shall be able to announce some decided additions to our knowledge on this subject. A sketch of the present situation of this remarkable branch of science may, therefore, interest the readers of the United Service Journal; the more so, as many of them, especially naval men, may have it in their power to promote our progress by their labours or their influence.

The popular opinion on this subject is, that the true theory of the tides was discovered by Sir Isaac Newton; that he showed this curious though familiar phenomenon to be a result of the attraction exerted by the moon upon the waters of the ocean and upon the earth itself; and that in this way the course of the tides, like the motions of all the bodies of the solar system, was shown to result from one great

* On the present state of the Science of the Tides. Directions for Tide Observations, Nos. 1, 2, and 3.

and pervading law—the universal, mutual attraction of matter. And so far the popular opinion is right; but there is a difference to be noticed with regard to what Newton and the Newtonian philosophy have done in reference to this subject, and to the other consequences of the law of universal gravitation. With regard to the motions of the earth, the moon, and the planets, those motions are not only accounted for, but all the circumstances and quantities of the motions are fully explained—so fully explained that they can be exactly calculated beforehand; and predictions of the future places of all the heavenly bodies can be delivered for any future period, however distant, which predictions are always verified with an accuracy truly remarkable. The power of calculation and prediction which we thus obtain is that which sets the seal of certainty and reality upon the theory, and makes it impossible for any intelligent and unprejudiced person who examines it, not to be entirely convinced of its truth.

Now, with respect to the tides, the case is hitherto very different from this. The tides are *explained* by the theory of gravitation; that is, it can be shown that a motion of the sea of that kind, governed mainly by the moon, would take place. But neither Newton, nor the Newtonians, nor any modern philosophers, have yet explained the amount and course of the tides, at any one place; nor can they calculate beforehand the time at which the tide will take place, and the height to which it will rise, with any pretensions to accuracy. A person, therefore, who should deny the doctrine of universal gravitation, so far as its application to the explanation of the tides, could not be convinced or refuted, as he might be in other cases, by showing the exact accordance of the results of calculations founded on the theory, with measurements obtained by observation. If we take a record of the times and heights of high water for a long period, we are not in a condition to show that they are what they ought to be, the theory being true; whereas, with regard to the astronomical phenomena which flow from gravitation, we can show this in the most complete and satisfactory manner.

It will probably occur to many of our readers that the effect of accidental circumstances upon the time of high water,—for instance, of wind and weather,—and of the form of the shore, when the tide has to enter harbours and rivers,—will account for a great difference between theory and observation, and indeed would lead us to expect such a difference. But difficulties of this kind may be got over almost entirely. If we observe the tides for a long period, the effect of the wind, upon the average, is very slight, or altogether disappears; and the obstacles and modifying causes which arise from the shore and bottom are the same every day; and therefore would not make the course of the tides irregular, though they may make the time and height different from what they would have been without such obstructions. These circumstances, therefore, do not relieve the theorist from the *onus* of showing that the course of the phenomena is in accordance with his assertions. The *mean* result of observations *ought* to agree with the calculated result of theory.

This responsibility, the Newtonian, if he is a fair and philosophical person, will not attempt to evade; but he will not be able to deny that the obligation has not yet been discharged: the agreement in *detail* of tide observations with the consequences of the moon's attraction, has never yet been shown. The present object, therefore, of the cultivators of this subject ought to be, to bring into view this agreement, that is, if there be an agreement; or, if not, to bring into view the disagreement of fact and theory, and to leave the theory to take the consequences in the best way it can. This, accordingly, is what some persons at present are endeavouring to do; and the collection of long series of exact tide observations, made at many various places, is one essential part of this undertaking, to which the readers of this Journal are invited to contribute.

But, in order to make this comparison, we must not only collect many and good observations, but we must also be able to trace the consequences of the theory, under the actual circumstances of the land and sea on the earth's surface; and this is by no means an easy matter. It is, indeed, so far from easy, that it does not appear possible to do it with great exactness at present: for, the form of the shores of the ocean is so complex and varied, that no calculation can apply to it; and the *depth* of the sea, which is an important element in the question, is absolutely unknown. And, even if we knew all these *data*, the mathematical calculation of the motions of fluids has not yet become so perfect and powerful a system, as to enable us to say what would be the result of the moon's attraction, combined with the earth's motion, on such a body of water; so that our comparison is hitherto defective at both ends: we want to compare calculation and observation, and we have not a sufficient command over either to do so.

We are not, however, yet liberated from our responsibility, as philosophers, of bringing theory and fact together. For though we cannot trace *exactly* the results of theory, we can obtain a general notion of what nature they will be; and we ought to be able to say whether they are of this nature or not. For the purpose of illustrating this, I will point out one view of the tides in which this comparison would be extremely interesting, and might be made without much difficulty, by a combination of efforts of different persons,—I speak of the manner in which the tide is distributed over the surface of the ocean, and the manner in which it moves from one position to another. For this purpose I must refer to the theory, but in a way not too abstruse for general comprehension.

The moon attracts every part of the earth, and those parts the most which are the nearest to her. Thus, the water under the moon, and the centre of the earth, are both attracted by her; but the water is more attracted than the centre, and therefore has a tendency to go away from the centre; which, if the centre and the water were equally attracted, it would not have. The water will, therefore, rise under the moon and form a protuberance: its convexity will rise higher than it would do if the moon did not attract it.

As the water under the moon is nearer the moon than the centre is, and consequently is more attracted, so the water on the opposite side of the earth is further off, and less attracted than the centre; and therefore is left, as it were, by the centre, which it would not be if the water and the centre were equally attracted. There will, therefore, be a protuberance of the water on the side of the earth which is turned from the moon, of just the same kind as that which is under the moon. The magnitude of these protuberances, will depend upon the mass of the moon and its distance; the nearer the attracting body is to the earth, the greater is the *difference* of its attractive power on the centre and the near or opposite side of the earth. These protuberances would be under the moon and directly opposite to her, if the earth were at rest, and if the whole surface were water. Neither of these things is so, and we must consider what difference will arise from an alteration of these conditions.

The earth revolves on its axis and carries the water with it; and the effect of this will be, that the protuberance will no longer be under the moon; it will *lag behind* the moon, if we suppose the moon to revolve round the earth. But if we suppose the ocean to be regularly diffused over the globe, this lagging will be always the same. If a small island exist in such an ocean, the two protuberances, and the lower water between them, will all pass the island in one day, and thus make two high and two low waters at the island; and these high waters will follow the passage of the moon at an interval of time depending on what I have called the *lagging* of the protuberance which forms the high water; and these protuberances would reach from one pole of the earth to the other, and thus bring high water at the same time to all places on the protuberant line. If we suppose, for the sake of simplicity, that the moon is always in the equator, the tide might be considered as a long wave, reaching from pole to pole, and moving round the earth, following the moon steadily and perpetually, and always at the same angular interval.

But it is very clear, that when we suppose the surface of the ocean to be interrupted by great continents, like those of the Old and the New World, this sort of motion of the waters cannot go on. If we suppose such a tide-wave as I have spoken of to travel across the Pacific, when it reaches the shores of Asia and Australia it must be utterly broken and dispersed among the large islands of that part of the globe, and its progress westward as one wave altogether interrupted. The Atlantic will not receive its tides by such a wave coming into it from the east; and those tides, and the tides of the whole of this part of the world, must take place in some other way.

Now in what way will the tides, considered on this large scale, take place on the earth, occupied as it really is, with land and sea? We may form some notion of the result, by observing the way in which the long swell of the sea travels into a small creek. The large wave extends across the creek, and the part which fills the opening breaks off and travels separately up the creek. In the same manner we may still imagine a tide-wave moving round the earth from the east to the west in the Southern Ocean (for there is there a complete circuit of water;) and we may conceive that this wave turns northward and then travels into the Indian seas, and that another part of it moves northward up the Atlantic, and after running the profile of its swell along the coast of Africa on one side, and of America on the other, brings the tides to our own shores.

The tide in the Atlantic will not, it may be said, depend entirely on the tide in the Southern Ocean, as we have supposed; for the moon would produce a tide in the Atlantic, even if there were no Southern Ocean. This is quite true; but the way in which the tide moves from one place to another will still be in the nature of the motion of a wave, as it was seen to be in the above explanation.

From this being understood and conceded, a very curious and important undertaking is, *to trace the motion of this wave along the various coasts of the ocean, by actually observing at what time, on a given day or days, it is at each place;* that is, in short, by observing the moment of high water at such places. This is what I have above referred to as a possible and interesting way of comparing the observation of the tides with theoretical views; and this is what I have tried and am now trying to induce several persons to assist in doing.

The line which the ridge of the tide-wave occupies at any moment, I have called a *cotidal line*, intending by that term to suggest its nature, namely, that it is the line drawn through all places having high water at the same instant. This line occupies a different position every hour, and a series of cotidal lines drawn on the surface of the globe for each hour of a given day (the day of new or full moon for instance) would exhibit the motion of the wave, just as in a plan of a battle, the successive places of the same battalion marked on the plan show the movements of the body during the engagement.

Some of the information which is required to enable us to cover the whole surface of the ocean with cotidal lines, has been brought together already; and though it is a mere scrap of that which we might wish, and but a small fraction of that which we hope before long to attain, it has led already to some curious conclusions. We will take two or three by way of specimen.

It appears, for instance, that the wave, the ridge of which is marked by the cotidal lines, does enter into the Atlantic from the south, and throws itself across that ocean, so as to extend from Brazil to the Gold Coast of Africa, bringing the tide to both at nearly the same time. This tide-wave then travels northwards, is much interrupted and disturbed about the Madeira and Cape Verd Islands, and, after washing the shores of Spain, Portugal and France, reaches the British Isles.

The general, or, as we may say, *natural* direction in which the tide-wave travels is from east to west, following the apparent daily motion of the moon. But in consequence of the position of the shores of Africa and America, the direction of this wave changes so, that its progress is *north*, as we have already seen. When it reaches the chops of the Channel, the tide-wave separates, one branch turns again and takes its way *eastward* up the Channel, thus moving opposite to its original direction. This is the branch which brings the tides to all points of the south coast of England as far as Dover, and, as it would seem, through the Straits of Dover to the North Foreland.

Another branch of the same tide travels along the west coast of Ireland and Scotland, and does not bend eastward till it reaches the Shetlands. But when it has thus turned the north point of Scotland, it not only turns to the east, but it afterwards turns to the south, and then travelling downwards, brings the tide to the whole of the east coast of England, as far as the mouth of the Thames. On reaching this opening, the tide again turns *westward*, and thus comes to London, after going through an entire circle in the way of change of direction.

The general direction of the motion of the tide-wave being from east to west, we might expect that, of all places in the world the most likely one for this direction to prevail in, would be the sea to the southward of Cape Horn, where there is an uninterrupted girdle of water round the earth. Yet it appears to be quite certain, from the observations of Captain King, (see his Sailing Directions,) that the tide is later and later as we take points more and more easterly on the south coast of Terra del Fuego; that is, the tide-wave in this part moves from west to east. It may easily turn out that this apparent anomaly prevails only near the shore, and that further out at sea the tide-wave moves in its *proper* direction; but the curious fact just mentioned shows how much caution, and how extended a collection of observations, are requisite, in order that we may draw our cotidal lines with any degree of accuracy.

There is one general rule which appears to hold respecting the positions of the cotidal lines, so far as they have yet been drawn. As we go out of the wide ocean into the narrower seas, these lines are more and more crowded; that is, the motion of the tide-wave is more and more slow. Thus in the Atlantic the velocity of the tide-wave is 600 or 700 miles an hour; in the Indian seas it is probably not a quarter of this. On the south coast of England the tide which is at the Lizard at half-past four, is at Dover at fifty minutes past ten. This gives six hours and a quarter, nearly, for the tide to travel from the Lizard to Dover, a distance of about 300 miles; or a velocity of fifty miles an hour. *Cœteris paribus*, the velocity is least in shallow water and contracted channels.

The reader may probably be startled at the mention of such a velocity as 700 miles an hour, or twelve miles in a minute. But he must recollect that this is not the velocity of the *water*, but of the *waves*—not the rate at which the *substance*, but that at which the *form* is transferred. An undulation may run rapidly along, while the undulating substance does not run on at all; as may be seen in the waves which run along a field of corn on a gusty day, or the undulation along a stretched chain of rope. The *water* which makes the tide at Dover is not *that water* which made the tide at the Land's End six hours before, though the *elevation of the water* has been in that time transferred in a regular manner past every intermediate point of the coast. The rate at which the wave travels is no more identical with the rate at which the water moves, than the rate at which intelligence is conveyed by a line of telegraphs is identical with the rate at which the arms of the telegraphs move.

I may hereafter return to this subject; for the present I fear I may have wearied my readers.

THE ESTABLISHMENT OF THE PLACE.

The interval at which high water follows the moon's meridian passage, or transit, at a given place, varies from day to day, (being affected by the semi-menstrual inequality.)

The *Vulgar Establishment* is the duration of this interval on the day of new or full moon.

The *Corrected Establishment* is the mean duration of this interval.

The "establishment" of any place is usually said to be the hour of high water on the day of new, or of full moon. The time of high water at London Bridge on September 28, (which was full moon,) was about two o'clock. It will be the same, or nearly the same, on every other day of new or full moon. Hence, two hours exactly, or nearly, is the establishment of London Bridge. In the same manner, at any other place, the time of high water falls very nearly at the same time on all the days when the moon is new, or full. At Portsmouth, this time is forty minutes past eleven; at Plymouth, thirty-three minutes past five; and thus, eleven hours and forty minutes is the establishment of Portsmouth, and five hours and thirty-three minutes the establishment of Plymouth.

This hour was called the *establishment* of the place, from an opinion that the differences of the tide hours at different places, depended solely upon the difference of the establishment; so that this hour being established, the whole course of the tides was settled also. This is not exactly true. If it were so, we could use tide tables of London to find the time of high water at Portsmouth, merely adding or subtracting the difference of the establishments of the two places; but in reality, this way of proceeding would lead us into error, as I have already stated.

It is not true, that the differences of tide times at different places, depend *solely* upon the differences of the establishments; they depend upon other differences also, as I shall endeavour to explain hereafter. The establishment may be considered as the starting point from which the tide hours set off every new or full moon; but these hours differ, not only in the point from which they start, but also in the pace at which they proceed; for this is, though in a smaller degree, different for different places.

The establishment is, however, much the most important of the circumstances which influence the tide hours at any place, and, therefore, deserves to be attended to in the first instance.

If we say that the establishment is "the time of high water on the day of new, or of full moon," the reader may naturally ask, *which* high water? since there are two on the same day. Are we to take that of the forenoon or that of the afternoon? On the full moon day of the 28th of September, the tide of London Bridge was at one hour and fifty-nine minutes in the morning, and at two hours and fifteen minutes in the evening. Which of these times is to be selected as the establishment?

The proper reply to this question would be, "*that* time is to be taken which corresponds to the exact moment of the full moon." But this introduces a new difficulty; for neither of the tides happens at the moment of the full moon. The full moon is at seventeen minutes past eleven in the afternoon; and it can rarely happen that the tide should occur at the very instant when the moon is new or full.

The reason why the tide hours of the forenoon and afternoon are different, is, that the tides are principally regulated by the moon, in consequence of which they fall later and later every half day, by about twenty-four minutes on an average, when we refer them to common time, that is, to the time of the *sun* passing the meridian. Hence, this perpetual difference of the hour will disappear, if we refer the tide to the time of the *moon* passing the meridian. Let us do this with respect to the tides above mentioned. The times of the moon's passing the meridian, were the afternoon of September the 27th, at eighteen minutes past eleven, and the morning of the 28th, at thirty-eight minutes past eleven. Thus the morning tide on the 28th (at fifty-nine minutes past one) was two hours and forty-one minutes after the moon's southing; and the afternoon tide of the 28th (at fifteen minutes past two) was two hours and thirty-seven minutes after the moon's northing. The difference is only four minutes. And, at whatever period of the day the tide had occurred, the interval between the moon's passage and the tide would have been the same, within a few minutes.

Hence, we describe the (vulgar) establishment to be "the interval at which high water follows the moon's meridian passage on the day of new, or full moon. And, taking this definition, it is not necessary to specify whether we mean the forenoon or the afternoon tide, because each of the two follows the next preceding southing or northing of the moon, at very nearly the same interval.

When I speak of the moon's northing, I mean her passing the meridian on the north side of the heavens, which will generally take place when she is below the horizon. And the time at which this occurs may easily be calculated when we know the time of the two successive southings between which this northing is intermediate.

This may serve as an explanation of the *vulgar establishment*, as described in the above "memoranda and directions."

The interval at which the tide follows the moon's passage across the meridian continues nearly the same, not only for the two tides of the same day, but also for all the tides of successive days. The tides are mainly governed by the moon; and, though the interval at which they follow the sun's transit, or noon, varies to all the hours of the day in the course of a fortnight, the interval at which they follow the moon's transit is only altered at the same place by about two hours at the most.

Thus, in the course of fifteen successive days, the intervals of the tide and the moon's transit are as follows:—

	h. m.			h. m.	
Sept. 28,	2 41	morning tide	Oct. 6,	0 20	afternoon tide
29,	2 36		7,	0 50	
30,	2 21		8,	1 42	
Oct. 1,	2 4		9,	2 12	
2,	1 48		10,	2 21	
3,	1 7		11,	2 18	
4,	0 43	afternoon tide.	12,	2 9	
5,	0 29				

And this interval will go on alternately decreasing and increasing for seven or eight days each way.

Now, this interval on the day of new, or of full moon is, as has been stated, the vulgar establishment; and if all the intervals were equal, the establishment might be got from the observation of the tide on any one day. But, in consequence of the continual increase and decrease of the intervals which has been mentioned, a correction is required in this way of obtaining the establishment, namely, the correction for the half monthly, or semimenstrual inequality, which will hereafter be explained.

Since the interval of tide time and moon's transit goes through all its changes in half a month, as has been mentioned, if we take the *mean* of such intervals for half a month, we shall have an interval which is independent of these half monthly changes. This mean interval will differ, by a small quantity, from the vulgar establishment; but, as it is independent of the half monthly inequality, which the vulgar establishment is not, I have called it the *corrected establishment.*

Thus, the mean of all the above intervals is one hour aud forty-seven minutes, which is the corrected establishment, while the vulgar establishment, as collected from these observations, is two hours and forty-one minutes. The corrected establishment is always less than the vulgar establishment, for reasons which will appear hereafter.

THE SEMIMENSTRUAL INEQUALITY.

It has already been said, that the time of high water is regulated mainly by the time of the moon's transit or southing. The establishment is the interval of time by which the tide follows the moment of the moon's southing, on the day of new or of full moon; and the interval at which the tide follows the moon's southing every other day, is *not very much* different from this. It may, however, be different to the amount of above an hour, and we have now to speak of this difference.

If at any port, for instance at London, we take the interval which elapses between the moon's southing and the time of high water, on every day from new to full moon, we shall have the following succession:—

Moon's Age.	Tide after Moon's Transit.	Time of Moon's Transit.	Tide after Moon's Transit.
Days.	h. m.	Hours.	h. m.
1	1 57	0	1 57
2	1 45	1	1 42
3	1 32	2	1 26
4	1 19	3	1 11
5	1 6	4	0 56
6	0 54	5	0 45
7	0 46	6	0 42
8	0 43	7	0 52
9	0 45	8	1 23
10	1 1	9	1 56
11	1 27	10	2 10
12	1 57	11	2 8
13	2 8		
14	2 10		
15	2 4		

We see, in the second column, that these intervals are unequal; but after half a month, (from new to full moon, or the reverse,) we come back to the original interval; and if we were to go on further to the sixteenth, seventeenth, and succeeding days of the moon's age, for instance, that is to the full moon, and second, third, and succeeding days from the full moon, we should have a recurrence of nearly the same intervals which we had on the day of new moon, and on the first, second, and succeeding days from the new moon.

In the above table, along with the intervals corresponding to the moon's age in days, I have placed the intervals corresponding to the hours of the moon's transit, or southing. In fact, by stating the hour of the moon's transit, we determine her age, and determine it much more accurately than by saying she is so many *days old*. For, if we say the moon is three days old, (or that it is the third day of the moon's age,) this *may* mean any period of a lunation which is more than two, and less than four days from the new moon; and it will be nearer to the one or the other of these limits, according as the new moon took place at a late or at an early period of that twenty-four hours which we call the first day of the moon. Therefore, when we only know that the moon's age is three days, we only know that the interval of the moon's transit and high water at London ought to be less than one hour and forty-five minutes, and greater than one hour and nineteen minutes. But if the moon pass the meridian at two o'clock, solar time, we know that she must be exactly thirty degrees of hour angle from the sun, and, therefore, that the interval of transit and tide should be exactly one hour and twenty-six minutes.* And the same reasoning applies every day, because the moon is, on

* In the whole of this section I leave out of consideration all inequalities except the semimenstrual; as those arising from declination, &c.

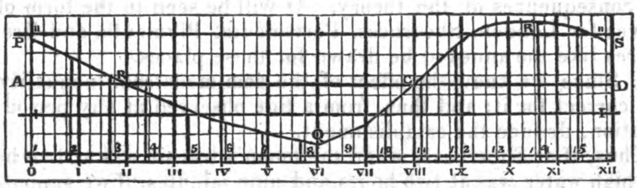
every day of the month, at a different angular distance from the sun; and passes the meridian at a different hour of the twenty-four. And, therefore, though it is a very inaccurate way of describing the moon's age to say she is so many days old; it is a very accurate way, to say she makes her transit at such an hour.

This way of marking the stage of the lunation, or semi-lunation, of which we have to speak, I shall adopt in future; and I recommend it to naval men, and to all persons who have to take into account the age of the moon, in cases where any sort of accuracy is required.

This being understood, I proceed to make some remarks on the inequality of the intervals of time between high water and the moon's transit; and for the reasons just stated, I shall take the second part of the above table, in which these intervals are referred to the corresponding hours of transit.

1. The inequality of the intervals goes through all its changes, or completes its *cycle*, in a half revolution of the moon, (in the passage from new to full moon, for instance;) it is hence called the *semimenstrual*, or half monthly *inequality*.

2. The change of the interval will be more easily comprehended, if the changing magnitude is expressed by spaces which can be looked at by the eye at one time.



Let a base line be taken, as 0 I. II. III. IV. V. VI. VII. VIII. IX. X. XI. XII. in the figure, and this being divided into twelve equal parts, let lines be drawn perpendicular to it, through all the points of division. Such lines are called *ordinates*. Let there be measured on these ordinates, from the base line, distances representing the twelve intervals in the last column of the above table; and let a curve line, as PBQCRS, be drawn through the extremities of all these measured lines: this curve line is the *curve of the semimenstrual inequality*.

3. This curve has, for all ports for which sufficient tide observations have hitherto been obtained, the same general figure. It has a *minimum* and *maximum*, or least and greatest ordinate, as at Q and R; or, as it may be otherwise expressed, it resembles the letter S laid along the line. If we proceed from the new (or full) moon, its height first diminishes, then increases, and then diminishes again.

4. If a line be drawn parallel to the base line, at a distance equal to the *mean* ordinate, the curve of the semimenstrual inequality will be *symmetrical* with respect to this line; and this line is called an *axis* to the curve.

Thus, the line ABCD, which is drawn at a distance from the base line representing one hour and twenty-six minutes, is an axis to the curve. It cuts the curve in the points B and C, and the parts before and after the point B are exactly similar above and below the line; and also the parts before and after the point C.

This mean ordinate represents, for any place, what I have called the *corrected establishment;* which is what Laplace has called the *fundamental hour* of the port. The *vulgar establishment* is represented by the ordinate at the beginning of the curve, on the first day of the moon's age, or when the time of the moon's transit is 0.

5. Though the curve is symmetrical on the two sides of the line AD, it is not symmetrical with regard to the two ends of that line. The distance BC is exactly half of AD, but the curve makes a smaller angle with the axis at B than it does at C.

This circumstance in the form of the curve corresponds to this fact; namely, that the intervals (of moon's transit and high water) increase more rapidly after their minimum than they decrease before it. They diminish from two hours and ten minutes to forty-three minutes in nine days, and increase again from forty-three minutes to two hours and ten minutes in six days.

This fact is hitherto found to be true, by experience, at all the places for which we have sufficient observations: it also agrees with the consequences of the theory. It will be seen in the form of the curves for London, Sheerness, Portsmouth, Plymouth, and Brest, if figures like the annexed be drawn for those places.

6. Since we know the effect of the semimenstrual inequality, we can correct for it; and thus, from a tide observed at any period of a lunation, deduce the establishment.

Thus, if at Sheerness, when the moon's transit was at two hours, the high water was at two hours and nine minutes, if we suppose the semimenstrual inequality to be the same as it is at London, we should reason thus:—The semimenstrual inequality makes the interval less by thirty-one minutes, when the moon's transit is at two hours, than when the transit is at 0 hours; (see the above table.) But in this case, the transit being at two hours, the interval is 0 hours and nine minutes. Therefore, when the transit is at 0 hours, the interval will be forty minutes; and 0 hours and forty minutes is the vulgar establishment of Sheerness.

This assumption, that the semimenstrual inequality is the same at all places for the same age of the moon, is not exact, in consequence of the difference of what I have called the *age of the tide.*

7. The intervals between the tide hours on successive days are unequal, in consequence of the semimenstrual inequality. These intervals are least when the tide is greatest, (at spring tides,) and greatest when the tide is least, (at neap tides.)

This will appear from the above table; for since the semi-lunation is fifteen days nearly, (fourteen and three-quarters more nearly,) the moon's time of transit, which increases by twelve hours in these fifteen days, will increase by forty-eight minutes, nearly, each day. We shall therefore have the following times of high water on each day, by finding the time of the moon's transit by the successive addition of forty-eight minutes each day, and the time of high water by adding to this the interval at which the tide follows the moon's transit.

I neglect here the difference between thirty days and an exact lunation; and I neglect likewise the inequalities of the moon's motion; for these enter into another part of the subject. I suppose also the new moon to occur at noon on the first day.

Moon's Age.	Time of Moon's Transit.	Tide after Moon's Transit.	Time of High Water.	Difference.
Day.	h. m.	h. m.	h. m.	h. m.
1	0 0	1 57	1 57	0 36
2	0 48	1 45	2 33	0 35
3	1 36	1 32	3 8	0 35
4	2 24	1 19	3 43	0 35
5	3 12	1 6	4 18	0 36
6	4 0	0 54	4 54	0 40
7	4 48	0 46	5 34	0 45
8	5 36	0 43	6 19	0 50
9	6 24	0 45	7 9	1 4
10	7 12	1 1	8 13	1 14
11	8 0	1 27	9 27	1 18
12	8 48	1 57	10 45	0 59
13	9 36	2 8	11 44	0 0
14	10 24	2 10	12 34	0 42
15	11 12	2 4	13 16	0 41
16	12 0	1 57	13 57	

It appears from the last column of this table, that the time of high water on successive days is, at springs, later by only thirty-five minutes each day; while at neap tides it is later by seventy-eight minutes, or more than double the former interval.

Nearly the same rule would be proved to hold at any other place.

This agrees also with the theory: according to which, the daily retardation of the tides at springs and at neaps, respectively, should be in the proportion of the sum of the lunar and solar tides to the difference of the same tides; where, by the lunar tide, I mean the tide which the moon would produce if the sun were not there; and, by the solar tide, the tide which the sun alone would produce. Hence, the lunar and solar tides are in the proportion of fifty-six and a half, to twenty-one and a half, or of five to two nearly.

The same rule would hold for the retardation of the tides from one *half* day to another, except in so far as it might be modified by the effect of the *diurnal difference of the tides.*

THE AGE OF THE TIDE.

The *circumstances* of each tide do not correspond to the places of the Sun and Moon at the time of that tide, but at a time one, two, or three days earlier; this distance of time is called the *Age of the Tide.*

Two such circumstances may be especially noted:

1°. The spring tide, or *highest* high water, is not on the half day of New or of Full Moon, but at a certain tide on some later half day.

2°. The interval of tide and moon's transit has not its *mean* value on the half day of New or of Full Moon, but for a certain tide at some later half day.

The distance of time from the New or the Full Moon to the time when the interval of tide and moon's transit has its mean value, is the *Age of the Tide*.

The age of the tide may be thus explained:

1. The mean ordinate of the curve of the semimenstrual inequality, or the mean interval of the moon's transit and the tide, takes place, at London, at the time when the moon's transit is about two hours, or her age about two days and a half from new or full.

This is also the time when the tides are highest; and since, by the theory, both the mean interval and the highest tides ought to correspond to new and full moon, we may suppose that this mean ordinate *corresponds* to the new and full moon, but that it does not *occur* till two days and a half after that time, in consequence of the length of time which is required to transmit the moon's effect upon the ocean to the port of London.

The length of time required for this purpose I have called the *Age of the Tide*. Mr. Lubbock, following Laplace, calls it the *Retard*.

2. The time which is required to transmit the moon's effect to different places is different. Thus, if we calculate it as we have done for London in the last article, it is a day and a half at Brest, and two days at Sheerness.

It appears also, that the tide hour is later and later at these places in the same order: thus, on the same half day the tide is at 0 hours and twenty-eight minutes at Brest, at nine hours at Sheerness, and at ten hours and nine minutes at London.

We may therefore suppose the tide to *travel* from Brest to London, and that the time of transmitting the moon's effect to each place depends on the time of the tide thus travelling to that place.

On this supposition, the tide would be earlier, and the time in which the effect of full moon reaches it would be smaller, as we go further back in the direction in which the tide hours are earlier. Thus, the tide at the Cape of Good Hope appears to be about twelve hours earlier than at Brest; we should expect, therefore, that the greatest tides, and the mean interval of tide and transit, would, at that place occur only one day after new and full moon, instead of a day and a half, as at Brest, or two days, as at London.

3. But our information with regard to this transit in the phenomena of the tides, is not sufficiently extensive and exact to enable us to reason upon it with confidence and accuracy. A good series of tide observations, continued for a few years, at any place in the southern hemisphere, would, on this account, be of singular value and interest.

4. The effect of the age of the tide upon the curve of the semimenstrual inequality, is not at all to alter the form of the curve itself, but to make the points B C, of intersection with the axis slide further and further from the new and full moon, as the age of the tide is greater. This change is apparent, if we draw the curves for Brest, Sheerness, and London, (as Mr. Lubbock has done in the Philosophical Transactions for 1833,) and then draw their axis.

5. A consequence of the different age of the tide for different places, is, that the tide tables which are good for one place cannot be applied to another, merely by addition or subtraction of certain hours and minutes, at least if much accuracy be wanted. The London and the Liverpool tide tables do not differ on the same day by a constant quantity; and neither of them apply exactly to other places on the coast.

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C

CONSIDERATIONS

ON THE

APPARENT INEQUALITIES OF LONG PERIOD

IN THE

MEAN MOTION OF THE MOON.

By SIMON NEWCOMB.

[From the American Journal of Science and Arts, Vol. L, Sept., 1870.]

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CONSIDERATIONS ON THE APPARENT INEQUALITIES OF LONG PERIOD IN THE MEAN MOTION OF THE MOON.

By SIMON NEWCOMB.

[Read to the National Academy, April, 1870.]

THE problem of determining the motion of the moon around the earth under the influence of the combined attraction of the sun and planets has, more than any other, called forth the efforts of mathematicians and astronomers. Nearly every great geometer since Newton has added something to the simplicity or the accuracy of the solution, and, in our own day we have seen it successfully completed in its simplest form, in which the earth, the moon, and the sun are considered as material points, moving under the influence of their mutual attractions. The satisfactory solutions are due to the genius of Hansen and of Delaunay. Working independently of each other, each using a method of his own invention more rigorous than had before been applied, they arrived at expressions for the longitude of the moon which, being compared, were found to exhibit an average discrepancy of less than a second of arc. No doubt could remain of the substantial correctness of each.

The solutions here referred to exhibit only inequalities of short period in the motion of the moon. But, it has long been known, from observation, that the mean motion of the moon is subject to apparent changes of very long period, and especially to a secular acceleration by which it has been gradually increasing, from century to century, since the time of the earliest recorded observations. If we inquire into the problem of these inequalities of long period, we shall find it seemingly no nearer a final solution than it was left by La Place, observation having since added more anomalies than theory has satisfactorily shown to exist.

The first inequality in the order of discovery was the secular acceleration. This was discovered about the middle of the last century by a comparison of ancient eclipses with modern ob-

* See Buff's paper in the Annalen d. Chem. u. Pharm., 4th supplement vol., 1865–6. Or, see his "Grundlehren der theoretischen Chemie."

servations. Its cause was first discovered by La Place, who showed that it was due to the effect of the action of the planets in changing the eccentricity of the earth's orbit.

The results of his computations agreed substantially with observations, and was therefore received with entire confidence until less than twenty years ago. The question being then taken up by Mr. John C. Adams, this eminent mathematician was led to the conclusion that La Place's result was nearly twice too large.

The same conclusion was reached independently by Delaunay, and gave rise to a remarkable discussion, the history of which is too familiar to be now recounted. It is now conceded that the value found by Adams and Delaunay is theoretically correct.

The new result no longer agreeing with observation, the difference is now accounted for by an increase in the length of the day. That this length is increasing is also known from theoretical considerations, but the data for its accurate determination are wanting.*

In the third volume of the *Mécanique Céleste* (Seconde Partie, Livre vii, Chapitre v) La Place discusses an apparent inequality of long period in the motion of the moon. The discussion is mainly empirical. The existence of the inequality is inferred from observations, these showing that the mean motion of the moon during the half century following 1756 was less than during the half century preceding. He then assumed that the inequality was due to the fact that twice the mean motion of the moon's node, plus the motion of its perigee, minus that of the sun's perigee was a very small quantity, less than two degrees per annum, and determined the coefficient of the varying angle solely from the observations. The result was that these might be satisfied by supposing the inequality of mean longitude

$$\delta l = 47''\cdot51 \; [\text{or } 15''\cdot39] \sin (2 \Omega_{\text{☽}} + \pi_{\text{☽}} - 3\pi_{\odot})$$

If, in this expression, we substitute Hansen's values of the elements, it becomes

$$\delta l = 15''\cdot39 \sin [173^\circ\, 26' + (1^\circ\, 57'\cdot4)\,(t - 1800)].$$

When in 1811 Burckhardt constructed his tables of the moon,

* The time and place when the discordance referred to was first distinctly attributed to the tidal retardation of the earth having been a subject of discussion, the following extract from an article on "Modern Theoretical Astronomy" in the North American Review for October, 1861 (vol. 93, p. 385), may not be devoid of interest.

"It seems to be well established that the new theory is inconsistent with the observations of ancient eclipses, and if it should prove to be correct, we may be driven to the conclusion, that a portion of the acceleration proceeds from some other cause than the attraction of gravitation, or that the length of the day is gradually increasing to an extent which has become perceptible from the cause to which we have already referred [the tidal retardation, p. 374]. If, as centuries roll by, the day should gradually increase, the moon would move a little farther in the course of a day than if no such increase should take place. Since, in our calculations, we suppose the day constant, the apparent acceleration would be greater than the real—precisely the effect observed. The difference can be entirely accounted for by supposing an increase of something less than one thousandth of a second per century in the length of the day, and a corresponding diminution in the lunar month."

omitted the sun's perigee from this argument by the author-
· of La Place, himself, who now attributed the inequality to
lifference of compression between the two hemispheres of the
rth. The function was also changed from *sin* to *cos* and the
efficient altered. The adopted term thus became

$$\delta l = -12''\cdot5 \cos \left[291^\circ\ 57' + (2^\circ\ 0'\cdot45)(t-1800)\right]$$
$$= \ 12''\cdot5 \sin \left[201^\circ\ 57' + (2^\circ\ 0'\cdot45)(t-1800)\right]$$

Succeeding investigators have regarded the theoretical coeffi-
nts of both of these terms as insensible. It does not seem
ely that there is any such difference between the two terres-
al hemispheres as could produce the second, but I am not
are that the coefficient of the first has ever been shown to be
sensible by any published computation. This coefficient is
the ninth order and the argument is,

In Delaunay's notation, $3D - 2F - l + 3l'$;
In Hansen's, $w - 3w'$.

The period is 184 years, and the large value of the ratio of
is period to that of the moon itself might render the coefficient
nsible. Both Hansen and Delaunay pronounce it insensible,
it neither publish their computations of its magnitude.

These terms have ceased to figure in the theory of the moon
nce Hansen announced that the action of Venus was capable
producing inequalities of the kind in question. So far as I
n aware, Hansen's first publication on this subject is that found
No. 597 of the Astronomische Nachrichten (B. 25, S. 325.)
ere, in a letter dated March 12, he alludes to La Place's coeffi-
ents, and says he has not been able to find any sensible coeffi-
ent for La Place's argument of long period. But on examin-
g the action of Venus on the moon he found, considering only
e first power of the disturbing force, the following term in the
oon's mean longitude:

$$\delta l = 16''\cdot01 \sin (-g - 16g' + 18g'' + 35^\circ\ 20').$$

g' and g'' being the mean anomilies of the moon, the earth
d Venus respectively. As this expression still failed to ac-
ount for the observed variations of the moon's longitude he
ntinued the approximation to the fourth power of the dis-
rbing force, and found that the terms of the third and fourth
rder increased the coefficient to $27''\cdot4$, the angle remaining un-
hanged, so that the term became

$$27''\cdot4 \sin (-g - 16g' + 18g'' + 35^\circ\ 20'),$$

ut this increase made the theory rather worse, and the term
epending on the argument of Airy's equation between the
arth and Venus was then tried with the result—

$$\delta l = 23''\cdot2 \sin (8g'' - 13g' + 315^\circ\ 30').$$

he introduction of this term seemed to reconcile the theory
ith observation.

Hansen finally remarks that these values of the coefficients are still subject to some uncertainty from his not having employed decimals enough in his computation.

In a letter to the Astronomer Royal, published in the Monthly Notices of the Royal Astronomical Society for Nov. 1854, Hansen gives a statement of the elements employed in his tables of the moon, and refers to the subject of these inequalities in the following terms:—

"The accurate determination of these two inequalities by theory is the most difficult matter which presents itself in the theory of the moon's motion. I have on two occasions and by different methods sought to determine their values, but I have obtained results essentially different from each other. I am now again engaged with their theoretical determination by a method which I have simplified, and hope to bring the operation to a definitive close. I have also applied to my tables some coefficients which are not free from empiricism but which I can justify by the circumstance that they represent the ancient as well as the modern observations with great exactness, and it may be expected that they will represent the future observations equally well."

Hansen's lunar tables were published in 1857.

The terms of long period finally adopted are

$$15''\cdot 34 \sin(-g - 16E + 18V + 30^\circ\ 12')$$
$$+21\cdot 48 \sin(8V - 13E + 274^\circ\ 14'),$$

V and E representing the mean longitudes of Venus and the earth. Changing them to mean anomalies the terms become

$$15''\cdot 34 \sin(-g - 16g' + 18g'' + 33^\circ\ 36')$$
$$+21\cdot 47 \sin(8g'' - 13g' + 4^\circ\ 44').$$

It appears that while the first term has been restored to what was substantially its original value, when only the first power of the disturbing force was included, the argument of the second term has been changed by 50°, the coefficient being but slightly changed.

In a letter to the Astronomer Royal, dated 1861, Feb. 2d, found in the Monthly Notices for March, 1861, Hansen again refers to this second term with the statement that its coefficient is one of those somewhat empirical. At the same time he has found the coefficient, by his last theoretical determination of it, by no means insensible, like Delaunay. He adds that in the comparison with observation he has never gone beyond Bradley, nevertheless his tables satisfactorily represent the ancient observations.

A well marked feature of Hansen's published works is the copiousness and perspecuity with which his theoretical calculations are laid down. But, so far as I am aware he has never published any computation of these inequalities except that part of the first inequality which depends on the first power of the disturbing force. This computation is found in vol. xvi of the Memoirs of the Royal Astronomical Society. In the second part of his "Darlegung" we find a general method of treating inequalities of long period, but—unless I have overlooked it—no computation of any particular inequality. Nor do we find any statements of the numerical results of Hansen's various computations except those already quoted.

The only geometer besides Hansen who has attacked the problem of these inequalities is Delaunay. His researches are published in full in the Additions to the Connaissance des Temps for years 1862 and 1863. For the first approximation to the first inequality his result is

$$16''\cdot 02 \sin(-l-16l'+18l''+35^\circ\ 20'\cdot 2)$$

a result practically identical with that of Hansen. The ulterior approximations change it to

$$16''\cdot 34 \sin(-l-16l'+18l''+35^\circ\ 16'\cdot 5),$$

so that they increase the coefficient instead of diminishing it as in Hansen's theory. The difference is however so small that the results may be regarded as identical.

But, in the case of the second inequality instead of reproducing the result of Hansen, he finds a coefficient of only 0″·27, a quantity quite insignificant in the present state of the question. We have thus an irreconcilable difference on a purely theoretical question.

I propose to inquire whether we have in either theory an entirely satisfactory agreement with observation. As a preliminary step to this inquiry I have prepared the following table of the mean longitude of the moon from the tables of Burckhardt and of Hansen respectively, for a series of equidistant dates, the interval being 3652·5 days, and the epoch 1800 Jan. 0, Greenwich mean noon. These dates are marked by the year near the beginning of which they fall. Column L_0 gives Burckhardt's mean longitude on the supposition of uniform motion, from the data given on the fifth page of the introduction to his tables. Next is given the acceleration of the mean longitude deduced from Table XLVIII. The inequality of long period is from Table XLIX. The sum of these three quantities gives the corrected mean longitude.

Hansen's mean longitude and secular acceleration are deduced in the same way from the elements given on page 15 of his *Tables de la Lune*. His terms of long period are deduced from Tables XLI and XLII, the constants being subtracted and the remainder reduced to arc by being multiplied by the factor 0″·004703. The last column of the table gives the correction to Burckhardt's mean longitude to reduce it to that of Hansen. That this difference is really the mean difference between the longitudes of the moon deduced from the two tables is shown by its agreement with the known difference at particular epochs. At the end of the British Nautical Almanac for 1862 is found a comparison of the two tables, from which it appears that Burckhardt's mean longitude was then greater than Hansen's by about 14″·2. The general agreement between 1750 and 1800, when both tables agreed with observations, shows that the difference of mean motion is certainly affected with no sensible error.

Year.	Burckhardt.								Hansen.							H.–B.
	L_0			Sec. Var	Long Period.	Corr. Mean Longitude.			L_0		Sec. Var.	Long Period.	Mean Longitude.			
	°	′	″	″	″	°	′	″	′	″	″	″	°	′	″	″
1630	100	19	28·0	+ 4·9	− 8·0	100	19	24·9	18	14·4	+38·5	−21·4	100	18	31·5	−53·4
40	347	6	45·4	+ 3·6	−10·8	347	6	38·2	5	36·3	+34·1	−20·0	347	5	50·4	−47·8
50	233	94	2·7	+ 2·5	−12·3	233	53	52·9	52	58·3	+ 30·0	−17·2	223	53	11·1	−31·8
60	120	41	20·1	+ 1·6	−12·3	120	41	9·4	40	20·3	+26·1	−13·1	120	40	33·3	−36·8
70	7	28	37·4	+ 0·9	−10·8	7	28	27·5	27	42·2	+22·5	− 8·1	7	27	56·6	−30·9
80	254	15	54·8	+ 0·4	− 8·0	254	15	47·2	15	4·2	+19·2	− 2·3	254	15	21·1	−26·1
90	141	3	12·1	+ 0·1	− 4·2	141	3	7·8	2	26·1	+16·1	+ 3·9	141	2	46·1	−21·7
1700	27	50	29·5	+ 0·0	+ 0·2	27	50	29·7	49	48·1	+13·3	+10·0	27	50	11·4	−18·3
10	274	37	46·8	+ 0·1	+ 4·4	274	37	51·3	37	10·0	+10·8	+15·6	274	37	36·4	−14·9
20	161	25	4·2	+ 0·4	+ 8·3	161	25	12·9	24	32·0	+ 8·5	+20·5	161	25	1·0	−11·9
30	48	12	21·5	+ 0·9	+11·0	48	12	33·4	11	59·9	+ 6·5	+24·2	48	12	24·7	− 8·7
40	294	59	38·9	+ 1·6	+12·4	294	59	52·9	59	15·9	+ 4·8	+26·4	294	59	47·1	− 5·8
50	181	47	56·2	+ 2·5	+12·2	181	47	10·0	46	37·9	+ 3·3	+26·9	181	47	8·1	− 2·9
60	68	34	13·6	+ 3·6	+10·6	68	34	27·8	33	59·8	+ 2·1	+25·7	68	54	27·6	− 0·2
70	315	21	30·9	+ 4·9	+ 7·8	315	21	43·7	21	21·8	+ 1·2	+22·9	315	21	45·9	+ 2·2
80	202	8	48·3	+ 6·4	+ 3·9	202	8	58·6	8	43·7	+ 0·5	+18·5	202	9	2·7	+ 4·1
90	88	56	5·6	+ 8·1	− 0·4	88	56	13·4	56	5·7	+ 0·1	+12·8	88	56	18·6	+ 5·2
1800	335	43	23·0	+10·0	− 4·7	335	43	28·4	43	27·7	0·0	+ 6·1	335	43	33·8	+ 5·4
10	222	30	40·4	+12·1	− 8·3	222	30	44·2	30	49·6	+ 0·1	− 1·1	222	30	48·6	+ 4·4
20	109	17	57·8	+14·4	−11·0	109	18	1·2	18	11·6	+ 0·5	− 8·4	109	18	3·7	+ 2·5
30	356	5	15·2	+16·9	−12·4	356	5	19·7	5	33·5	+ 1·2	−15·4	356	5	19·3	− 0·4
40	242	52	32·5	+19·6	−12·2	242	52	39·9	52	55·5	+ 2·1	−21·6	242	52	36·0	− 3·9
50	129	39	49·9	+22·5	−10·6	129	40	1·8	40	17·5	+ 3·3	−26·5	129	39	54·3	− 7·5
60	16	27	7·2	+25·6	− 7·6	16	27	25·2	27	39·4	+ 4·8	−29·8	16	27	14·4	−10·8
70	263	14	24·6	+28·9	− 3·8	263	14	49·7	15	1·4	+ 6·5	−31·3	263	14	36·6	−13·1

Burckhardt's tables have been selected for this comparison because they have been extensively compared with observations made before 1700. The additions to the Connaissance des Temps for 1824 contain a paper by Burckhardt himself giving a comparison of his tables with observations of occultations made by Flamstead, Hevelius and others, between 1637 and 1700. The general result of this comparison is that the mean longitude of his tables could hardly have been more than a very few seconds in error in the year 1670. But, the preceding table shows that for this epoch Hansen's mean longitude is 30″ less than Burckhardt's. Therefore, unless we suppose Burckhardt's investigation to be affected with some egregious systematic error we must admit that the mean longitude of Hansen's tables for the epoch 1670 is about 30″ too small.

Desiring an independent test of this conclusion I have selected certain observations which, with the data available, seemed well fitted to answer this purpose and compared them directly with Hansen's Tables.

They are

1. Occultation of Aldebaran, 1680, Sept. 13, observed at Greenwich by Flamstead.
2. Occultation of the same star 1680, Nov. 7, observed at Greenwich by Flamstead, and at London by Halley.
3. Total eclipse of the sun 1715, May 3, observed at London, Greenwich and Wanstead by Halley, Flamstead and Pound.

To compute the occultations of Aldebaran the mean position for 1680·0 was derived from Le Verrier's Tables (Annales de l'Observatoire, Tome II) correcting the right ascension by $+0^s\cdot01$, and was as follows:

$$\alpha(1680) = 4^h\ 17^m\ 37^s\cdot01$$
$$\delta\ \ldots\ldots +15^\circ\ 49'\ 11''\cdot8$$

The corrections for reduction to apparent place are

for Sept. 13, $\Delta\alpha = +2^s\cdot90$; $\Delta\delta = +1''\cdot1$
Nov. 7, $\Delta\alpha = +4\cdot18$ $\Delta\delta = +2\cdot4$

The following geocentric positions of the moon were derived from Hansen's Tables.

Date (Julian Cal.)	Sept. 13.	Sept. 13.	Nov. 7.	Nov. 7.
	h m s	h m s	h m s	h m s
Gr. Mean Time,	15 0 53	16 12 53	7 50 39	8 48 15
☽'s Longitude,	64° 54′ 24″·3	65° 37′ 20″·4	64° 33′ 11″·6	65° 9′ 49″·6
" Latitude,	−4 45 29·8	−4 48 10·6	−4 39 26·9	−4 40 48·0
" Parallax,	0 59 30·0	0 59 28·8	1 1 18·5	1 1 17·8

From these data we derive the following times for the immersion and emersion of Aldebaran for the dates in question. The observed times have been concluded from the observed altitudes and clock times given by Flamstead in the *Historia Celestis*, kindly furnished me by Prof. Winlock. They differ but little from the results of Flamstead himself, when the latter are corrected for the equation of time.

	Computed.			Observed.			C—O.
	h	m	s				s
Sept. 13, Immersion,	15	2	49	15	0	53	+116
Emersion,	16	10	5	16	9	12	+ 53
Nov. 7, Immersion,	7	51	47	7	50	43	+ 64
Emersion,	8	48	16	8	47	12	+ 64

The great difference between the results of the two phases of the first occultation gives rise to a suspicion of error in the observations or the data of reduction. The second observation is confirmed by that of Halley in London, he having observed the immersion at $7^h\ 50^m\ 9^s$, and noticed that the star was "newly emerged" at $8^h\ 47^m\ 1^s$. His place of observation was probably twenty-five or thirty seconds west of Greenwich, and therefore his observation agrees well with that of Flamstead. The discordance between the observed and computed times, of this second occultation indicates a correction of about +32″ to Hansen's mean longitude at the epoch 1680, and the first may be considered as confirming this correction in direction, if not in amount.

For the eclipse of May 3, 1715 we have the following computed and observed times. I have assumed Halley's station to be in latitude 51° 31′ and longitude 25^s west. Pound's is taken in accordance with his own statement to be in latitude 51° 34′, and longitude 8^s west. These agree pretty well with Flamstead's statements that Wanstead is seven or eight miles N. by E. from Greenwich,* and that Crane Court is half a minute of time West of Greenwich.

Halley at London.

	Computed.			Observed.			C—O
	h	m	s	h	m	s	s
First contact,	20	2	35	20	2	37	— 2
Beginning of Totality,	21	5	52	21	5	39	+13
End of "	21	9	3	21	9	2	+ 1
End of Eclipse,	22	16	55	22	16	37	+18

Pouna at Wanstead.

	Computed.			Observed.			C—O
	h	m	s	h	m	s	s
Eclipse first perceived,	20	3	18	20	3	15	+ 3
The total immersion,	21	6	38	21	6	6	+32
The emersion,	21	9	48	21	9	26	+22
The just end of the eclipse,	22	17	42	22	17	10	+32

The only information I have respecting Flamstead's observations is contained in a letter of his found in Baily's 'Life and Correspondence of Flamstead, p. 315, from which it appears that his times differ only a few seconds from Halley's, instead of differing by the half minute required by the difference of meridians. An obvious slip of the pen, (*later* being written instead of *earlier*) makes it doubtful in which way the "few seconds" are to be counted. It can, however, be fairly inferred from his statement that his observations diverge from the tabular times as much or more than Pound's.

The discordance of the results of first and last contact may be attributed to this cause: that with their imperfect telescopes the observers did not begin to see the moon until several seconds after the actual commencement of the eclipse, and lost sight of it a few seconds before the actual end. The discordance in the duration of totality indicates with a high probability that the computed shadow path falls a few miles too far north. In this case the mean of the results for beginning and end of totality

* Baily's Flamstead, p. 316 p. 328.

will be about right, and we have for the excess of computed times

Halley's observations,	+ 7ˢ
Pound's,	+ 27
Flamstead's,	+ 30 ±

I infer from these results that the correction to Hansen's mean longitude at the epoch 1715 is about +11″.

Comparing the corrections thus found for the epochs 1680 and 1715, we find they are substantially those required to reduce Hansen's mean longitude to Burckhardt's. I conclude, therefore, that no egregious systematic error has crept into the researches by which Burckhardt sought to show that the epoch of his tables was substantially correct during the latter half of the seventeenth century, and that the difference between the mean longitude of Hansen and Burckhardt during that period represents approximately, at least, errors of Hansen's mean longitude.

The observations of the moon made at the observatories of Greenwich and Washington during the last ten years, indicate a tabular deviation of a remarkable character. From 1850 to 1862 we find the moon slowly running ahead of the tables, until the latter required a correction of plus two seconds in longitude to make them agree with observation. But this correction, instead of continuing to increase as all analogy would have led us to anticipate, suddenly began to diminish, so that since 1862 the moon seems to have been falling behind the tables at the rate of a second a year. This is shown by the following exhibit of the corrections to Hansen's mean longitude, or right-ascension, deduced from the meridian observations of the two observatories.

Year.	Correction given by Greenwich.	Correction given by Washington.	Mean.	Corr. mean.
	″	″	″	″
1850	+0·3	−1·3	0·0	+1·0
51	+1·5	+0·6	+1·3	+2·7
52	+0·9	----	+0·9	+2·4
56	+1·0	----	+1·0	+1·4
57	+1·5	----	+1·5	+1·4
58	+2·0	+1·5	+1·8	+1·3
62	+2·4	+2·4	+2·4	+0·9
63	+2·2	+1·2	+1·7	+0·5
64	+0·1	−1·0	−0·4	−1·2
65	−1·1	−2·4	−1·7	−2·1
66	−2·2	−2·5	−2·4	−2·4
67	−3·9	−4·1	−4·0	−3·6
68	−4·4	−4·5	−4·5	−3·6
69	----	−5·5	−5·5	−4·3

The corrections here given as those of Greenwich are, previous to 1859, derived from the comparison found in the Greenwich observations for 1859. From 1863 forward they are derived from a paper by Mr. Dunkin in the Monthly Notices of the Royal Astronomical Society for April, 1869. The work of only the four principal observers is therefore included in the comparison. The object of this comparison being not so much to determine the absolute correction to the epoch of the tables as to show the changes of this correction, it is better to reject the results of the observers whose labors were discontinuous. In the case of the Washington observations, such a selection could not be made: the results given are therefore an indiscriminate mean of all. The systematic personal differences are however found to be very small.

That these corrections are real will not, I conceive, be disputed. To suppose them due to errors of observation, would be to suppose that six or eight long practiced observers divided between the two hemispheres, all progressively changed their habits of observing in the same way, and to nearly the same amount, through a period of seven or eight years.

A portion of the observed discordance may arise from a small error in Hansen's value of the coëfficient depending on the ellipticity of the earth, which is more than a second greater than the values derived by previous investigators, either from theory or observation. The last column of the preceding table shows what the correction would be if Hansen's coëfficient were $1''\cdot5$ smaller than it is.

From all these comparisons it would appear that the problem of the inequalities of long period in the moon's mean motion is really no nearer such a solution as will agree with observation, than when it was left by La Place. By a partially empirical correction, Hansen has succeeded in securing a very good agreement during the period 1750–1860, but, if the results of the preceding examination are correct, this has been gained only by sacrificing the agreement for the century previous to 1750, and for the years following 1860. This failure to reconcile theory with observation must arise from one of two sources. Either:

(1) The concluded theory does not correctly represent the mean motion of the moon. Or:—

(2) The rotation of the earth on its axis is subject to inequalities of irregular character and long period.

The first hypothesis admits of two explanations. We may suppose either that the mean motion of the moon is subject to change from some other cause than the gravitation of the known bodies of the solar system, or that the effect of this gravitation is incorrectly calculated, and that theory and observation will be reconciled by a correct calculation.

There are difficulties in the way of accepting either of these explanations. In reference to the first it may be remarked that anomalies of mean motion cannot be accounted for by a deviation from the received law of gravitation inversely as the square of the distance, because the anomalies produced by such deviation would be regularly progressive, and would be most sensible in the secular motion of the moon's perigee. The comparison of the theoretical and observed values of this motion is, perhaps, the severest test to which the Newtonian law has yet been subjected. That the anomalies proceed from the attraction of unknown bodies passing through the system seems extremely improbable, since, if they were distant, they would affect the earth and planets more than the moon, while the closer passage of bodies could scarcely escape detection. Still, this explanation does not admit of being mathematically disproved. If we attribute the deviation to the impact of meteoric matter, we must suppose the moon to have encountered such matter in quantities nearly incredible.

These three causes exhaust those on which we can base the first explanation, unless we invalidate the third law of motion. For, by that law, matter moves only by the influence of other matter. Other matter can affect the motion of the moon only by impact and gravitation. The gravitation of known bodies. the gravitation of unknown bodies, and the impact of matter is therefore an exhaustive enumeration.

We pass now to the second explanation of the first hypothesis, namely, errors or omissions in the theoretical computation of the effect of gravitation. The wide difference between the conclusions of Hansen and Delaunay suggests the possibility that there may be inequalities still overlooked. We have however the assurance of Hansen that there are none, and we shall find it extremely difficult to introduce any periodic terms whatever which will represent the observed deviation of the moon from the tables during the past ten years, without discordance during the century previous, when the agreement of Hansen's tables with theory is believed to be quite close. It is however hardly worth while to dwell upon this explanation until we have a more rigorous theory of the inequalities of long period produced by gravitation.

Considering that the reconciliation of theory and observation is not very probable, the second hypothesis may become worthy of serious consideration. If we accept it we must admit that between the years 1860 and 1862 the rotation of the earth was so accelerated that our reckoning of time is already eight or ten seconds ahead of what it would have been had the day remained invariable. Such an acceleration could proceed only from a change in the arrangement of the matter of the earth. The possibility of this effect being produced by changes in the quantity of ice accumulated around the poles has, I believe, been pointed out by geologists. But the effect of this cause could scarcely be sensible. But, if we admit that the interior of the earth is a fluid, and also admit that general changes in the arrangement of this fluid are possible, we have all that is necessary to account for considerable changes in the rotation of the outer crust. That this fluid, admitting its existence, is not in a state of entire quiescence is rendered probable by the phenomena of volcanoes and earthquakes. If we suppose a large mass of it to move from the equatorial regions to a position nearer the axis, a mass from the latter position taking its place, the following effects will follow:—

1. A diminution in the angular velocity of the surface of the fluid, accompanied by a corresponding increase in the velocity of the axial portion. The velocity of the outer crust will then be gradually retarded by friction.

2. The gradual transmission of the increased rotation of the central mass to the surface by friction and viscosity. The motion of the crust will then be gradually accelerated. The velocity of rotation finally attained will be greater or less than the original velocity, according as the radius of gyration of the fluid mass is diminished or increased by the change in the arrangement of the fluid.

I conclude, from this discussion, that we have reason to suspect that the motion of rotation of the crust of the earth is subject to inequalities of an irregular character, which, in the present state of science, can be detected only by observations of the moon. This suspicion can be neither confirmed nor removed until we have more positive knowledge than we now have of the possible inequalities which may be produced in the mean motion of the moon by the action of gravitation.

The operation of calculating these inequalities, though complicated and difficult, is certainly within the powers of analysis. When it is completely and thoroughly done, we may ascertain whether the result can be made to represent observations. If so, well; the length of the day is not variable, and the future positions of the moon can be safely predicted. If not, it will follow either that the motion of the moon is affected by other causes than the gravitation of the known bodies of the solar system, or the day is irregularly variable.

By the end of the present century, if not sooner, we shall have an independent test of the latter hypothesis, in the agreement of the observed and theoretical times of the transits of Mercury and Venus. If the hypothesis is a true one, the irregularities may range over half a minute of time in the course of a century, and this quantity might be detected even by meridian observations of the planets in question.

3

Hansen über die Bestimmung der Figur des Mondes, in Bezug auf Aufsätze der Herren Newcomb und Delaunay darüber

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KÖN. SÄCHS. GESELLSCHAFT DER WISSENSCHAFTEN

MATHEMATISCH-PHYSISCHE CLASSE.

SITZUNG AM 11. FEBRUAR 1871.

3

P. A. Hansen, *Ueber die Bestimmung der Figur des Mondes, in Bezug auf Aufsätze der Herren Newcomb und Delaunay darüber.*

Herr *Newcomb* will gefunden haben, dass die von mir in meiner Abhandlung über die Figur des Mondes*) aufgestellte, und durch die Untersuchung der Mondbeobachtungen bestätigte Ansicht, dass der Mittelpunkt der Figur des Mondes nicht mit dessen Schwerpunkt zusammenfällt, der logischen Begründung entbehre. Sein Aufsatz über diesen Gegenstand scheint zuerst in *Silliman's American Journal for November* 1868 gestanden zu haben, von wo er in *The London, Edinburgh and Dublin Philosophical Magazine etc.* No. 246 *January* 1869 übergegangen ist. Herr *Newcomb* verspricht in diesem Aufsatze eine kritische Prüfung der logischen Begründung meiner Doctrin zu geben, und wenn ich in der That eine solche in seinem Aufsatze hätte erkennen können, so wäre ich längst darauf eingegangen, aber da ich die Logik, die in seinem Aufsatze enthalten ist, gar nicht verstehen kann, so hielt ich es für überflüssig darauf zu antworten.

Nun hat aber Herr *Delaunay*, Mitglied der Academie der Wissenschaften des französischen Instituts in Paris, sich bewogen gefunden, Herrn *Newcomb* zu secundiren, und ist noch weiter gegangen. Er versucht in seinem Aufsatz, welcher sich in den *Comptes rendus etc. de l'Académie des Sciences* No. 2 vom 10. Januar 1870 befindet, darzuthun, dass der Schwerpunkt des Mondes mit dem Mittelpunkt desselben zusammenfalle, indem er darauf ausgeht zu beweisen, dass die Oberfläche des Mondes aus Niveauschichten bestehen müsse. Er vergisst freilich hiebei,

*) Sur la figure de la Lune. In den Memoirs of the Royal Astronomical Society. Vol. XXIV.

dass früher das Gegentheil bewiesen worden ist, aber da man in Frankreich so gerne von der Ansicht ausgeht, dass von dort aus nur Vortreffliches zu erwarten sei, und es ausserhalb Frankreichs auch hie und da Personen giebt, die geneigt sind gleiche Ansicht zu hegen, so halte ich dafür, dass es nicht überflüssig sein möchte, auf die Beschaffenheit der Schlüsse, die *Delaunay* in seinem Aufsatze anwendet, aufmerksam zu machen. Ich würde dieses schon früher gethan haben, wenn ich früher auf diesen Aufsatz aufmerksam gemacht worden wäre. Bei dieser Veranlassung kann ich aber nicht umhin, auch den Aufsatz von *Newcomb* zu beantworten. Wenden wir uns zuerst zu diesem.

Die Ansicht, dass der Mittelpunkt der Figur des Mondes nicht mit dem Schwerpunkt desselben zusammenfällt, ist gleichwie die ganze Theorie des Weltgebäudes eine Hypothese, so lange sie nicht durch die Beobachtungen ihre Bestätigung gefunden hat. So waren, um von den vielen vorhandenen Beispielen nur zwei anzuführen, die *Keppler*'schen Gesetze und das *Newton*'sche Attractionsgesetz Hypothesen, bis dahin, wo sie durch die Beobachtungen ihre Bestätigung erhalten hatten.

Um erkennen zu können, ob meine oben angeführte Hypothese mit der wahren Beschaffenheit des Mondkörpers übereinstimmt oder nicht, musste ihre Einwirkung auf die Beobachtungen ermittelt werden, und diese habe ich durch ein Theorem gegeben, welches sich mit wenigen Worten, und sehr geringem Aufwande an mathematischen Betrachtuugen klar beweisen lässt. Dieses Theorem bildet eine der mathematischen Wahrheiten, gegen welche jedes Auftreten durchaus vergebliche Mühe ist. Man erkennt aus demselben, dass, wenn meine Hypothese in der That stattfindet, alle Ungleichheiten der Mondbewegung in den Beobachtungen in einem constantenVerhältnisse vergrössert oder verkleinert erscheinen müssen, und zwar findet eine Vergrösserung statt, wenn der Mittelpunkt der Figur des Mondes der Erde näher liegt als der Schwerpunkt, so wie eine Verkleinerung im entgegengesetzten Falle. Es ist hiedurch die Erforschung des Stattfindens der Hypothese der Beobachtung zugänglich gemacht.

Wollte man nun diese Erforschung für sich allein, und unabhängig von der Bestimmung derjenigen übrigen unbekannten Grössen der Mondbewegung vornehmen, die auch nur durch die Zuziehung der Beobachtungen erhalten werden können, so würde man einen logischen Fehler begehen, und fast scheint es, dass Herr *Newcomb* sich eine solche unlogische Bestimmung vorgestellt hat, denn er spricht in einem Theile seines Aufsatzes von solcher Bestimmung einzelner Coefficienten der Mondstörungen, und führt numerische Werthe solcher Bestimmungen an. Nun weiss freilich Jeder, dass man früher sich mit solchen einzelnen Bestimmungen der Coefficienten der Mondbewegung begnügt hat, aber Niemand hat bis dahin dieses Verfahren ein logisches genannt. Man hat sich dessen blos bedient, weil man meinte, auf andere Weise diese Aufgabe nicht bewältigen zu können. Es ist aber in der That dieses Verfahren schon deshalb unlogisch, weil bei demselben die Rückwirkung der einen Bestimmung auf die andere entweder ganz übergangen, oder nur höchst mangelhaft und daher unrichtig berücksichtigt wird.

Die vollständige Bestimmung der Rückwirkung einer jeden Unbekannten auf alle übrigen Unbekannten ist aber in jeder Aufgabe, die auf mehr als Eine Unbekannte führt, vom wesentlichsten Belange.

Die einzig logische Art der Bestimmung der Mondbewegung ist von der oben beschriebenen sehr verschieden. Man muss zuerst alle Coefficienten, die sich durch die Theorie sicher bestimmen lassen, durch die Theorie genau berechnen, und nur zur Bestimmung derjenigen, welche sich auf diese Weise nur unsicher oder gar nicht erhalten lassen, die Beobachtungen anwenden. Die Bestimmung der letztgenannten Coefficienten, deren Anzahl sehr klein ist, darf sich auch nicht auf jeden einzelnen, mit Absonderung von den übrigen, erstrecken, sondern man muss nach der Berechnung des Einflusses eines jeden dieser Coefficienten (der Differentialquotienten derselben) auf die Beobachtungen, die linearischen Gleichungen bilden, die die Gesammtheit der Wirkung aller Unbekannten ausdrücken. Man darf hiebei, wo es angemessen erscheint, zuerst vorläufige, nahe richtige Werthe der Unbekannten mit den Beobachtungen vergleichen, worauf die Unbekannten der eben beschriebenen linearischen Gleichungen die Verbesserungen dieser vorläufig angenommenen Werthe enthalten werden.

Der vorhandene Vorrath an Beobachtungen giebt auf diese Weise ein System von linearischen Gleichungen, deren Anzahl weit grösser ist, als die der Unbekannten, welche nun durch Anwendung der Methode der kleinsten Quadrate zu bestimmen sind.

Auf diese Art habe ich die Mondtafeln berechnet. In dem

System von linearischen Gleichungen, von welchem eben die Rede war, habe ich in jeder Gleichung auch das Glied, welches den allgemeinen Factor der Ungleichheiten enthält, den das eben angeführte Theorem verlangt, mit aufgenommen, und dessen numerischen Werth zugleich mit den numerischen Werthen der übrigen Unbekannten, oder deren Verbesserungen, durch die Methode der kleinsten Quadrate bestimmt. Ich meine, dass Niemand in diesem Verfahren etwas Unlogisches wird finden können. Auch selbst den Umstand, dass ich, wie man aus meinen früheren Veröffentlichungen weiss, bei der Berechnung der Unbekannten die eben erwähnten Bedingungsgleichungen in Gruppen getheilt habe, kann Niemand, der mit der Praxis bekannt ist, mir zum Vorwurf machen, da er so häufig angewandt wird, und angewandt werden muss, um die Anwendung der Methode der kleinsten Quadrate ausführbar zu machen.

Das erklärte Verfahren habe ich auf zwei von einander ganz unabhängige Beobachtungsreihen angewandt, nemlich auf die Greenwicher und die Dorpater Beobachtungsreihe, und jede dieser beiden Reihen hat für den in Rede stehenden Factor nahe gleiche Werthe gegeben. Ich bin noch weiter gegangen. In Bezug auf diesen Factor lassen sich die Einzelnheiten der Rechnung auf verschiedene Arten variiren, und um die möglichste Sicherheit in die Bestimmung desselben zu legen, habe ich mir die Mühe nicht verdriessen lassen, diese Bestimmung auf verschiedene Arten auszuführen. Jedes Mal habe ich diesen Factor grösser als Eins erhalten, wenn gleich der absolute Werth desselben, wie nicht anders erwartet werden konnte, aus den verschiedenen Bestimmungen desselben etwas verschieden ausfiel. Ich sage hiemit nichts Neues, da ich dieses schon vor dem Erscheinen der Mondtafeln öffentlich (in den *Monthly Notices* der *R. A. S.*) ausgesprochen habe.

Auf diese Umstände gestützt, darf ich im Gegensatze zu den Herren *Newcomb* und *Delaunay* behaupten, dass ich den Unterschied zwischen dem Mittelpunkt der Figur des Mondes und dessen Schwerpunkt l o g i s c h b e g r ü n d e t h a b e. Dieser Satz kann nicht durch blose Meinungsäusserungen umgestossen werden, sondern könnte nur dadurch bekämpft werden, dass man aus genaueren Beobachtungen, als die, welche mir zu Gebote standen, ein anderes Resultat gefunden hätte.

Herr *Newcomb* spricht in seinem Aufsatze von der Evection, führt an, dass das Hauptglied dieser Ungleichheit mit der ersten Potenz der Excentricität der Mondbahn multiplicirt ist, und meint, dass ich dieses übersehen habe. Er benutzt diese Meinung zur Lobeserhebung der sogenannten analytischen Entwickelung der Mondstörungen, die von Einigen versucht worden ist, von welcher ich aber schon vor Jahren nachgewiesen habe, dass sie zum Theil auf divergirende, und im Uebrigen auf solche Reihen führt, die in dieser Beziehung zweifelhaft sind. Mein Verfahren hingegen, welches auf schnell convergirende Reihen führt, setzt er jenem nach. Es kommt mir vor, als wolle sich diese Ansicht des Herrn *Newcomb* nicht recht mit der Logik vertragen.

In Bezug auf die Evection ist das Folgende hier zu bemerken. Wenn man die Elemente einer Planetenbahn verbessert, so betrachtet man das Differential der Mittelpunktsgleichung in Bezug auf die Excentricität als den Coefficienten der Verbesserung dieser, und bei den Planeten ist dieses auch in der Regel ausreichend, nur bei dem Neptun könnte eine Ausnahme nöthig werden. Bei der Verbesserung der Mondelemente ist dieses aber unzureichend, es muss vielmehr auch das Differential der Evection, wegen des beträchtlichen Werthes des Coefficienten derselben, in Bezug auf die Excentricität der Mondbahn mit in Betracht gezogen werden. Es reicht hiebei aus, den Quotienten des jedesmaligen Betrages der Evection durch diese Excentricität den Werthen des obengenannten Differentials der Mittelpunktsgleichung hinzuzufügen. So habe ich es gemacht, und ebenso haben es Andere auch schon früher gemacht; die Unterlassung dieser Hinzufügung würde die aus der Rechnung hervorgehende Verbesserung der Excentricität der Mondbahn wesentlich unrichtig machen. Ich habe dieses auch schon im Art. 15 meiner oben angezogenen Abhandlung über die Figur des Mondes durch die Worte angedeutet: »En vérité, pour compléter les expressions (25), il faut encore ajouter des termes dépendants de quelques inégalités lunaires, et du mouvement du périgée et du noeud, mais ici je ne parle pas de ces inégalités, je dirige seulement l'attention aux expressions (24), qu' on n'a pas considérées jusqu' à présent.«

Hier ist also allerdings etwas übersehen worden, aber nicht von mir, sondern von Herrn *Newcomb*, sein Haupteinwand gegen meine Logik fällt hiemit von selbst vollständig weg.

Man bekommt auf die oben beschriebene Art schon von selbst den scheinbaren Werth der Evection, und diese trennt sich von den Coefficienten des in Rede stehenden Factors von selbst ab; aber wenn man hieraus schliessen wollte, dass die Entfernung des Mittelpunkts der Figur des Mondes von dessen Schwerpunkt keine Wirkung auf die Evection ausübe, oder dass die theoretische Evection mit der beobachteten übereinstimme, so würde man einen grossen Fehler begehen. Sei e die wirklich stattfindende Excentricität der Mondbahn, so müssen alle Störungsglieder mit dieser Excentricität berechnet werden. Wenn also k den von den grossen Halbachsen und den mittleren Bewegungen des Mondes und der Erde abhängigen Theil des Hauptgliedes der Evection, und k', k'', etc. die analogen kleineren Glieder, so wie e' die Excentricität der Erdbahn, und γ den Sinus der halben Neigung der Mondbahn bezeichnen, dann bekommt der theoretische Werth der Evection den Ausdruck

$$e\,(k + e^2k' + e'^2k'' + \gamma^2k''' + \text{etc.}).$$

Nennt man hierauf p den Factor der Ungleichheiten des Mondes, welcher durch die Entfernung des Mittelpunkts der Figur vom Schwerpunkt veranlasst wird, so ist der scheinbare, oder der durch die Beobachtungen sich darstellende Werth der Evection =

$$pe\,(k + e^2k' + e'^2k'' + \gamma^2k''' + \text{etc.}).$$

Diese Werthe sind einander also nicht gleich, sondern verhalten sich zu einander wie $1 : p$, und ebenso verhält es sich mit allen Ungleichheiten der Mondbewegung*). Die im Vorhergehenden beschriebenen Rechnungen bestimmen neben den andern nur durch Beobachtungen bestimmbaren Grössen, auch p und pe, womit schon alles erlangt ist, wessen man bedarf. Ich habe mich aber nicht damit begnügt, sondern, wie schon oben angeführt ist, die Bestimmung von p auch auf andere Arten ausgeführt. Meine zweite Bestimmung von p beruht auf den folgenden Grundsätzen. Nachdem, wie eben erklärt wurde, p und pe erhalten worden sind, kann man aus diesen Werthen e selbst berechnen. Dies vorausgesetzt, habe ich den so erhaltenen

*) Dieser Aufsatz war schon längst niedergeschrieben, als ich Nachricht von einem neuen Aufsatze des Herrn *Newcomb* bekam, in welchem er auf den Schluss kommt: »The theoretical evection will agree with that of observation, nothwithstanding a separation of the centers of gravity and figure of the moon.« Herr *Newcomb* hat also in der That den Fehler begangen, den ich oben supponirt habe, ohne zu ahnen, dass ihn irgend Jemand begehen könnte.

Werth von e in die Evection substituirt, und die Wirkung dieser Substitution auf die schon gegebenen Unterschiede zwischen den Beobachtungen und den vorhergehenden Berechnungen berücksichtigt. In den auf diese Weise entstehenden neuen Bedingungsgleichungen muss selbstverständlich der Werth der Evection dem bisherigen Werthe des Coefficienten von p hinzugefügt werden, und die Auflösung dieser neuen Gleichungen gab eine zweite Bestimmung von p und pe, die sehr nahe mit der vorher erhaltenen übereinstimmte. Wenn man dieses Verfahren näher überlegt, so wird man finden, dass in dem Falle, in welchem die Anzahl der vorhandenen Gleichungen dieselbe ist, wie die Anzahl der unbekannten, daraus zu bestimmenden Grössen, diese beiden Bestimmungen auf identische Werthe der Unbekannten führen müssen. In dem Falle hingegen, welcher hier vorliegt, in welchem die Anzahl der Gleichungen grösser ist, als die der Unbekannten, verhält sich die Sache anders, hier kann man nur auf identische oder nahe identische Werthe kommen, wenn sich die betreffenden Unbekannten mit Sicherheit aus den gegebenen Gleichungen bestimmen lassen, und dieses wird desto mehr der Fall sein, je mehr die Anzahl der Gleichungen die der Unbekannten übersteigt. Noch grösser wird die Sicherheit, wenn, wie von mir geschehen ist, zwei von einander unabhängige Beobachtungsreihen angewandt werden. Man sieht hieraus, dass ich die Bestimmung des in Rede stehenden Factors nicht leicht genommen, sondern zur sicheren Bestimmung desselben mir alle erdenkliche Mühe gegeben habe. Der zuletzt erklärten Berechnungsart ist es zuzuschreiben, dass die in der Einleitung der Mondtafeln angegebene Evection der Anwendung des allgemeinen Factors mit unterliegt.

Die grössten Ungleichheiten in der Mondbewegung sind ausser der Mittelpunktsgleichung und der Evection, die Variation, die jährliche Gleichung und die parallactische Gleichung. Diese drei Ungleichheiten sind alle in ihren Hauptgliedern von der Excentricität der Mondbahn unabhängig, und enthalten also nicht den Factor pe, sondern nur den Factor p; vermöge ihrer beträchtlichen Grösse sind sie für sich schon vollkommen geeignet, eine gute Bestimmung des fraglichen Factors zu gewähren.

Hören wir, was Herr *Newcomb* darüber sagt. Es heisst in seinem Aufsatze wörtlich: »But the value of this perturbation (der Variation) has not been accurately determined from observation, because attaining its maxima and minima in the moon's octants, it is complicated with the moon's semidiameter and parallactic inequality.« Herr *Newcomb* wird gewiss behaupten, dass dieser Satz logisch richtig ist, aber ich muss gestehen, dass ich die Logik, die darin enthalten sein soll, nicht verstehen kann. Es ist mir unmöglich zu begreifen, dass eine solche Verbindung oder Abhängigkeit zwischen dem Mondhalbmesser und der parallactischen Gleichung in den Octanten des Mondes bestehen soll, die die genaue Bestimmung der Variation verhindert. Betrachten wir das oben beschriebene System von linearischen Gleichungen, und nehmen an, dass man ausser den übrigen Unbekannten, die es enthält, auch den Coefficienten der Variation durch dasselbe bestimmen wolle. In allen einzelnen Gleichungen dieses Systems von Gleichungen ist der Werth des Coefficienten der Verbesserung des Mondhalbmessers fast derselbe, und nur geringer Veränderung unterworfen, denn er ist der Aequatoreal-Horizontal-Parallaxe des Mondes proportional. Er ist von der Conjunction bis zur Opposition positiv, und von da bis zur nächsten Conjunction negativ. Der Coefficient der Verbesserung der Variation hingegen ist periodisch, er ist dem jedesmaligen Werthe dieser Ungleichheit selbst proportional, die von einem negativen Maximum bis zu einem positiven stetig wächst, und von da bis zum selbigen negativen Maximum wieder abnimmt. Von der Conjunction bis zur ersten Quadratur ist dieser Coefficient positiv, und von da an bis zur Opposition negativ, während durch diesen ganzen Zeitraum hindurch der Coefficient der Verbesserung des Mondhalbmessers positiv bleibt, und seinen Werth wenig ändert; in der zweiten Hälfte der Lunation wiederholen sich diese Unterschiede zwischen den beiden in Rede stehenden Coefficienten in entgegengesetzter Reihenfolge.

Der Coefficient der Verbesserung der parallactischen Ungleichheit ist zwar, gleichwie der des Halbmessers, von der Conjunction bis zur Opposition positiv, und von da bis zur nächsten Conjunction negativ, allein er ist periodisch wie der der Variation, geht von einem negativen Maximum stetig bis zu einem positiven, von da wieder stetig bis zum negativen Maximum und so fort. Er bietet also schon in jeder Hälfte der Lunation die bedeutendsten Verschiedenheiten, sowohl von dem des Halbmessers als von dem der Variation dar. Die jährliche Gleichung endlich ist auch periodisch, hat aber eine weit längere Periode als jene Ungleichheiten.

Ich komme jetzt auf einen alten Satz, gegen dessen Logik wohl Niemand etwas wird einwenden können. Dieser Satz sagt in Bezug auf jedes System von Gleichungen, also auch in Bezug auf das oben beschriebene, und hier in Betracht kommende System von linearischen Gleichungen: »Je grössere Verschiedenheiten die Coefficienten der Unbekannten, in Vergleichung mit einander, in den verschiedenen Gleichungen darbieten, desto grössere Sicherheit bietet dieses System von Gleichungen in der Bestimmung der Unbekannten, die es enthält, dar; und im Gegentheil, je geringere derartige Verschiedenheiten vorkommen, desto geringere Sicherheit gewährt diese Bestimmung.«

Nun haben wir aber eben gesehen, dass die Coefficienten der Verbesserungen des Mondhalbmessers, der Variation und der parallactischen Ungleichheit in jeder Hälfte der Lunation, in den verschiedenen Punkten der Mondbahn, die grössten Verschiedenheiten von einander darbieten, und folglich lassen sich diese beiden Unbekannten aus dem oben beschriebenen System von linearischen Gleichungen mit Sicherheit bestimmen.

Diese Schlussfolge ist nun zwar das Entgegengesetzte der oben angeführten *Newcomb*'schen, aber ich hege dennoch die Hoffnung, dass Herr *Newcomb* in deren Ableitung keinen Verstoss gegen die Logik wird finden können.

Da nun der Coefficient des in Rede stehenden Factors, zufolge der vorhergehenden Darstellung des ersten Verfahrens, in seinen Haupttheilen der Summe der oben genannten Ungleichheiten, nebst der jährlichen Gleichung, entspricht, während der Factor, welcher die Verbesserung der in den Störungsrechnungen angewandten Sonnenparallaxe bestimmt, blos der parallactischen Gleichung nahezu folgt, so lässt sich jeder dieser beiden Factoren sicher bestimmen, und sie sind nur mit der Unsicherheit behaftet, die von den unvermeidlichen Beobachtungsfehlern herrührt, und womit jede unserer Bestimmungen von Daten, die in der Natur stattfinden, unabwendlich behaftet ist. Im gegenwärtigen Falle, wo das oben beschriebene zweite Verfahren sehr nahe denselben Werth des fraglichen Factors gegeben hat, lässt sich schliessen, dass die Beobachtungen selbst, und die grosse Anzahl derselben, die angewandt worden ist, hinreichende Genauigkeit besitzen, um ihrerseits die Genauigkeit, die überhaupt durch die angewandten Gleichungen erreicht werden kann, nicht in Frage zu stellen.

Herr *Newcomb*, welcher in seinem Aufsatze die Logik in den Vordergrund stellt, schliesst denselben mit den Worten: »The hypothesis (nemlich die von mir erklärte und hier besprochene Theorie der Figur des Mondes) is therefore devoid of logical foundation.« Nach den vorstehenden Auseinandersetzungen überlasse ich Jedem die Beurtheilung dieses absprechenden Ausspruchs.

Ich komme nun zu dem Eingangs erwähnten Aufsatze von *Delaunay*, dessen Erläuterung sich mit wenigen Worten geben lässt. Herr *Delaunay* referirt über den Aufsatz von *Newcomb*, findet keine Veranlassung gegen dessen Inhalt Bedenken zu erheben, sondern meint vielmehr, ohne weitere Gründe hinzuzufügen, dass meine Theorie durch den Aufsatz von *Newcomb* sehr erschüttert (fortement ébranlée), wo nicht gar vollständig vernichtet (tout à fait anéantie) worden sei. »Schnell fertig ist man mit dem Wort.« Nun, da Herr *Delaunay* den eben wiederholten Satz blos ausgesprochen und gar keinen Versuch zu dessen Begründung unternommen hat, so brauche ich mich in Bezug darauf blos auf das Vorhergehende zu beziehen.

Im Uebrigen vergleicht Herr *Delaunay* den Mondkörper mit dem Erdkörper, spricht sich dahin aus, dass dort dieselben Umstände stattfinden müssen wie hier, nimmt an, dass der Mond durch allmäliche Erkaltung vom flüssigen Zustande in den festen übergegangen sei, und kommt endlich auf den Schluss, dass »la surface du globe s'éloigne fort »peu de la surface de niveau que les eaux déterminent.« Herr *Delaunay* scheint zur Zeit, wo er diesen Aufsatz schrieb, ganz vergessen zu haben, was zwei mit Recht hoch berühmte frühere Mitglieder der Academie, von welcher er jetzt ein Mitglied ist, *Lagrange* und *Laplace*, in Bezug auf die Figur des Mondes ermittelt haben. So sagt *Laplace* unter anderm nach der Entwickelung der Theorie des Mondkörpers in der *Méc. cél. Livre V*, pag. 370 (der alten Ausgabe) »d'où il suit que la lune n' est pas »homogène, ou qu' elle est éloignée d'avoir la figure qu' elle prendrait, si elle était fluide«, und eben daselbst pag. 372 »la lune »n'a donc point la figure d' équilibre qu' elle aurait prise, si elle »avait été primitivement fluide.« Die Bezugnahme auf die Schriften der eben genannten Gelehrten, *Lagrange* und *Laplace*, genügt vollständig, um dem Aufsatz des Herrn *Delaunay* seinen richtigen Platz anzuweisen.

Ich kann diesen Aufsatz nicht schliessen ohne über die Mondtheorie im Allgemeinen etwas zu sagen. Die sogenannte analytische Entwickelung der Mondstörungen, oder die Darstellung derselben durch Reihen, die nach den Potenzen des Verhältnisses der mittleren Bewegungen des Mondes und der Erde fortschreiten, die von Mehreren, auch von *Delaunay*, versucht worden ist, muss als ein unlogisches Verfahren bezeichnet werden, da die Convergenz dieser Reihen nicht bewiesen ist, und nicht bewiesen werden kann. Dass wenigstens einige dieser Reihen divergiren, habe ich schon früher gezeigt, und in Folge dessen ist es mehr als wahrscheinlich, dass die übrigen Reihen, die von diesen divergirenden Reihen mit abhängen, zum Wenigsten Reihen sind, durch welche man nicht jede gewünschte Genauigkeit erhalten kann. Sollten diese Reihen in der That convergiren, so ist ihre Convergenz jedenfalls sehr schwach. Nun ist es aber bekannt, dass wenn man eine schwach convergirende Reihe bei einem Gliede abbricht, welches an sich klein ist, und welches ich das Ende nennen will, der Rest der Reihe viel grösser sein kann, wie dieses Endglied.

Bis jetzt giebt es nur zwei Verfahrungsarten zur Berechnung der Mondstörungen, die als logisch richtig bezeichnet werden können, nemlich die Methode der unbestimmten Coefficienten, ohne Auflösung der Nenner in Reihen, welche *Laplace* und *Damoiseau* angewandt haben, und die Methode der successiven Annäherungen der numerischen Werthe, die fortgesetzt werden muss, bis die Resultate zweier auf einander folgenden Annäherungen mit einander übereinstimmen, oder wenigstens nur unbedeutend von einander abweichen; es ist dieses das Verfahren, welches ich zuerst angegeben und angewandt habe. Das zweitgenannte Verfahren ist schon um deswillen dem erstgenannten vorzuziehen, weil man dabei jede überflüssige Combination vermeiden und sich stets durch Controlen der richtigen Ausführung der numerischen Rechnungen versichern kann. Dahingegen geräth man bei der Anwendung der Methode der unbestimmten Coefficienten auf die Berechnung der Mondstörungen auf den unangenehmen Umstand, dass man nicht sogleich die Wirkung der verschiedenen Combinationen auf die Bestimmung der unbekannten Coefficienten beurtheilen kann, und daher Gefahr läuft, entweder Combinationen zuzuziehen, die sich schliesslich als unmerklich ausweisen, oder Combinationen wegzulassen, die merklichen Einfluss haben. *Damoiseau's* Entwickelungen bieten Beispiele beider Arten dar. Auch ist zu erwägen, dass diese Methode auf Gleichungen höherer Grade führt, die in einander verflochten sind, und deren schliessliche Auflösung nicht ohne Schwierigkeiten ist.

Die Bestimmung der Coefficienten derjenigen Mondstörungen, die kurzer Periode angehören, kann jetzt als abgemacht betrachtet werden, aber nicht ebenso verhält es sich in Bezug auf die Störungen von langen Perioden, in Bezug auf welche immerhin möglich ist, dass bis jetzt noch etwas unentdeckt geblieben ist, welches Veranlassung geben kann, dass die Epoche in den Mondtafeln, nach wie vor, von Zeit zu Zeit einer Verbesserung bedürfen wird. Ja es kann nicht einmal zu den Unmöglichkeiten gezählt werden, dass noch unbekannte Kräfte, oder solche, die sich jetzt noch der Möglichkeit der Berechnung entzogen haben, auf den Mond einwirken. Niemand weiss dieses bis jetzt! und Niemand kann das Gegentheil davon beweisen!

Ich habe mir zur Ermittelung der Ungleichheiten langer Periode in der Bewegung des Mondes die grösste Mühe gegeben, eine grosse Anzahl derjenigen Argumente untersucht, von welchen vermuthet werden konnte, dass sie nicht unmerkliche Coefficienten haben würden; ich habe diese Untersuchungen wiederholt zu verschiedenen Zeiten während der Bearbeitung der Mondtafeln aufgenommen, und eine Anzahl von Gliedern erhalten, deren Coefficienten nicht ganz unmerklich sind. Von diesen habe ich die ganz kleinen, die nur sehr geringe Wirkung ausüben können, weggelassen, die grösseren aber den Tafeln einverleibt. Anders kann Niemand es machen, und es muss abgewartet werden, ob es Jemand gelingt, etwas wesentlich Neues in dieser Beziehung zu finden. Bis jetzt hat von einer solchen Auffindung noch gar nichts verlautet.

Herr *Newcomb* hat vor einiger Zeit einen Aufsatz über die Störungen langer Periode veröffentlicht, aber in diesem Aufsatze ist kein Anknüpfungspunkt zu weiteren Entdeckungen in dieser Theorie zu finden.

Corrections of the present Theory of the Moon's Motions, according to the Classic Eclipses.

By Prof. G. SEYFFARTH, A.M., Phil. & Theol. D.

Multiformi Luna ambage torsit ingenia contemplantium et, proximum ignorari sidus, indignantium.—*Pliny, H. N.* ii 6, 12.

INTRODUCTION.

The present disquisitions were, four years ago, called forth by the learned treatise, "Historical Eclipses," reprinted in *Nature*, New York, July 25, 1872, to which my attention was directed by Dr. C. H. F. Peters, Director of Litchfield Observatory, Hamilton College, Clinton, N. Y. The meritorious author of the treatise, J. R. Hind, Director of Bishop's Observatory, Twickenham, Eng., made the historical chronology of the Romans, Greeks, and Babylonians, as set down in Petavius's *Doctrina Temporum*, and *bona fide* repeated down to Clinton in all later chronologies, the groundwork of his computations of ancient eclipses; and, moreover, he presumed Hansen's Lunar Tables, principally based on Ptolemy's Almagest, to be perfectly correct. To both very much divulged prejudices I remonstrated in an extensive letter of Feb. 11, 1873, which, soon after, was transmitted to Prof. Hind. Compliance with the added request to publish my communications in an astronomical journal seems to have been prevented by circumstances down to this day. In the mean time many friends of history, even distinguished astronomers, being occupied with re-examining the usual theory of the moon, desired, for the promotion of science, the publication of my disquisitions concerning the true dates of ancient eclipses and the resulting amendments of our Lunar Tables. These particulars, apart from others, may excuse the final appearance of these historico-astronomical investigations.

In the next place the reader has to bear in mind that all, both historical and astronomical dates, to be mentioned hereafter, refer to the astronomical method of counting the years, and not to the so called historical one, because the former is the most practical and the only true one. The historians commence, in consequence of Beda Venerabilis, the original Dionysian Era too late by one year, and augment all dates preceding the Christian Era by a unit; and hence they refer Christ's birth to a wrong year. The astronomical year 400 B.C. is, according to the historians, the 401st B.C., and so forth.

It is an axiom that no theory of the moon's motions can be correct, as long as it does not correspond with the times and magnitudes of the most reliably ascertained eclipses of old. The most trustworthy ancient eclipses, however, are those mentioned in the classical works of the Romans and Greeks; for, their authors were, in nearly all instances, eye-witnesses, otherwise reporters, of what earlier eye-witnesses had perceived. Thus Livy, Cicero, and others, mention the eclipses recorded in the *Annales Maximi*, and this great work, still existing in Tiberius's days, contained all remarkable events of Roman history which the annalists of the Capitol had, day by day, once recorded. The classical authors, moreover, were reasonable and honest men; they were able to speak truth, and willing to do it. What would they have gained by telling falsehoods, by improvising eclipses which nobody had seen, or by referring them to wrong years, seasons, days, and hours? Of this character are, for instance, Thucydides, Xenophon, Aristophanes, Herodotus, Pindar, Plutarch, Josephus, Philostratus, the Roman chroniclers, Livy, Cicero, Tacitus, Pliny, and the like. All these ancient authors deserve confidence as long as the impossibility of their traditions is not clearly demonstrated. This is and must be the stand-point of all historians.

Formerly, it is true, the eclipses in Ptolemy's Almagest were considered to be the most reliable ones, but erroneously; for, Ptolemy, 140 A.C., had not, with his own eyes, observed those ancient eclipses; and their particulars are not the result of Babylonian observations, but the fruits of Ptolemy's computations, as will be seen below. Had Babylonian astronomers themselves observed the minutes of those eclipses, the times and magnitudes of the latter, as specified in the Almagest, would agree with each other and with the classic eclipses. Instead of this, careful computations of Ptolemy's 19 lunar eclipses, by means of Hansen's Lunar Tables, have brought to light that one of them finished prior to the rising of the moon, and that another obscuration of the moon amounted only to a quarter of an inch, which nobody would have perceived with the naked eye. Paradoxes similar to these are coming. Granting that the Babylonian eclipses were exactly described in Ptolemy's Almagest; granting that Hansen was right in deducing from the same eclipses the secular accelerations of the moon's motions and other elements of his Tables,—how is it that the latter do not correspond with the ascertained Roman and Greek eclipses? The obscuration of the sun in —400, July 1, e.g., which was, according to the *Annales Maximi*, a total one in Rome, amounted, according to Hansen"s theory, to 2′ 34″ only. How came it to pass that all the Lunar Tables, from Ptolemy down to Damoiseau, based both on the Almagest and modern observations, proved incorrect some years after their construction? The reason is that the *terminus a quo*, the Babylonian eclipses in the Almagest were wrong ones; that Ptolemy had referred them to wrong years; that the longitudes of the moon, her Nodes and Apsides, were in 721 B.C. other ones than those determined by means of the Almagest.

In short, it is evident that, in establishing a true theory of the moon's motions, either the eclipses in the Almagest or else those in the Classics must be given up. *Tertium non datur*. It is true that, sometimes, the astronomers determined the dates of Greek and Roman eclipses *a priori*, and in spite of the actual history and chronology, and only by the instrumentality of Lunar Tables based on the eclipses in the Almagest; but this was obviously a gross mistake. For all events of Greek and Roman history are at present, by infallible historical and mathematical certainties, so accurately fixed that none of them can be referred to a date later or earlier by one, or ten, or twenty years. These evidences, finally, are confirmed by an authority which every astronomer will respect. In 1857, Prof. Airy was still fully convinced of the correctness of Hansen's theory of the lunar motions; for in that year he determined, by means of the said Tables, the dates of three total eclipses which he (but erroneously) referred to 309, and 555, and 583 B.C. (See Transactions of the R. Astron. Soc. 1857, vol. xviii. p. 92, and Month. Not. vol. xvii. p. 233.) Yet, a few months ago, our newspapers report as follows: "In his last report Prof. Airy devotes a few words to the great work he has been engaged in, namely, the preparation for the *formation of Lunar Tables*, according to a *new treatment of the theory* by which he hopes to be able to give greater accuracy to the final results, by means of operations which are entirely numerical throughout the work. Considerable progress has been made in these numerical developments, and he expects, at least, to put *his theory* in such a state that there will be no danger of its entire loss in the event of his death."—This is, indeed, a gratifying confirmation of my iterated researches concerning the secular accelerations of the moon's motions. First in 1846 I essayed to harmonize the classic eclipses with the usual theory of the moon based on the Almagest. (See the author's *Chronologia Sacra*, p. 281 to 358.) The same was done, but much more carefully, in Seebode, Jahn, and Klotz's "Archiv für Philologie," 1848, p. 586; in Jahn's "Astronomische Unterhaltungen," 1853, p. 172; in "Göttinger Gelehrte Anzeigen," 1855, No. 125; in my "Berichtigungen der alten Geschichte und Zeitrechnung," 1855, p. 92; in "Transactions of the St. Louis Academy of Science," 1860, p. 385. Twenty years ago I predicted, without being a prophet, that Hansen's Lunar Tables would, after forty or fifty years, prove as incorrect as Damoiseau's Tables did in 1851, on occasion of the total eclipse of the sun in Germany; and from the publication of Hansen's Tables to 1875, less than twenty years having elapsed, the incorrectness of those Tables comes to light.

These arguments will suffice for understanding that the theory of the secular accelerations of the moon, her Nodes and Apsides, mainly depends upon the classic eclipses, and not upon Ptolemy's computations in the Almagest; for the ancient eclipses which, according to reliable eye-witnesses, coincided with sunrise, or sunset, or certain hours of the day, determine the real longitude of the moon on the respective hours. To this class of ancient eclipses refer, e.g., the solar eclipse in —478, Feb. 27, 15h. 30m., perceived, during sunrise, at Smyrna, and that in —752, May 25, 16h., which was seen in Rome 2 hours and about 30 minutes after sunrise. Further, the eclipses which were total in certain localities determine the real longitudes of the moon's Nodes on the respective days. From the very small eclipses witnessed by ancient authorities we learn how far the longitude of the moon's Nodes must be diminished in order to obtain a corresponding obscuration of the sun. Thus the very small eclipse of the sun in —420, Jan. 18, 2h., ♉ 17° east of the sun, was invisible in Athens, and yet the eye-witness Aristophanes saw it. Finally, many of the twenty-nine total solar eclipses mentioned in the classics were, according to our Lunar Tables, annular ones; and by means of them the usual secular acceleration of the Apsides can be corrected. To this class, e.g. the eclipses in —581, Mar. 27; —400, July 1; —360, May 12; —306, June 13. belong.

Since, then, the classic eclipses are very important in establishing a correct theory of the moon's motions, the next task must be, first, to collect, at least to A.D. 400, all reports of the classic authors referring to an eclipse either of the sun or the moon, and specifying the localities, and magnitudes, and hours of the respective eclipses; in the second place, to reduce the latter to their real years. The chronology of ancient eclipses is inseparably connected with the historical Chronology of the Romans, Greeks, Babylonians, Egyptians, Chinese; and, in this respect, history has made considerable progress since Ptolemy, especially since Petavius, whose chronology is notoriously based upon Ptolemy's erroneous Historical Canon.

New Astronomical and Historical Subsidiaries of Ancient Chronology and History.

Since the year 1627, in which Petavius's *Doctrina Temporum*, the basis of all later Chronological Tables down to Clinton and Fischer, made its appearance, a great many of both astronomical and historical materials have come to light, by which all events of Roman, Greek, Babylonian, Egyptian, Hebrew, and other histories, especially the dates of ancient eclipses, are incontrovertibly fixed, as will be seen in the author's "Astronomia Ægyptiaca," 1833; in his "Chronology of the Roman Emperors" (Gettysburg Quarterly Review, 1872, p. 47), and the other aforecited works (p. 403). It will be sufficient to specify only, and as briefly as possible, the following:

1. *Planetary Configurations*, *Lectisternia*, *Pulvinaria*, *Ἱεραὶ κλῖναι*, that is to say, representations of the ancient seven planets—Saturn, Jupiter, Mars, Sun, Venus, Mercury, and Moon—together with the Signs and smaller parts of the Zodiac with which the former were conjoined on certain days of certain years. Nearly all these autoptical observations were performed on the cardinal day preceding the historical event which they referred to. These planetary configurations are the most solid fundamentals of ancient chronology, because none of them returns twice

during a period of 2146 years, in which the procession of the fixed stars amounts to 30 degrees, one sign of the Zodiac; and because the ancients, being destitute of the Copernican System, could not calculate earlier places of the seven planets. All epochs of ancient history determined by a planetary configuration are fixed with mathematical certainty. We mention the following ones only:

Sixteen Egyptian monuments, representing the planetary configuration, observed on the day of the summer solstice in —2780, previous to the beginning of the first Canicular period on July 19th, the time of Menes' arrival in Egypt. Hence the Canicular periods of 1460 Julian years commenced in —2780, —1320, + 140, and not, as Petavius imagined, one year earlier. Together with the same dates, the Apis periods of 25 Egyptian years, each of 365 days only, commenced; and hence these periods recommenced, during the period from —1320 to +140, in all years which being divided by 25 give the remainder 20, e.g. in —520, —495, —320. This is very important, because several events of Persian and Greek histories are linked to the epochs of Apis periods. With the same July 16 in —2780, moreover, the Egyptian period of 30 years, called τριακονταετηρίς, so often mentioned on Egyptian monuments, had begun. To-wit, in —2780, July 16, a close conjunction of Mars with Saturn took place, and this conjunction returned after 30 Egyptian years, whereby several epochs of Egyptian and Greek histories are mathematically fixed. The renewals of this Triacontaeteris occurred, during the period from —1320 to +140, in such years, of which the number being divided by 30 leaves the remainder 0, e.g. in —210, in which Ptolemæus Epiphanes, "the lord of the Triacontaeteris," was born. A copy and the explanation of the planetary configuration of —2780 will be found in the author's "Berichtigungen," etc.

The Olympian Altars. Pausanias (v. 14) and the Scholiast of Pindar (Ol. v. 10, x. 59) narrate that at the beginning of the Olympiads six altars were erected, and each of them contained two statues, one of a planetary, and one of a zodiacal god. This planetary configuration, expounded in the same "Berichtigungen," p. 230, refers to —777, March 29, the day of the vernal equinox, preceding the first Olympian games. Consequently the latter were celebrated in June of the year —777, and hence "Ol. i. 1" signified, conformably to all ancient eras, the first year subsequent to the end of the quadriennial period of the first Olympiad. Hence the second Olympian games were delivered in —773; and it was a deplorable mistake, committed by Petavius, to commence the Olympiads two years earlier, —775, instead of —773. The consequence of this blunder was that Petavius and his followers antedated all events of Greek history in general by two years. All Olympian games were repeated every four years, namely, prior to the summer solstice, in such years before Christ, of which the number being divided by 4 gives the remainder 1; but after Christ, in all years, of which the number being divided by 4 leaves a remainder of 3, e.g. in —1 and +3. It is, moreover, a strange phenomenon, not yet explained, that several fathers of the church, and some later authors, commenced the Olympiads two years earlier. But these are exceptions to the rule. (See Ideler's Chronologie, ii. p. 465.)

The Statue of the Olympian Zeus. Subsequent to the battle at Marathon, the Greeks, applying the gold taken from the Persians on occasion of the Marathonian battle, erected a grand statue to Zeus, the deliverer of the country; and on the pedestal of his statue the planetary configuration referring to the same battle was represented. This astronomical monument, described by Pausanias (v. 11, 3), concerns the autumnal equinox, Sept. 25 in —489, as will be found in my "Berichtigungen," etc. p. 234. The date of the battle, August 6th in —488, is confirmed, as will be seen below, by the solar Calendar of the Greeks. Consequently, Petavius has antedated the battle, and the reigning-time of Xerxes by one year.

The Parthenon frieze in Athens contains the planetary configuration concerning the battle at Salamis, subsequent to the battle at Thermopylæ. The latter Herodotus (vii. 206) refers to the celebration of the Olympian games, and to the Archonship of Kalliades, i.e., according to Petavius, to —479, but the Parian Marble puts the battle in the following year. The latter is confirmed by the aforesaid planetary configuration, observed on the winter solstice in —479, and by Thucydides (i. 18), who counts ten years from the battle at Marathon, on Aug. 6 in —488, to the battle at Salamis in —478, Sept. 23. Consequently, Petavius has again antedated these events by one year.

The planetary configuration (Solin. Pol. i. 18) *referring to the foundation of Rome* demonstrates that Rome was founded in —752, and not, as Petavius imagined, in —753. (See Seebode, Jahn, and Klotz's Archiv f. Philol. 1848, p. 596.)

The Lectisternium (Liv. v. 13; Dion. xx. 9), viewing the Bruma in —396, evidences that the tribuni Ginucius, Pomponius, etc., ruled in —395 and not in —397. Petavius, having shortened the history of the Roman kings by one year, arbitrarily intruded in —331 a consular year, which is not to be found either in Livy or other annalists. (See "Berichtigungen," etc. p. 229.)

The Lectisternium (Liv. xxii. 10) puts beyond any question that the Coss. Geminus and Flaminius ruled in —215, and not in —216. (See "Berichtigungen," etc. p. 226.)

The Ara Albani, representing the nativity of Emperor Augustus, demonstrates that Cicero was consul in —62, and not in —63; that, accordingly, Augustus was born one year later than Petavius stated. (See "Berichtigungen," etc. p. 239.)

The Puteolian Basis, the nativity of Tiberius, refers his birth to —40, and not to —42. Hence the consuls Blancus and Lepidus officiated two years later than Petavius and his adherents presumed. (See "Bericht." etc. p. 223.)

The Puteale Capitolinum refers the birth of Claudius to the year —8. (See "Bericht." p. 244.)

The Ara Gabinia, the nativity of Vespasian, states that he was born in the year +9. ("Bericht." p. 208.)

The Ara Capitolina, the nativity of Caligula, argues that his birth and the respective consuls Germanicus and Capito belong to the year +14, and not, as Petavius brought out, to +12. (See "Berichtigungen," etc. p. 226.)

The Borghesian Candelabre (*Ara*) represents the nativity of Claudius like the *Puteale Capitolinum*, but takes into view the preceding vernal equinox, March 22, —7. (See "Bericht." etc. p. 248.)

The nativity of Galba, represented in "Memoires des Sciences," Paris, 1709, p. 110, Pl. i., refers to the year —1, Sept. 22.

The nativity of Cæsarion (Rosellini's Monum. del Egitto, vol. iv., Pl. cccxlix.) puts Cæsarion's birth in —45; consequently, Cæsar's expugnation of Alexandria in the last month of —46, and not in —47.

All these astronomical monuments concur in demonstrating that the usual chronology of the Egyptians, Romans, and Greeks, antedates the concerned events, respectively, by one or two years.

2. *The Solar and Lunar Calendars of the Greeks.*—Many ancient authors, e.g. Theodorus Gaza (Petavius's Uranol. c. 9), Censorinus (De d. n. 18), witness that the Greeks used not only lunar, but also both solar months and tropic years. The latter concerned the civil life of the Greeks, but their festivals were celebrated according to the former; for instance, the Olympian games. The latter, it is well known, were performed from the 11th to the 16th days of the lunar month preceding the summer solstice (Thucyd. v. 49. 50; Scholiast to Pind. Ol. iii. 35). The new moons commencing the lunar months of the Greeks and Romans were the days on which the first crescent after sunset became visible, viz. commonly 24, sometimes 48 hours after the astronomical conjunction of the moon with the sun; hence the full moons were, like the days, 24 to 48 hours after the astronomical full moons. Since, moreover, the lunar year contained only 354 days, the Greeks and Romans had, every two or three years, to add a thirteenth lunar month, a second Poseideon or December, in order to keep the lunar year in harmony with the seasons of the tropic year. The solar year of the Greeks was first discovered by Halma (Chronologie de Ptolemée, p. 40) in an ancient manuscript; and I do not understand how it came to pass that Clinton, Fischer, and other modern chronologers, knew nothing about this very important calendar, as follows. We join the names of the Macedonian months, because the latter commenced with the same days of the Julian year. (Demoth. D. C. Or. G. i. 280.)

Attic.	Macedonian.	Julian.	
Gamelion,	Appellæus,	December	4.
Anthesterion,	Audynæus,	January	3.
Elaphebolion,	Peritius,	February	2.
Munychion,	Dystrus,	March	4.
Scirophorion,	Artemisius,	May	3.
Hecatombæon,	Dæsius,	June	2.
Metageitnion,	Panemus,	July	2.
Boëdromion,	Lous,	August	1.
Pyanepsion,	Gorpiæus,	August	31.
Mæmacterion,	Hyperberetæus,	September	30.
Poseideon,	Dius,	October	30.
Five or six additional days,		November	29.

As the ancient Romans commenced the months with the appearance of the crescent subsequent to the astronomical new moon, and, accordingly, each day with sunset, 6 hours prior to the Julian day, beginning at midnight; it is natural that our Menologium commences, e.g. Pyanepsion with Sept. 1, and so in all other cases. In order to harmonize the Greek days, likewise beginning with sunset, with our civil days, we have the Julian dates in the preceding table diminished by a unit. The Spartan months began, as we learn from Thucydides (v. 19, iv. 118; Plut. Nic. 28), two days later.

The leap-years of the Greeks were, as Censorin (De d. n. 18) reports, the same in which the Olympian games were celebrated. In such years, the first three months of the Greek year commenced with the following day of the Julian months, as specified in the Table.

By means of this Solar Calendar of the Greeks, e.g. the following events of Greek history are incontrovertibly fixed; first, the years in which Archon Apseudes ruled. Diodor (xii. 36) reports that, during the archonship of Apseudes, Meton commenced his Lunar year and Lunar cyclus of 19 years with the 13th day of Scirophorion, that is, as our Table shows, with the 15th day of May, Julian style. In —428 the new moon happened on May 13th about 7 o'clock after noon, and consequently the crescent became visible on May 15th after sunset. Since then no new moon coincides twice with May 13th during a period of 19 years; the archonship of Apseudes, extending from July in —429 to July in —428, is mathematically fixed. Petavius and his adherents put Apseudes earlier by one year, but erroneously. By the way, this fact demonstrates that Ideler's exposition of Meton's Lunar Calendar is wrong, because he referred Apseudes to a wrong year. This result, moreover, is confirmed by Ptolemy (Alm. iii. 2, p. 162, 163. H.), who reports that, during the same year of Apseudes, Meton and Euctemon found the summer solstice coinciding with sunrise on June 27 (Phamenoth 21st), because the summer solstice happened, according to our Solar Tables, on the same day at 5 o'clock a.m , during the year —428

These two facts involve a result of great importance. The Peloponnesian war commenced, on the part of the Athenians, with their naval expedition against Sparta, which took place in the early spring during the archonship of Apseudes' predecessor, viz. Pythodor I. (Thuc. ii. 2; Diod. xii. 36; Argum. Medeæ; Schol. Av. 998); accordingly, in January of the year —429, and not, as Petavius imagined, in —430. This is confirmed by the nearly total eclipse in —429, Jan. 26, 21h. (Thuc. ii. 28), observed during the embarkation of the Athenians. The Peloponnesian war it is well known, came to an end with the destruction of the Piræus on March 19th (Munichion 16th), as Plutarch's Lys., Xenophon's Hell. ii. 4, 43, etc., witness, in the course of the archonship of Pythodor II.; and this year is confirmed by the solar eclipse in —401, Jan. 18. Accordingly, Pythodor II. must have ruled from July in —402 to July in —401 (Xen. Hell. ii. 3, 1). Now, from the spring in —429 (Pythodor I.) to the spring in —401 (Pythodor II.) really 28 years elapsed, and consequently the Peloponnesian war lasted 28, and not 27 years, as Petavius "*post ingentem laborem*" brought out: to-wit, the latter knew not that the first chapters of Xenophon's Hellenica, containing the history of an entire year, are lost; that Thucydides (v. 26) expressly testifies to the Peloponnesian war commencing with the expedition against Sparta one year after the destruction of Potidæa by the Spartans, and finishing with the destruction of the Piræus, lasted "four times seven years"; that, inclusive Endius (Thuc. viii. 9), Xenophon (ii. 3, 10) specifies, for the same period, 29 annual Ephori of the Spartans; that the Parian Marble, as every historian knows, counts, for the same time, 28, and not 27 archons. I do not understand how Petavius arrived at the conclusion that "the good Xenophon erred" (*bonus Xenophon erravit*).

The same Calendar, moreover, serves to fix many other epochs of Greek history, as follows: Herodotus (vi. 106, 120) reports that the battle at Marathon was fought on the 6th day of Boëdromion, that is, according to our Calendar, on August 6th, namely, "three days after the full of the moon." During a period of 19 years, it occurs only once that 3 days prior to August 6th a full moon takes place, which was the case in —488. The astronomical full moon happened on July 31st, the civil of the Greeks (p. 408) two days later, because the crescent had become visible first two days after the real conjunction of the moon with the sun. Subsequent to this full moon the Spartans marched out, namely, on August 3rd, and they arrived on the battle-field "after three days"; consequently on August 6th, but "after the end of the battle," as Herodotus narrates. Accordingly, the battle at Marathon belongs to —488 (Arch. Phœnippus), and not, as Petavius imagined, to —489 Thus, in the same time, the year is fixed in which Xerxes occupied Attica, viz. in —478, for Thucydides (i. 18) bears witness that the latter occurred "ten years after the Marathonian battle" in —488, to which it is likewise referred by the planetary configuration on the pedestal of the Olympian Zeus (p. 406).

Further, the eye-witness Aristophanes (Nubes, 580) testifies that in the tenth year of the Peloponnesian war (—420) both a very small eclipse of the sun and a total of the moon were, in the early spring, a short time prior to Kleon's orderly election as strategus, perceived in Athens; and the Scholiast in Scaliger's Synagoge (Euseb. 1658, p. 431) reports that the former took place on the 16th day of Anthesterion, i.e. according to our Calendar (p. 408), on January 18th. Since it happens very seldom to see two eclipses within 15 days, and because only after 19 years a solar eclipse coincides again with January 18th, which was the case in —420, two hours after noon, the 10th year of the Peloponnesian war and the archonship of Aristophanes are fixed with mathematical certainty.

Furthermore, Thucydides (viii. 20) specifies 21 days from the eclipse of Nicias to the capture of the Attic army in Sicily (Thuc. vii. 50; Clinton F. H. ii. 70), and the latter event happened on the 27th day of the Spartan month Carneius, the 29th of Metageitnion (Plut. Nic. 28; Thuc. iv. 118); accordingly on the 30th day of July, Julian style (p. 408). Therefore Nicias's eclipse, perceived 21 days prior to Metageitnion 29th, belongs to the 8th day of Metageitnion, i.e the 8th of July; and on this very day, 7h. 45m. after noon, an eclipse of the moon took place in —410, and not, as Petavius imagined, in —411. Since this was the 20th year of the Peloponnesian war (Thuc. vii. 18), the latter must have commenced in —429. Petavius, again, has antedated by one year all events reported by Thucydides.

The ten Attic Pritany ruled each 36 days, and their office commenced on the 1st day of the lunar Hecatombaeon (Corsini F. A. ii. 26). An inscription, referring to Archon Glaucippus (Boeckh's Corp. Insc. i., Pt. ii., Nos. 107 & 108), parallels the dates of the lunar months of this year with the dates of the solar ones as follows: The 1st day of the 2nd Pritany coincided with the 8th day of the solar Metageitnion (July 10), the 13th day with the 21st, the 17th with the 25th, the 22nd with the 30th of Metageitnion, the 23rd with the 1st of Boëdromion, the 24th with the 2nd, the 36th with the 14th day of the solar Boëdromion. According to our solar calendar the 8th day of Metageitnion commenced on July 10th, and consequently the 1st Pritany must have begun 36 days prior to the 8th day of the solar Metageitnion, that is to say, on July 3rd, being the 1st day of the lunar Hecatombæon. Indeed, the first astronomical new moon during the archonship of Glaucippus took place in —408, on the 1st day of June about noon, and the crescent appeared on June 3rd after sunset, with which the 1st Pritany commenced to officiate. This inscription, therefore, puts beyond any question that Archon Glaucippus ruled in —408, and not, as Petavius imagined, in —409.

A similar inscription (Corpus Insc., Pt. ii., No. 11, p. 50) reads as follows: *'Επὶ Νικοδόρου ἄρχοντος ἐπὶ τῆς Κεκροπίδος ἕκτης Πρυτανείας, Γαμελιῶνος ἑνδεκάτῃ, ἕκτῃ καὶ εἰκόστῃ τῆς πρυτανείας κ. τ. λ.* Hence, during the archonship of Nicodorus the 26th day of the 6th Pritany coincided with the 11th day of Gamelion (Dec. 14). Accordingly the 1st Pritany must have ruled earlier by 206 days ($5 \times 36 + 26 = 206$), i.e. since May 23rd, and this day must have been the first day of the lunar Hecatombaeon. This was the case only in —312, for the astronomical conjunction took place on May 20th about midnight, and the crescent appeared on May 22d after sunset, with which the lunar month Hecatombaeon began. Consequently Archon Nicodorus must have ruled since July in —312, and not, as Petavius fancied, two years earlier.

Alexander the Great was, notoriously, born on the 6th day of Hecatombaeon (Dæsius), i.e. June 7th, Archon Elpines, Ol. 106, 1, namely, "during the Olympian games" (Plutarch Al. 3; Cic. De div. i. 23). The latter being celebrated from the 11th to the 16th day of the lunar Hecatombæon preceding the day of the summer solstice (Thuc. v. 49. 50; Schol. Pind. Ol. iii. 35), the respective new moon must have preceded Alexander's birth-day (Jun. 7) by about 11 days, and this was the case only in —353; for during this year the astronomical new moon happened on May 25 about 11 p.m., and the crescent became visible on May 27th, and hence the Olympian games were in —353 celebrated from June 5th to 9th. Thus Alexander was born on the 2d day of the Olympian games in —353, and not in —355, as Petavius brought out. All these astronomical certainties concur in demonstrating that Petavius has antedated all events of Greek history, all Olympian years, and all archons, from —489 to —407, by one, thence by two years, as the archonship of Nicodorus and Alexander's birth evidence.

The latter is confirmed by the lunar eclipse preceding the battle at Arbela; for Arrianus (Al. iii. 7, 6) and Cicero (De div. i. 53) report that this small eclipse, 11 days prior to the battle at Arbela, happened in Pyanepsion (Aug. 31 to Sept. 30) "a short time before sunrise, whilst the sun stood in Leo." About that time only one lunar eclipse occurred in August and a short time previous to sunrise, viz. that in —328, Aug. 31, 5 hrs. after midnight in Arbela (☊ 12° west of the sun). Consequently the battle at Arbela belongs to the year —328, and not, as Petavius imagined, to —330.

3. *The Solar Calendars of the Hebrews.*—Formerly it was universally believed that the Hebrews used only lunar months, but the contrary has come to light. See the author's "Chronologia Sacra," etc. p. 26–68, and "Zeitschrift der Deut. Morg. Ges." 1848, p. 344, and "Berichtigungen," etc. p. 14. There the matter having been discussed *in extenso*, we only briefly mention the principal proofs. Josephus parallels very often the Greek solar months with the Hebrew months. The Hebrew months were solar ones in Syria, Arabia, Ascalon, Gaza, etc. Greek and Roman authors, especially Josephus, the Books of the Maccabees, and the New Testament, refer Saturdays to certain days of Hebrew months, which would have been impossible provided the latter were lunar months. Even the Talmud bears witness that the Hebrews, prior to the destruction of Jerusalem by Titus, used only both solar months and a tropic year. Moreover, the Old and New Testaments—especially Josephus, Philo, and Hebrew inscriptions—demonstrate that the Hebrews celebrated their feasts according to their ecclesiastic year, commencing 17 days prior to their civil year (Minjan Shtaroth). The commencement of the ecclesiastic year is, apart from other arguments, fixed by

Eusebius (Hist. Ecc. iii. 4), Chrysostomus (L. iv. De sacerdot. i. 7), and others, who report that Dionysius Areopagita, while travelling in Ethiopia, perceived, A.D. 33, an eclipse of the sun on the 14th day of Nisan, which eclipse, by the way, was invisible in Palestine, and it differed from the obscuration of the sun during the crucifixion of Christ. Further, the commencement of the civil year is fixed by Josephus, who reports that the civil months began in the midst of the ecclesiastic months (*κατὰ σελήνην*), for *σελήνη* signifies very often the full of the moon, especially the 17th day after the astronomical new moon, and, in general, the middle day of all months of the tropic year. This is confirmed by the *σάββατον δευτερόπρωτον*, that is to say, the *second first day* of the year, the newyears day of the civil year (Ev. Luke, vi. 1), which day is still celebrated among the Jews like the Sabbath at the beginning of the ecclesiastic year. Hence the months of the Hebrews, since the Babylonian captivity, commenced on the days of the Julian Calendar, as follows. It is, however, to be remembered that the Hebrews commenced the day six hours prior to the beginning of our days.

Ecclesiastic Year.	Civil Year.	Julian Year.
1. Nisan		March 6.
	Nisan	March 22.
2. Ijar		April 5.
	Ijar	April 21.
3. Sivan		May 5.
	Sivan	May 21.
4. Thammuz		June 4.
	Thammuz	June 20.
5. Ab		July 4.
	Ab	July 20.
6. Elul		August 3.
	Elul	August 19.
7. Thishri		September 2.
	Thishri	September 18.
8. Marcheshvan		October 2.
	Marcheshvan	October 18.
9. Kislev		November 1.
	Kislev	November 17.
10. Tebeth		December 1.
	Tebeth	December 17.
11. Shebat		December 31.
	Shebat	January 16.
12. Adar		January 29.
	Adar	February 14.
Intercalary days		March 1.
	Intercalary days	March 17.

By means of these Calendars many epochs of the Old and New Testaments, of the Books of the Maccabees, and even of Roman history, are determined. For instance, the Talmud (Tract Thaan. fol. 29, 1) bears witness that Titus "destroyed the temple on the 9th day of Ab, on a Saturday"; consequently on July 28th, "just after a new class of the priests had entered the temple," that is, on July 27th at sunset. This day being only A.D. 71 a Saturday, it is evident that Jerusalem was taken in 71, and not in 70 A.D.; accordingly, that Vespasian reigned one year later than Petavius brought out: for Jerusalem was, notoriously, destroyed in the 2d year of Vespasian. Further, Josephus (B. J. v. 9, 4; Ant. xiv. 4, 3 & 16, 4) and other authors report that during Cicero's consulate (Ol. 179, 1) Pompeius captured the temple of Jerusalem on a "Saturday" and "on the 10th day of Tishri," i.e. Sept. 11th, which was only in —61 a Saturday. Consequently Cicero ruled one year later than Petavius imagined; and Ol. 179, 1, commenced in June —61, and not two years earlier. Remember that the Hebrew day commenced on the preceding Julian day with sunset.

4. Transits of Venus, the so-called self-combustions of Phœnix.—The ancient traditions concerning that famous myth will be found collected in the "Zeitschrift d. Deut. Morgenl. Ges.," 1849, p. 63, of which the summary is as follows:—The ancients distinguished two Phœnixes, depicted in many copies of the sacred records of the ancient Egyptians, and accompanied with their respective names, *Bennoh* and *Choli.* The latter name agrees with the Hebrew name *Chol* (Job 29: 18), the Coptic "*Alloe*, Phœnix"; the former (*Bennoh*) is obviously the Coptic *Bene* (Jer. viii. 7), the Latin *Venu*(*s*), ancient *Benu*(*s*), probably related with the Coptic *Wein*—splendere, pulchrum esse. Hence Hermapion translates the image of the Phœnix on the Flaminian Obelisk by *φοινίκιος*, i.e. pulcher, venusius. Accordingly Bennoh signified Venus, but Choli was the planet Mercury, called the "wrong Phœnix"; and their combustions mean their transits before or behind the sun's disk. Venus, it is well known, crosses the solar disk in case the distance of her nodes from the sun amounts to less than 1° 49′; otherwise Venus transits south or north from the borders of the sun. The ancients, however, being destitute of telescopes, it was extremely difficult to determine real transits of Venus before or behind the sun's disk. Their only help was to observe, with the naked eye, the latitude of Venus a short time prior to her conjunction with the sun, and hence it came to pass that the ancients sometimes took close conjunctions of Venus with the sun for transits of the former. For the same reason the ancients determined very different periods of the reappearance of Phœnix. By means of such ancient transits of Venus many epochs of Roman history are incontrovertibly fixed.

First, Pliny (H. N. xxx. 3; x. 2) narrates that the year U.C. 657, i.e. —95, coss. Licinius and Cn. Cornelius, was the 215th year of the Phœnix period. Consequently a transit of Venus must have occurred in —309 (95 + 214 = 309), coss. Bubulcus Brutus III. and Aim. Barbula II. Pliny's authority is Manilius, the notorious Roman astronomer. Indeed, in —309, Nov. 22d, the longitudes of the sun and Venus were 7^s 26°, and that of the ☋ was likewise 7^s 26°. Accordingly Venus traversed in —309 nearly the centre of the sun, but behind it, because the former was in its superior conjunction with the sun. This astronomical fact in U.C. 443 puts beyond the reach of controversy that Rome was founded in —752, and not, as Petavius made out, in —753; that the consuls Licinius and Cornelius as well as Brutus III. and Barbula II. ruled one year later than formerly was believed.

Further, in Tacitus (Ann. vi. 28) we read: "P. Fabio et L. Vitellio coss. post longum sæculorum ambitum Phœnix in Ægyptum venit præbuitque materiam doctissimis indigenarum et Græcorum, multa super eo miraculo disserendi." About that time only one close conjunction of Venus with the sun was possible, viz. A.D. 36, May 31, on which day the ☊ of Venus lay 6° west of the sun. During this conjunction the distance of Venus from the sun amounted to 38′ only. Consequently the said consuls ruled one year later than Petavius stated.

Aurelius Victor (Claud. iv. 12): "hujus (Claudii) anno sexto," says he, "DCCC. urbis, mire celebratus visusque apud Ægyptum phœnix"; and Pliny (H. N. x. 2) reports: "Cn. Valerius phœnicem devolasse in Ægyptum tradidit, Q. Plautio, Sex. Papinio coss. (A.D. 38); Allatus est in urbem, Claudii principis censura, a. u. DCCC.; et in Comitiis propositus, quod actis testatum est." Suidas (v. Φοίνιξ) and Salinus (c. 36) narrate the same. About that time only one close conjunction of Venus with the sun was possible, namely, A.D. 48, May 29; for the ☊ of Venus lay only 5° west of the sun; therefore Venus stood, during the conjunction, 32′ only from the sun's northern borders. In the same year, as Aurelius Vict. (Claud. iv. 12) reports, "in Ægæo mari repente insula ingens emersit nocte, qua defectus lunæ acciderat," namely, A.D. 48, June 14, 6h., which confirms Venus's transit A.D. 48. All these reports put beyond question that the foundation of Rome, as well as the consulates and the reigning of Claudius, are to be postdated by one year.

Since, however, so many hypotheses exist concerning the myth of Phœnix, it will be necessary to add some new proofs clearly evidencing that the *true Phœnix* signified the planet Venus, and that its combustions were transits of Venus. Thus, for instance, Lepsius (Vorbedingungen zur Entstehung einer Chronologie, etc. Berlin, 1848, p. 180) imagined the Phœnix to signify the human soul, purified, during the period of its transmigrations, in animal bodies. Hence he concluded that the Phœnix period, commencing with the Canicular periods, contained exactly 1500 years, sometimes however 1,000 years, sometimes 500, sometimes only 250 years. This chimera, however, is inconsistent with all ancient reports concerning the Phœnix, and it is apparently refuted by all epochs of ancient history to which a reappearance of the Phœnix is linked. The true Phœnix, represented by a crane (Jer. viii. 7) is simply Venus, as its Coptic name *Bene*, and the hieroglyphic term *Bennoh*, and many other circumstances demonstrate. Further, the Egyptian sacred records, e.g. Lepsius's Todtenbuch (xxx. 81) themselves enumerate the planets, according to their apparent celerities, in the following order: [Saturn,] Jupiter, Mars, Venus, Mercury; of which, the latter two are called "Bennoh" and "Choli." The same records (iv. 13) call the same Bennoh "the greatest of the stars," and this clearly denotes Venus. The said hymns, moreover, refer to Bennoh what the Greeks and Romans referred to Venus, viz. all objects of beauty, e.g. "the four gamuts (musical scales) of the Muse of the seven tones." Besides, Strabo (x. 3, p. 474) tells us that "the ancients veiled their physical conceptions by riddles, and added myths to their scientific contemplations." Thus, e.g., the 12 works of Hercules signified the effects of the sun, during the year, in the 12 signs of the Zodiac and the corresponding 12 months. The myth, according to which Hercules, being still in his cradle, killed two serpents, signifies the sun's victory over the two houses (signs) of Saturn (serpents), near the point of the winter solstice. The myth, according to which Typhon (the water) killed Osiris (the main-land), and Pontus (the sea) overcame Demarus (*adam-arez*, the earth), refer to the deluge. The myth allegorizing Jupiter (the sun) to burn Semele (the vineyard), whereupon he saved and perfected Bacchus in his thighs, simply contains a mythical description of the origin of the vine, and the like. Therefore the myth of the Phœnix burning itself in Heliopolis (the sun), must involve a similar contemplation of a natural phenomenon, and not the transmigrations of the human souls. The ancients expressly say that only one specimen of the fowl (planet) Phœnix (Bennoh) is in existence, and "nobody had seen it eating."

5. *The Seasons of the Greeks*, discussed *in extenso* in the author's "Berichtigungen," pp. 67 & 262, are very important in correcting the common Greek history, and establishing a correct chronology of the Greek eclipses, especially those mentioned by Thucydides and Xenophon. Plutarch (Symp. iii. 7, 1; viii. 10) certifies that the Greek year was divided into "two equal parts," viz. θέρος and χειμών, of which the former commenced subsequent to the winter solstice, or, according to the aforementioned Calendar, with the month Anthesterion (Jan. 3d). Hence the Greek summer contained the six months January, February, March, April, May and June, and the following months belonged to the Greek winter. Moreover, that θέρος was divided into the spring (ἔαρ) and the shorter θέρος,—the former comprising January, February, March, the latter April, May, June. Χειμών likewise was subdivided into ὀπώρα (July, August, September), and the shorter χειμών (October, November, December). Plutarch's reports are confirmed by Thucydides and Xenophon; for Thucydides (iv. 52, 117; v. 20; vii. 19, etc.) expressly says that the spring (ἔαρ) belonged to the semi-annual θέρος. This division of the Greek year, moreover, is easily proved; for in Homer's days and later times Sirius rose heliacally, i.e. prior to sunrise, on July 25th, and Orion rose on July 10th. Both risings Homer (Il. xxii. 17), Aristotle (Probl. xxvi. 4), and Theophrast (De vent. p. 414)

refer to ὀπώρα, and hence Homer (Il. v. 3) calls Sirius expressly ἀστηρ ὀπωρινός, the star of the autumn. Consequently the season of autumn (ὀπώρα) must have contained the months July, August, and September. Accordingly all other Greek seasons must have begun with the aforesaid months. Even the Greek name of our January, viz. Ἀνθεστεριών, the month of flowers, demonstrates that the Greek spring commenced three months prior to our spring. Further, Thucydides (v. 49. 50) testifies that χειμών began soon after the Olympian games, which were always held in θέρος, and a short time prior to the summer solstice. Thucydides, moreover, refers the rising of Arcturus after sunset to the middle of θέρος, and in that time Arcturus rose about the end of March, and hence θέρος must have contained the months from January to July. Again, Plutarch (Symp. iii. 7, 1; viii. 10, 3) bears witness that Anthesterion, commencing with January 3d, was "the first month after χειμών." Still further, Harpocratio (v. Μαιμακτ.) tells us that χειμών commenced with the month Maemacterion, of which the first day, as we have seen (p. 408), coincided with September 30th, Julian style. Hence this quarter of the Greek year must have been followed by ἔαρ, beginning with January 3d. Moreover, Xenophon (Hell. vii. 5, 14) testifies that the Mantinean battle "on the 12th day of Scirophorion" (May 15th) happened in θέρος, and prior to χειμών. Add to this that Thucydides (iv. 52) certifies the small solar eclipse in the 8th year of the Peloponnesian war to have taken place "soon after the beginning of θέρος," and about that time only one small eclipse of the sun was possible in the early θέρος, viz. that in —420, Jan. 28th, as we shall see hereafter. Finally, Plutarch (Æm. Paul. c. 16) narrates that the battle near Pydna was preceded by a lunar eclipse in the last days of the summer (θέρος φθίνοντος), and about that time only one eclipse of the moon coincided with the end of θέρος, viz. that in —166, June 10th; conseqnently χειμών must have commenced with Metageitnion (July 2d), whilst θέρος began on January 3d.

These arguments will suffice to convince every intelligent reader that the Greek seasons θέρος and χειμών, of six months each, commenced respectively on January 3d and July 2d. By means of these Greek seasons many epochs of Greek and Roman history are fixed. For instance, the consuls Æm. Paullus II. and Lic. Crassus must have ruled in —166, and not, as Petavius imagined, in —167; because the battle near Pydna being fought during the same consulate, happened, as we have seen, in —166. This date confirms the aforesaid result, that, down to Julius Cæsar, all events of Roman history, as determined by Petavius, have to move down one year.—Again, the eye-witness Thucydides (ii. 28) testifies that the solar eclipse observed in Athens in the course of the first year of the Peloponnesian war, Arch. Pythador I., happened in *θέρος* soon after noon. About that time only one eclipse agrees with Thucydides, viz. that in —429, Jan. 22, 22h. P. T. Petavius, on the contrary, recurred to the eclipse in —430, Aug. 3, 5h. 30m. after noon; but, alas! this eclipse belongs to *χειμών*, and not to *θέρος*.—The eye-witness Aristophanes (Nub. 580) testifies that, in the course of the 10th year of the Peloponnesian war, on occasion of Cleon's election as strategus, during the early spring, a partial eclipse of the sun and a total one of the moon happened; of which, the former, according to the scholiast in Scaliger's Synage, took place on Jan. 18th (Anthesterion 16th). These two facts demonstrate that the events of the Peloponnesian war narrated by Thucydides happened one year later than Petavius made out.—Xenophon (Hell. ii. 3, 4) bears witness, that, during *θέρος* of the last year of the Peloponnesian war, an eclipse of the sun was perceived which Petavius referred to —403, Sept. 2d. But, alas! this eclipse of Sept. 2d belonged to *χειμών*, and not to *θέρος*. The eclipse under consideration happened two years later, viz. in —401, Jan. 18, 9 o'clock a.m. Thus the seasons of the Greeks, apart from all other arguments, evince that the Peloponnesian war commenced in —429, and ended in —401; that the same lasted 28 full years, as Thucydides and Xenophon testify; that all events narrated by Thucydides come down by one year; that the first part of Xenophon's Hellenica is, at present, missing; and that all events narrated by Xenophon, all Greek archons and Olympian games subsequent to the year —407, are to be postdated by two years.

6. *Greek and Roman Inscriptions and Coins.*—Subsequent to Petavius's "Doctrina Temporum," 1627, many Greek and Ro-

man inscriptions and coins have come to light which, not being subjected to learned or unlearned alterations, are decisive in fixing the years of Greek and Roman history. We specify the following:

The Fasti Capitolini. This catalogue of Roman consuls, going down to Tiberius, originated from the Annales Maximi, the work of the Capitolian annalists, who had, from the beginning of Roman history down to Tiberius, recorded, day by day, all remarkable events of Roman history. This precious monument refutes the usual Roman history and chronology in two principal points. First, it demonstrates that Petavius shortened the period of the Roman kings by one year, and authoritatively intruded a consular year in —331, u.c. 421. The consuls L. Papirius Cursor with C. Poetil. Libo II., whom Solinus (c. 40) mentions again five years later, have never ruled, as the Fasti Capitolini, Livy, Diodor, and Cassiador, bear witness. (See the author's "Berichtigungen," p. 56). In the second place, Petavius shortened the ruling time of Julius Cæsar by one year; the latter must have ruled six years, and not five years. For the Fasti Capitolini bear witness together with Livy that Cæsar was dictator in the course of six consecutive consular years; and even Josephus (Ant. xviii. 2, 2), Cicero's letters of this time, Dio Cassius, and other authors, count six years trom Cæsar's crossing the Rubicon to his death. (See the author's "Berichtigungen," p. 52.)

The Ancyran Marble, written by Augustus himself, evidences that Cæsar died in —41, and not in —43; and that Augustus died, not A.D. 14, but A.D. 16. For the Ara Albani, the nativity of Augustus (p. 407), furnishes evidence that Augustus was born in —61, coss. Cicero and Antonius, whilst the former delivered the fourth Catilinaria, and "Capricornus rose heliacally" (Sueton. Aug. 94), consequently in February of —61, especially, according to the erratical lunar calendar of the Romans, ix. Kal. Oct. Moreover, Cicero's consulate is fixed, as we have seen (p. 415), by the capture of the Hebrew temple on Sept. 11 in —61, a "Saturday." Now, according to the Ancyran Marble, Augustus himself tells us that, subsequent to Cæsar's assassination, he came from Apollonia to Rome whilst he was 19 years and some months old (annos undeviginti natus). Accordingly Cæsar must have died in —41, and not in —43 (61 — 20 = —41), and this date is confirmed by many other ancient reports. (See the author's "Berichtigungen," p. 52.) Since Augustus, moreover, being born in —61, died aged 77 years (Joseph. Ant. xviii. 2, 2), he must have died A.D. 16, and not, as Petavius taught, two years earlier (—61 + A.D. 16 = 77). Further, the following inscriptions evidence that Claudius reigned only 12 years, instead of 13 years, as Ptolemy's Canon and Petavius state. In Gruter's Thesaurus (p. 238, no. 39) and Wolf's Suetonius (no. 2 & 3) the following inscription will be found: "T. Claudius, Drusi filius, Cæsar Augustus Germanicus, Pont. Max., Trib. pot. V., Imp. X., P. p., Cos. design. IIII.," etc. The first Tribunitia potestas of Claudius commenced on that day on which his predecessor Caligula died, i.e. A.D. 43, Jan. 24th, and consequently his 5th Trib. pot. began A.D. 47, Jan. 24th, in which year Claudius became Cons. designatus IV. Since all consuls, as is well known, were designated six months previous to the beginning of their consulates, Claudius must have been consul quartum A.D. 48. Petavius, on the contrary, refers the fourth consulate of Claudius to A.D. 49, and gives the consules suffecti C. Valerius Asiaticus, associated with M. Junius Silanus, the whole of the year 48. The same is proved by another inscription (Gruter's Thesaur. p. 238, no. 39; Wolf's Sueton. no. 3), which reads as follows: "Ti. Claudius, Aug. German., pont. max., Trib. pot. V., imp. XI., p. p., cos. IIII.," etc. For this inscription refers to the days from January 1st to January 24th, A.D. 48, in which Claudius already officiated as consul, whilst his 6th Tribunitia potestas commenced later, on January 24th, A.D. 48. Add to these authorities the decree of Claudius in Josephus's Ant. xx. 1, 2: *Κλαύδιος Καῖσαρ Γερμανικὸς, δημαρχικῆς ἐξουσίας τὸ πέμτον, ὕπατος ἀποδεδειγμενος τὸ τέτραρτον—ἐγράφη πρὸ τεσσάρων Καλανδῶν Ἰουλίου ἐπὶ ὑπάτων Ρούφου καὶ Πομπηΐου Σιλάνου.* For this decree concerns June 27, A.D. 47, during which Claudius, being invested with his fifth Tribunitia potestas, became Cos. des. IV. Accordingly Claudius was Cos. IV. A.D. 48, and Rufus with Silanus were, A.D. 47, coss. suffecti instead of Vinicius II. with Statilius Taurus Corvinus. The conclusion therefore is that the latter being coss. suffecti, the following consuls ruled only one year later than Petavius brought out. That, moreover, Claudius reigned only 12, and not 13 years, is confirmed by numismatics and epigraphics, because no coin and no inscription referring to Claudius's supposed 13th year are in existence.

Another class of inscriptions evidences that Vespasian likewise reigned one year less than, down to this day, was generally believed. Gruter's Thesaurus (p 270, 2; 243) and Eckhel's D. N. (vi. 343) contain the following inscriptions: "Imp. Cæsari Vespasiano Aug., Pont. Max., Tribun. pot. VIII., imp. XVII., Cos. VIII., des. IX., censori.," etc.; and, "Pontifici Max. .... Trib. pot. .... Imp. XVII., Cos. VIII., des. VIIII., conservatori," etc. Of the latter inscription Gruter obtained two copies, made in different times and by different persons. These inscriptions apparently demonstrate that Vespasian, like Claudius, administrated two consecutive consulates. Petavius, on the contrary, puts between these 8th and 9th consulates of Vespasian a whole year, the consules suffecti Commodus Verus and Novius Prisius. Accordingly, Vespasian must have reigned one year less than Petavius imagined; he must have governed nine years, instead of ten years. This is confirmed by Eutropius, and both by numismatics and epigraphics. There exists not one coin, and not one inscription, vindicating the supposed 10th year of Vespasian.

The coins of Julius Cæsar concerning the introduction of his solar calendar, which happened 2 months and 15 days prior to Cæsar's assassination, represent a crescent, because the first Julian year had commenced with the appearance of the crescent. The same crescent is, moreover, testified by Macrobius (Sat. i. 14). It was only in —41 that a crescent became visible in Rome on the first day of the first Julian year. Since, then, during a period of 19 years, only one new moon coincides with January 1st, and only in —41 a crescent was to be seen on the beginning of January, it is clear enough that the Julian Calendar was not introduced, and that Cæsar died, not in —43, but —41. According to Petavius, the first day of the Julian Calendar would have commenced twenty-two days prior to the new moon, which stands in direct opposition to both Macrobius and the coins.

The *calendrical inscriptions* according to which the archons Glaucippus and Nicodorus respectively ruled later by one or two years, have been spoken of in the premises (p. 411–12).

The *Parian Marble*, which specifies the most important events of

Greek history, and refers them to the epoch —261, demonstrates that the archons down to the year —407 ruled one year, the following two years, later than Petavius stated. Since, moreover, Persian history is connected with the Greek, the same monument evidences that all events of Persian, Median, Assyrian, and Babylonian histories, connected with certain archons, likewise happened respectively one or two years later than Ptolemy's Historical Canon says. Accordingly, the eclipses in the Almagest, being linked to certain years of the same kings and archons, must necessarily be referred to other dates than those cited by Ptolemy.

7. *A number of Solar and Lunar Eclipses unknown to Petavius.*—Since the year 1627, in which Petavius's Doctrina Temporum appeared, several ecliptic new and full moons, unknown to Petavius, have come to light, which are well qualified to correct the present history and chronology of the Greeks, Romans, Egyptians, and other nations of old. We specify the following only:

The total eclipse of the sun in Rome, u.c. 450, —400, July 1st, 17h. 45m., mentioned by Cicero (De rep. i. 16), which work was discovered about sixty years ago. The same is the case with the Armenian Eusebius, who mentions several eclipses formerly unknown.

The solar eclipse preceding the conquest of Nineveh by Cyrus, in —532, Jan. 27th, 15h. 45m. P. T., described by Xenophon (Anab. iii. 4, 7).

The total eclipse of the moon, and the very small one of the sun, at Athens, in —420, Jan. 18, 2h., and Feb. 2, 6h., of which the former happened on the 16th day of Anthesterion (Jan. 18). Aristoph. Nubes 581, and the Scholiast in Scaliger's Synagoge.

The total eclipse of the sun at Thebes, Bœotia, in —465, Dec. 20. (Pindar in Dionys. Hel. p. 167, 18 Sylb.)

The total eclipse of the sun in Pekin on the day of the autumnal equinox in —2192, of which the date is fixed by a planetary configuration. (Gaubil's Histoire de l'Astron. Chinoise, Paris, 1732, vol. 2, p. 140.)

The partial eclipse of the moon, observed during the year of the first Canicular period, at Tanis, Egypt, in —2780, May 22d, 13h. (Suidas, v. *Απις*; Plutarch Symp. Q. 1. p. 718; De Iside, p. 368; Strabo xvii. 555; Ammian xxii. p. 245, etc.)

With these new chronological resources are to be numbered

8. Pingrés Computations of all Ancient Eclipses, visible in the Old World from 1000 *B.C. to A.D.* 2000.—These computations, published in "Histoire de l'Académie R. des inscriptions et belles-lettres," T. xlii. p. 78, Paris, 1786, and "L'art de vérifier les dates," vol. i. p. 243, Paris, 1818, and p. 147, Par. 1819, rely on Halley's Tables referring to Paris time. In all instances Pingré determined the magnitudes of the lunar eclipses, and, so far as the solar eclipses are concerned, he described the curves of the total shadow of the moon, viz. at the beginning, at noon, and at the end of each eclipse. These computations, relying on Halley's Tables, it is true, have been declared to be inexact (Bode's Astron. Jahrbuch, 1820, p. 202), and yet they are very useful in determining the dates of ancient eclipses. It is true, moreover, that Halley's and La Hire's Tables, if applied to modern eclipses, prove incorrect, but they quite sufficiently agree with all the old eclipses; for the *terminus a quo* of Halley's Tables were the eclipses in the Almagest, and the same are the bases of Burckhardt's, Damoiseau's, and Hansen's Tables. Pingré's computations, at any rate, furnish, with few exceptions, the dates of all ancient ecliptic full and new moons mentioned by Roman, Greek, and other authors.

These astronomical and historical auxiliaries, unknown to Petavius and his followers, viz. planetary configurations, transits of Venus, calendrical inscriptions, observations of the solstices, eclipses referred to certain days of the tropic year, the solar calendars of the Greeks and Hebrews, new Greek and Roman inscriptions, Julius Cæsar's coins, the seasons of the Greeks,—these new resources of ancient chronology, I say, will suffice to re-estabish the true ancient history and the chronology of the eclipses mentioned in the classic and other historical works. For all ancient eclipses are linked to certain years of the Æra urbis conditæ, or to that of the Olympiads, or to certain years of the Babylonian and Roman kings and emperors, or to certain consuls and archons; and by the very same historical and mathematical certainties numberless events of Roman, Greek, and Babylonian histories are incontrovertibly fixed.

Before, however, entering into a closer examination of the true dates of ancient eclipses, it will be necessary to prove the incorrectness of the present theory of the moon's motions, and to determine *approximately* the corrections of the principal statements of the usual Lunar Tables. We confine ourselves to the secular accelerations of the moon, her Nodes and Apsides, because the solution of the whole of the problem belongs to professed astronomers only. In all the following computations I shall apply Lalande's Tables owing to their handiness, and because they sufficiently agree with Damoiseau's and Hansen's Tables so far as the old Babylonian, Greek, and Roman eclipses are concerned.

Approximate Corrections of the present Theory of the Principal Motions of the Moon.

It is a well known fact that the total eclipse of the sun observed in Germany A.D. 1851, happened later than Damoiseau's Tables, inclusive of Airy's correction, had predetermined, and that in that year the longitude of the moon's Node was somewhat shorter. The late Prof. Moebius, Director of the Leipzig Observatory, and myself observed this eclipse, and we found that both the beginning and ending of the obscuration happened 57 seconds later, and that the obscuration of the sun amounted in Leipzig to less than 10 inches, as was predicted by means of the said Tables. Prof. D'Arrest, however, who observed the same eclipse in Danzig, found that the obscuration of the sun commenced and finished only 56 seconds later than Damoiseau's Tables, corrected by Airy, had predetermined. Now, granting the mean motions of the moon, her Nodes and Apsides, to depend, as all astronomers maintain, on the law of gravitation, it follows that the difference of the computation and the observation of the eclipse A.D. 1851 is to be put on account of the secular accelerations of the moon's motions, erroneously derived from the Babylonian eclipses in the Almagest. The following compuations will, in the first place, demonstrate that on occasion of all ancient ecliptic new moons the longitude of the nodes must have been somewhat shorter than Damoiseau's Tables state. The total eclipse of the sun "u.c. 350, Nonis Junîs," i.e. —400, July 1st, 17h. 57m. mean Roman time, belongs to the most reliably ascertained eclipses of the ancients, because it is corroborated by the Annales Maximi, Ennius, and Cicero (R. P. i. 16). Pingré (Art de vér. les dates) refers the conjunction to July 1st, 17h. 45m. mean Paris time, and this result agrees, with trifling exceptions, with the following computation, the work of Prof. D'Arrest, who applied Damoiseau's Tables and Airy's corrections of the latter. The letter *t* signifies time.

Long.	☉	95° 0′ 30″ 7	+ 2′ 23″ 8. t.
Long.	☽	94 53 49 5	+ 33 16 8. t.
Lat.	☽	0 29 31 0	+ 3 2 1. t.
Sunrise in Rome		16h. 27m. (local time).	
Conjunction in long.		17h. 57m. 59s. Paris time.	
		18 38 34 Roman time.	
☉ and ☾		95° 1′ 1″ 8	
Lat.	☾	—0 28 51 6	
Paral.	☾	0 57 21 5	
Rad.	☾	15 38 1	
Paral.	☉	8 4	
Rad.	☉	15 48 2	

According to this computation the southern obscuration of the sun amounted to 2′ 34″ 2, i.e. nearly to 1 inch only. This result clearly demonstrates that the secular acceleration of the moon's Nodes, adopted in our Tables, is incorrect, and that the computed longitude of the ☊ must be diminished. Supposing the ☊ was 7° west of the point to which Damoiseau refers it, then the following figures result, as the computation of Mr. Heym, then Adjunct of the Leipzig Observatory, shows. The applied Tables were those of Carlini and Damoiseau.

New moon	17h. 57m. 2s. mean Paris time.
Long. of ☉ and ☽	3^s 4° 55′ 4
Motus hor. ☉ in long.	2 40
Motus hor. ☽ in long.	33 38
Lat. ☽	+32 8
Motus hor. ☽ in lat.	3 03
Long of ☊	3 6 029 (1° 46′ E.)
Long. ☊ —7°	2 28 520 (6° W.)
Paral. ☽	57′ 62
Rad. ☽	15 72
Rad. ☉	15 80
Siderial time	6h. 25m. 51s.
Obliquity of the ecliptic	23° 46′ 5

According to these statements the obscuration of the sun commenced at 15h. 43m., and ended at 17h. 29m.; its middle was 16h. 36m.; whilst 10 inches of the sun's northern limb were covered in Rome. Hence it is evident that the western distance of the ☊ from the sun must have been about 5° only. According to Lalande's Tables the longitude of the sun was likewise $3^s\ 3°\ 58'$, and that of the ☊ $3^s\ 5°\ 5'$.

The total eclipse of the moon, and the partial one of the sun, in the 10th year of the Peloponnesian war, i.e. –420, of which the latter was seen on January 18th, both being observed by the eye-witness Aristophanes (Nub. 581), belong to the most reliably ascertained eclipses of ancient history; and yet the ecliptic new moon in –420, Jan. 18, 2h., was, according to our Tables, invisible on our globe, because the ☋ lay 17° west of the sun. Even Pingré states that on the said day no solar eclipse was possible in the old world. Consequently the longitude of the ☋ must have been shorter by about 5°, which agrees with the aforesaid eclipse.

Further, the eye-witness Thucydides testifies that the nearly total eclipse of the sun in –429, January 26th, 22d, happened, in Athens, after noon (*μετὰ μεσημβρίαν*); consequently the longitude of the moon must have been shorter, and the conjunction must have been later by about three hours.

Herodotus reports that the total eclipse of the sun in Smyrna (Sardes), noticed by the whole Persian army, in –478, Feb. 27, 15h. 30m., coincided with sunrise in Sardes. According to our Tables, however, this eclipse was over prior to sunrise in Sardes; consequently the longitude of the moon must have been shorter, and, again, the conjunction must have been retarded by about 3 hours and 20 minutes.

The whole Roman history bears witness that Rome was founded on the day of the Roman parilia (the vernal equinox), and that during the building of the city an eclipse was seen within the third hour after sunrise. Tarutius narrates that the same eclipse was observed at Teos in Ionia; our Tables, on the contrary, state that this eclipse was invisible in Rome, even in Asia Minor. In order to obtain a corresponding eclipse, the longitude of the moon in –752, May 25, 16h. must be shortened, and the conjunction postponed by about 3 hours 50 minutes.

Finally, many eclipses of the sun, reported by ancient authors to have been total, were, according to the present theory of the moon's motions, annular ones. On occasion of the total eclipse in –400, as we have seen (p. 424), the radius of the sun was greater than that of the moon by 10 seconds.

Add to these the following eclipses: –360, May 12, 3h. 15m., at Thebes, Bœotia; –306, June 13th, 22h. 45m., a little east of Syracuse; –201, Oct. 18, 23h. 30m., near Carthage, and so forth, which were likewise, according to ancient authorities, total, but, according to our Tables, annular ones.

These examples will be sufficient for understanding that the present theory of the moon's motions principally fails concerning the secular accelerations of the moon, her Nodes and Apsides.

Now, remembering, on the one hand, that the present theory of the moon relies on the Babylonian eclipses in Ptolemy's Almagest, and that the times and magnitudes of those eclipses, specified in the Almagest, are the results of Ptolemy's computations; that, on the other hand, the dates of the classic eclipses are mathematically fixed,—the problem remains to bring our Lunar Tables into harmony with the reports of the classic authors. For this purpose we proceed in the following way.

Since the total eclipse in A.D. 1851 commenced and ended, at Danzig, 56 seconds later than our Tables had predetermined, we presume the real longitude of the moon on that day to have been shorter by 13 seconds, which gives a retardation of 24 seconds. The longitude of the Perigeum may be diminished by 17 seconds, which effected the conjunction to come later by 32 seconds. These corrections would explain that the said eclipse commenced and finished later by 56 seconds than it was expected. Moreover, since the central shadow of the moon then traversed unexpectedly higher degrees of latitude of our globe, we presume the longitude of the moon's Nodes to have been, during the conjunction, shorter by 37 seconds. These quantities increase, of course, proportionally to the squares of times; and hence we obtain the following

TABLE OF APPROXIMATE CORRECTIONS.

Epochs.	Long. Moon.			Long. Apsides.			Long. Nodes.			Time ☾.		
+ 1800	– 0°	0′	13″	– 0°	0′	17″	– 0°	0′	37″	+ 0h.	0m.	24s
+ 1700	0	0	52	0	1	42	0	2	28	0	1	36
+ 1600	0	1	58	0	2	37	0	5	33	0	3	36
+ 1500	0	3	28	0	4	39	0	9	52	0	6	24
+ 1400	0	5	26	0	7	16	0	15	26	0	10	0
+ 1300	0	7	49	0	10	28	0	22	12	0	14	24
+ 1200	0	10	40	0	14	16	0	30	13	0	19	36
+ 1100	0	13	55	0	18	42	0	39	28	0	25	36

Epochs.	Long. Moon.	Long. Apsides.	Long. Nodes.	Time ☾.
+ 1000	— 0° 17′ 37″	— 0° 23′ 24″	— 0° 49′ 57″	+0h. 32m. 24
+ 900	0 21 46	0 29 15	1 1 40	0 40 0
+ 800	0 26 20	0 25 23	1 14 37	0 48 24
+ 700	0 31 20	0 42 25	1 28 48	0 57 36
+ 600	0 36 48	0 49 43	1 44 13	1 7 36
+ 500	0 42 40	0 57 21	2 0 52	1 18 24
+ 400	0 47 42	1 5 51	2 28 45	1 28 20
+ 300	0 55 44	1 14 49	2 37 52	1 42 24
+ 200	1 2 56	1 24 17	2 58 13	1 55 36
+ 100	1 10 33	1 34 48	3 19 48	2 9 36
+ 0	1 18 36	1 45 42	3 42 37	2 24 24
— 100	1 27 6	1 57 6	4 6 40	2 40 0
— 200	1 30 22	2 9 22	4 31 57	2 56 24
— 300	1 45 23	2 21 42	4 58 28	3 13 36
— 400	1 54 51	2 34 51	5 26 13	3 31 36
— 500	2 5 11	2 48 37	5 55 12	3 50 24
— 600	2 16 6	3 2 57	6 26 25	4 10 0
— 700	2 27 2	3 17 33	6 56 52	4 30 24
— 800	2 38 47	3 43 15	7 29 33	4 51 36
— 900	2 50 42	3 49 32	8 3 28	5 13 36
— 1000	3 3 7	4 6 14	8 38 37	5 36 24
— 2200	6 •6 1	8 12 18	17 16 37	11 12 24
— 2300	6 24 7	8 36 18	18 3 58	11 45 36
— 2800	8 1 1	10 36 46	21 44 32	14 43 36
— 3400	10 11 39	13 42 22	28 52 13	18 43 36

All these figures are not at all rigorous—they are only approximate; for, in the first place, the Constant, which is in all instances to be taken into account, is entirely neglected. Moreover, since other elements of our Tables are naturally connected with the secular motions of the moon, her Apsides and Nodes, it is self-evident that the former are likewise liable to corresponding alterations. Nevertheless the preceding Table will be useful in fixing the dates and magnitudes of all ancient eclipses observed by the Romans, Greeks, Babylonians, Egyptians, and Chinese. I claim only to have collected and fixed the classic eclipses down to A.D. 400, and to have corrected the principal elements of our Lunar Tables approximately.

The Actual History of the Romans.

The historical chronology of the Romans having been discussed in the author's "Chronologia Sacra," 1846; "Berichtigungen," 1855; "Summary of Recent Discoveries," 1857 (2d ed. 1859); "Göttinger gelehrte Anzeigen," No. 125, 1855; "Chronology of of the Roman Emperors," Gettysburg Quarterly Review, 1872, p. 47; "Chronologie der Römischen Kaiser," Rudelbach's Zeitschrift, Halle, 1872, p. 50; "Probst's Theologische Monatshefte," Allentown, 1872, p. 168, and in the premises,—it would be superfluous to repeat *in extenso* all that has been done in this respect; it will be sufficient to remember, in short, the principal proving arguments. The following Table shows the difference of Petavius's and the author's Roman histories and chronologies. Since the consuls, preceding the introduction of the solar calendar of Julius Cæsar, commenced to rule on different days of the lunar calendar, and since the dates of Roman eclipses depend on the times of the consulates, the days on which the earlier consuls commenced to rule are, in accordance with Livy, specified in all instances. The asterisk (*) joined to the years of the *æra urbis conditæ* denotes the Olympian games celebrated in the same years. The names of the consuls are abbreviated. *T* signifies total eclipse, *P* partial one.

ROMAN KINGS, AND THE RESPECTIVE ECLIPSES.	Reigns.	Petav.	Seyf.	u. c.
Romulus born........................	....	—772	—771	—19
1. ⊙ T. ☋ 14° E., Nov. 19, oh. 45m. Plut. Romul. 12.				
Foundation of Rome on the parilia..Rom.	I	753	752	+ 0
2. ⊙ ☊ 1° E., May 25, 16h. Cic. Div. ii. 37; Plut. Rom. 12.				
Romulus dies after 37 years, during a solar eclipse........................	XXXVII	716	715	37
3. ⊙ T. ☊ 2° E., Jun. 5, 21h. 15m. Cic. R. P. i. 16; Livy i. 16; Plut. Rom. 22; Dion. H. ii. 56; Sen. Ep. xviii. 5, 31; Lamp. c. 2.				
Numa Pompilius reigns	I	716	715	37
Tullus Hostilius reigns during 32 years ..	I	671	670	82
4. ⊙ T. ☊ 1° E., Jan. 11, 18h. Livy vii. 28; Comp. Livy i. 31............	XXVIII	643	642	110
Tarquinius Superbus's 32d year	XXXII	582	581	170*
5. ⊙ T. ☊ 2° E. in Miletus, March 27, 17h. 45m. Pliny H. N. ii. 12, 9.				
ROMAN CONSULS, AND THE RESPECTIVE ECLIPSES.				
Val. Max. & Ver. Tricostus since the Kal. Aug. u.c.300		454	453	300
Trib. mil. Publ. Cornelius since the Idus Decembres.		402	401	351*
6. ⊙ T. in Rome, ☊ 1° 5′ E., July 1st, 17h. 45m. Cic. R. P. i. 16		401	400	350
Coss. Mart. Rutilus and T. Manlius since the Kal. Quintiles....................................		342	341	411*

ROMAN CONSULS, ETC. (*continued.*)	Petav.	Seyf.	U. C.
7. ⊙ T. in Rome, ☋ 10° E., Sept. 25, 18h. Livy vii. 28; J. Obseq. c. 22	—341	—340	—412
L. Papirius II. and Poetilius Venno, Kal. Jul.; Livy viii. 19	329	328	423
App. Claudius and L. Volumnius, since the Kal. Jan.	294	293	458*
8.? ⊙ ☋ 12° E., March 23, 23h. Livy x. 23.			
Liv. Salinator & Æm. Paullus, since the Idus Mart..	218	217	534*
9. ☽ ☋ 3° W. in Mysia (25° W.), Mar. 9, 4h. Polyb. v. 58, p. 383.			
Corn. Scipio & Semp. Longus, since the Idus Martiæ	217	216	536
10. ⊙ small, in Sardinia, ☋ 5° E., Feb. 11, 2h. 30 m. Livy xxii. i.			
Nero and Salinator, since the Idus Martiæ..........	206	205	547*
The Olympian games celebrated. Livy xxvii. 35, xxviii. 7; Polyb. xi. 5.			
Serv. Scæpio and C. Servilius, since the Idus Martiæ	202	201	550*
11. ⊙ T. ☊ 3° W., near Carthage, Oct. 18, 23h. Zonar. Ann. ix. 14.			
Cl. Nero and Serv. Pulex, since the Idus Martiæ....	201	200	551
12. ⊙ small, near Cumæ, ☋ 13° E., March 3, 23h. Livy xxx. 38............................	200	199	
T. Flaminius and Paitus Cato, since the Idus Martiæ	197	196	555
13. ⊙ small, in Rome, ☊ 3° W., Aug. 6, 15h. 30m. in —197. Julius Obseq. c. 48.			
14.? ⊙ small, ☊ 11° W., July 25th, 21h. 45 m., in —196. Jul. Obseq. c. 48.			
Purpurio and Marcellus; Isthmia æst. celebr. Livy xxxiii. 32	194	193	558
Corn. Scipio and C. Lælius, since the Idus Martiæ..	189	188	563
15. ⊙ ☋ 4° E., July 16, 20 h. Livy xxxvii. 4.			
Manlius and M. Fulvius, since the Idus Martiæ	188	187	564
16. ⊙ ☊ 3° W., —186, Jan. 10th, 23h. 30m. Livy xxxviii. 36 (2-3 p.m.)....................	187	186	565
Q. Martius and Cn. Servilius, since the Idus Martiæ.	168	167	585
17. ☽ T. ☋ 3° E., in Macedonia. Cicero De R. P. i. 15; Plutarch. Æmil. 17; Valer's M. xi. 1.			
L. Æm. Paullus and C. Licinius, since the Id. Mart.	167	166	586
18. ☽ ☋ P. 5° W., in Macedonia, June 10, 13h. 30m. Livy xliv. 37 (from 2-4 a.m.); Pliny H. N. ii. 12; Quintil. i. 10, 49.			
C. Marius and C. Flavius, since the Kal. Januar....	103	102	649
19. ⊙ T. ☋ 15° E., Dec. 2, 19h. Jul. Obs. 103(3d h.) Rome.			
C. Marius and L. Valerius, since the Kal. Januar...	99	98	653
R. Antonius and A. Posthumius, since the Kal. Jan..	98	97	654
L. Aurelius and L. Manilius, since Kal. Jan. (vi. Id. Dei)	64	63	688
Horatius born, his coss. Suet. v. Hor. p. 52. Horat. Carm. iii. 21, 1; Epod. 13, 8; Epp. i. 20, 27 (See —5).			
L. Jul. Cæsar and C. Marius Fig., since Kal. Jan. ..	63	62	689
M Tullius Cicero & Caj. Antonius, since Oct. 13 in —	63	62	690
20. ☽ T. ☊ 5° 57′ E., Oct. 27, 8h. Cicero De Cons. s. ii. 17.			
D. Silanus and L. Licinius, since Kal. Jan. (Nov.)..	62	61	691*

ROMAN CONSULS, ETC. (*continued.*)	Petav.	Seyf.	u. c.
21. ☉ Mar. 27, 4h. 15m. ☊ 0° W. Jul. Obseq. c. 123	—61	—60	—692
C. Jul. Cæsar and Marcus Calpurnius	58	57	694*
L. Calpurnius and A. Gabinius	57	56	695
P. Cornelius and Q. Cæcilius Metellus	56	55	696
Cn. Cornelius and L. Marc. Philippus	55	54	697
Cn. Pompeius and M. Licin. Crassus	54	53	698*
L. Domitius and Appius Claudius	53	52	699
Cn. Domitius and Marc. Valerius Mesalla	52	51	700
Cn. Pompeius III. and Q. Cæcilius Metellus, v. Kal. Mart.	51	50	701
Serv. Sulpicius and M. Cl. Marcellus, Kal. Jan. (Dec. 30, —50)	50	49	702*
Luc. Æm. Paullus and C. Cl. Marcellus, Kal. Jan. (Dec. 19, —49)	49	48	703
C. Cl. Marcellus and L. Corn. Lentulus, Kal. Januar. (Dec. 8, —48)	48	47	704
22. ☉ T. in Rome, ☋ 12° E., Jan. 3, 21h. Lucan's Phars. i. 535; Dio 41, 14; Petron. Sat. c. 122.			
23. ☽ T. in Rome, ☊ 3° W., Jan. 18, 9h. Lucan's Phars. i. 535.			

CONSULS AND EMPERORS.	Emp.			
J. Cæsar Dictator I.	I	48	47	704
J. Cæsar Consul II. and P. Serv. Vatia, Kal. Jan., Nov. 27 in —47	II	48	46	705
J. Cæsar Dict. II., Kal. Jan., Nov. 16, —46	III	47	45	706*
J. Cæsar Dict. III., Kal. Jan., Nov. 5, —45	IV	46	44	707
J. Cæsar Dict. IV., Kal. Jan., Oct. 24, —44	V	45	43	708
J. Cæsar Dict. V., Kal. Jan., Oct. 13, —43	VI	44	42	709
Oct. 13, —43, ann. confusionis begins.				
J. Cæsar Dict. VI., since the 1st Julian Jan.	VII	43	41	710*
Jan. 1st, the crescent visible.				
24. ☽ T. in Asia, ☊ 7° E., Mar. 13, 1h. 45m. Ovid Met. xv. 769.				
25. ☉ T. in Asia, ☊ 7° W., Mar. 27, 11h. 45m. Serv. ad Geo. i. 467; Ovid Met. xv. 789; Tibull. ii. 5,75; Joseph. A. xiv. 22; Euseb. Chron. p. 197 ad Ol. 184, 3.				
Julius Cæsar dies on Mar. 15... Augustus	I	43	41	710*
Pansa and Hirtius, since the Julian Jan. 1	II	42	40	711
26. ☉ ☋ 14° E., in Rome, Aug. 10, 16h. 15m. Dio 47, 40.				
Plancus and Lepidus, since Jan. 1st of the Julian year	III	41	39	712
27. ☉ ☋ 6° E., in Rome, July 30, 18h. 15m. Dio 47, 40; p. 529 R.				
L. Antonius Pietas and P. Servil. Vatia	IV	40	38	713
Calvinus II. and Pollio. Olympian games celebrated, Joseph. An. xiv. 14, 4; xv. 10, 1; B. J. i. 9, 13; his coss.	V	39	37	714*
28. ☉ ☊ 9° W., in Rome, Jan. 13, 21h. 30m. Euseb. Chron. ii. 197, ad Ol. 185, 2.				

CONSULS AND EMPERORS (*continued*).	Emp.	Petav.	Seyf.	u. c.
Censorinus and Calvis. Sabinus	VI	−38	−36	−715
Pulcher and Flaccus	VII	37	35	716
Agrippa and Gallus	VIII	36	34	717
29. ☉ ☊ 7° W., in Rome, Oct. 31, 22h. Fasti Sic. ad Ol. 185, 4.				
Poplicola and Nerva	IX	35	33	718*
Cornificius and Pompeius	X	34	32	719
Libo and Antonius	XI	33	31	720
Augustus II. and Tullus	XII	32	30	721
Ahenobarbus and Sosius	XIII	31	29	722*
Augustus III. and M. Valerius Messalla Corvinus	XIV	30	28	723
30. ☉ ☋ 10° E., in Rome, Jan. 4, 19h. Fasti Sic. ad Coss. Augustus III. and Corvilius (Corvinus).				
Augustus and Crassus	XV	29	27	724
Augustus and Apuleius	XVI	28	26	725
Augustus and Agrippa II.	XVII	27	25	726*
Augustus and Agrippa III.	XVIII	26	24	727
Augustus and Taurus	XIX	25	23	728
Augustus and Silanus	XX	24	22	729
Augustus and Flaccus	XXI	23	21	730*
Augustus and Muræna	XXII	22	20	731
Marcellus and Aruntius	XXIII	21	19	732
Lollius and Lepidus	XXIV	20	18	733
Apuleius and Nerva	XXV	19	17	734*
Saturninus and Vespillo	XXVI	18	16	735
Marcellus and Lentullus	XXVII	17	15	736
Furnius and Silanus	XXVIII	16	14	737
Ahenobarbus and Scipio	XXIX	15	13	738*
Libo and Piso	XXX	14	12	739
Crassus and Augur	XXXI	13	11	740
Nero and Varus	XXXII	12	10	741
Messalla and Appianus	XXXIII	11	9	742*
Tubero and Maximus	XXXIV	10	8	743
Antonius and Africanus	XXXV	9	7	744
Germanicus and Capitolinus	XXXVI	8	6	745
Censorinus and Gallus	XXXVII	7	5	746*
Horatius dies his coss. aged 58 years. (Sueton. v. Hor. (see —63.)				
Nero and Piso	XXXVIII	6	4	747
Balbus and Vetus	XXXIX	5	3	748
Augustus and Salla	XL	4	2	749
Sabinus and Rufus	XLI	3	−1	750*
Lentulus and Messalinus	XLII	2	+0	751
31. ☽ ☊ 0° W. in Jerusalem, Jan. 9, 11h. 30m. Joseph. Ant. xvii. 6, 4.				
Augustus and Silvanus	XLIII	−1	+1	752
Lentulus and Piso	XLIV	+0	+2	753
C. Cæsar and L. Aim. Paullus	XLV	+1	3	754*
Vinicius and Varus	XLVI	2	4	755
Lanio and Servilius	XLVII	3	5	756
Catus and Saturninus	XLVIII	4	6	757
L. Valerius Messalla and Cn. Corn. Cinna Magnus	XLIX	5	7	758*

CONSULS AND EMPERORS (*continued*).	Emp.	Petav.	Seyf.	U. C.
32. ☉ ☋ 15° E., in Rome, Feb. 5, 23h. Dio lv. 22, p. 390 St., his cons., and the 48th year of Augustus.				
Æm. Lepidus and L. Aurel. Nepos......	L	—6	—8	—759
Silianus and Creticus..................	 LI	7	9	760
Camillus and Quinctilianus.............	 LII	8	10	761
Sabinus and Camerinus..................	... LIII	9	11	762*
Dolabella and Silanus..................	LIV	10	12	763
Lepidus and Taurus....................	 LV	11	13	764
Cæsar and Capito.......................	LVI	12	14	765
Silius and Plancus.....................	...LVII	13	15	766*
Pompeius and Apuleius..................	..LVIII	14	16	767
Augustus obit on Aug. 19.....Tiberius	I			
33. ☉ ☋ 0° E., August 20, 16h, in Asia. Euseb. Chr. Arm. ad Ol. 198, 1; Hieron. Chron. p. 157; Dio lvi. 29, p. 472 St.				
Drusus and Flaccus.....................	II	15	17	768
34. ☽ T. ☊ 8° E., near Laybach, Jan. 30, 8h. Tacit. i. 28; Dio lvii. 4, p. 522 St.				
Taurus and Libo........................	 III	16	18	769
Rufus and Flaccus......................	 IV	17	19	770*
The Olympian games celebrated in the 3d yr. of Tiberius. Ol. 199, 1; Euseb. Chron. Arm. ad Ol. 199, 1; Cramer's Anecd., Paris, p. 15.				
Tiberius and Germanicus...............	 V	18	20	771
Silanus and Flaccus....................	 VI	19	21	772
Messalla and Cotta.....................	 VII	20	22	773
Tiberius and Drusus....................	... VIII	21	23	774*
Galba and Agrippa......................	 IX	22	24	775
Pollio and Veter.......................	 X	23	25	776
Cetheius and Varro.....................	 XI	24	26	777
Isauricus and Agrippa..................	 XII	25	27	778*
Gethulicus and Sabinus.................	... XIII	26	28	779
Crassus and Piso.......................	... XIV	27	29	780
Silanus and Nerva......................	 XV	28	30	781
R. Geminus and F. Geminus..............	... XVI	29	31	782*
Quartinus and Longinus.................	...XVII	30	32	783
Tiberius and Sejanus...................	..XVIII	31	33	784
35. ☉ T. ☋ 8° E., in Nicæa, Bithynia, Sept. 11, 22h. 30m. Euseb. Chr. i. p. 77, ii. p. 202, ad Ol. 202, 4, and the 19th year of Tiberius; Paul. Diac. Hist. misc. 7 to the 6th hour of the day; Fasti Sic. p. 222 P.				
Ahenobarbus and Vitellius..............	... XIX	32	34	785
Galba and Felix........................	 XX	33	35	786*
F. Persicus and Vitellius..............	... XXI	34	36	787
Gallus and Nonianus....................	...XXII	35	37	788
Plautius and Papinius..................	..XXIII	36	38	789
Proculus and Nigrinus..................	.(XXIII)	37	39	790*

CONSULS AND EMPERORS (*continued*).	Emp.	Petav.	Seyf.	U. C.
Tiberius ob. March 16th .... Caligula	I			
Julianus and Aprenas	II	−38	−40	−791
Caligula II. and Cæsia	III	39	41	792
Caligula III. alone	IV	40	42	793
Caligula IV. and Saturninus	(IV)	41	43	794*
Caligula ob. January 24th .... Claudius	I			
Claudius II. and Largus	II	42	44	795
Claudius III. and Vitellius	III	43	45	796
36. ⊙ (great) ☊ 0° E., Rome, July 31, 22h. Dio lx. 26, p. 76.				
Crispinus and Taurus	IV	44	46	797
Vinicius Quartinus and Corvinus	V	45	47	798*
Coss. suffecti Caj. Val. Asiaticus and M. Jun. Silanus	....	[46]		
37. ☽ ☋ 8° W., in Rome. Dec. 21, 16h. 45m. Cassiodor. his coss. and to the 5th year of Claudius. Seneca Q. N. ii. 26, his coss.				
Claudius IV. and L. Vit. Nepos III.	VI	47	48	799
38. ☽ ☊ 7° W., June 14, 6h., in Greece. Aur. Victor: Claud. iv. 22, to the 6th year of Claudius, and to u.c. 800, in which Phœnix appeared. Dio lx. 29, his coss.				
Vitellius and Vipsanius	VII	48	49	800
Gallus and Verannius	VIII	49	50	801
Vetus and Nervilianus	IX	50	51	802*
Claudius V. and Orfitus	X	51	52	803
Faustus and Titianus	XI	52	53	804
Torquatus and Antoninus	XII	53	54	805
Marcellus and Aviola	XIII	54	55	806*
Claudius ob. Oct. 13th .... Nero	I			
Nero and Vetus	II	55	56	807
Saturninus and Scipio	III	56	57	808
Nero II. and Piso	IV	57	58	809
Nero III. and Messalla	V	58	59	810*
Vipstanus and Fontejus Capito	VI	59	60	811
39. ⊙ T. ☊ 1° W., Campania, Oct. 12, 19h.; Armenia 10h. 30m. Pliny H. N ii. 70 (72) his coss.; Tacit. A. xiv. 12; Dio lxi. 16, p. 36 St.; Euseb. Chron. ad Ol. 209, 2; Hieron. ad Ol. 209, 3.				
Nero IV. and Lentulus	VII	60	61	812
Paitus and Turbilianus	VIII	61	62	813
Celsus and Gallus	IX	62	63	814*
Regulus and Rufus	X	63	64	815
Bassus and Frugi	XI	64	65	816
Silianus and Atticus	XII	65	66	817
Telesinus and Paulinus. The Olympian games postponed by Nero	XIII	66	67	818*
40. ⊙ ☊ 3° W., Rome, May 31, 3h. Philost. V.A. 4. 43, p. 183 Ol., his coss.				
Capito and Rufus	XIV	67	68	819

CONSULS AND EMPERORS (*continued*).	Emp.	Petav.	Seyf.	U. C.
41. ☽ T. ☊ 2° E., Rome, May 5, 12h.; and				
42. ☽ T. ☊ 2° W., Rome, Oct. 28, 18h. 30m. Dio xlv. 8, p. 180 St.; xlv. 11, p. 184, ad Coss. Galba & Crispinus(?); Zonar. xi. 16, p. 574.				
Trachalus and Italicus	.. [XIV]	68	69	820
Nero ob. June 9th .... Galba	 I			
Galba and Rufinus	...... [I]	69	70	821
Galba ob. January 15th	 I			
Otho ob. April 16th .... Vitellius	 I			
Vitellius ob. Dec. 20..July 1, Vespasian	 I			
Vespasian II. and Titus	 II	70	71	822*
43. ☽ P. ☋ 8° E., Rome,.March 4, 8h. Pliny H. N. ii. 13 (10) his coss.				
44. ⊙ P. ☊ 8° W., Rome, Mar. 19, 22h. Pliny l. l. his coss.				
Vespasian III. and Nerva	 III	71	72	823
Vepasian IV. and Titus II.	 IV	72	73	824
Domitian and Messalinus	 V	73	74	825
Vespasian V. and Titus III.	 VI	74	75	826*
Vespasian VI. and Titus IV.	 VII	75	76	827
Vespasian VII. and Titus V.	... VIII	76	77	828
Vespasian VIII. and Titus VI.	 IX	77	78	829
Coss. suff. Verus and Priscus	..... [X]	[78]		
Vespasian IX. and Titus VII.	 X	79	79	830*
Vespasian ob. June 23d .... Titus	 I			
Titus VIII. and Domitian VII.	 II	80	80	831
Nonius and Verucossus	 III	81	81	832
Domitian XVII. and Clemens. Olympian games .... Domitian	... XIV	95	95	846*

Subsequent to Titus, the emperors and consuls reigned, according to Petavius, in the same years to which the author refers them. This is, apart from other arguments, demonstrated by the celebration of the Olympian games in the 14th year of Domitian, and by all the later ones; for the Olympian altars (p. 405) mathematically determine that the Olympian games were celebrated in such years *before Christ,* which being divided by 4 give the remainder 1; but *after Christ,* in such years, which being divided by 4 leave the remainder 3; and of this character is A.D. 95, etc.

Chronology of the Roman Eclipses down to Titus.

We proceed now to the following questions: First, to what years of the Christian and ante-Christian eras do the Roman eclipses really belong? Secondly, in what localities and in what hours of the day were they observed by ancient eye-witnesses? Further, of what magnitudes were they according to classical reports? Finally, respecting the presumed corrections of the secular accelerations of the moon, her Nodes and Apsides (p. 429), are they approximately correct? In answering these questions, it is an indispensable duty to cite the respective passages of the classics, as has been done more explicitly in Seebode, Jahn & Klotz's Archiv f. Philol., 1848, p. 586. We follow the chronological succession of the Roman Eclipses.

1. Plutarch (Rom. c. 12) reports that about six months prior to Romulus's birth a total eclipse of the sun (ὁ ἥλιος ἐξέλιπε παντελῶς) happened on the 19th day of November (Χοιὰκ τρίτῃ καὶ εἰκάδι), during the 3rd hour (τρίτης ὥρας), Ol. 2, 1. The same total eclipse (τὸν ἥλιον ἐκλιπεῖν ὅλον) is mentioned by Dionysius Halicar. (ii. 56). Since Rome was founded in –752 on the Parilia, as we have seen (p. 407), whilst Romulus was "19 years of age," the eclipse under consideration belongs to Nov. 19th in –771. The Olympiads being in later times counted from –775, the date Ol. 2, 1, likewise refers to –771. Moreover, about that time only one eclipse was possible on Nov. 19, viz. that in –771, 45m. past noon. According to the present theory of the moon, as Pingré's computation states, the conjunction took place in —771, Nov. 19th, oh. 45m. P. T. The ☋ lay 14° E. of the sun, and the obscuration of the sun was invisible south of the 27° N. Lat. It is to be remembered that Plutarch relies on the report of the famous astronomer Tarutius, and that the astronomers commenced the day with noon; therefore the said 3rd hour commenced at 3 o'clock p.m. According to our correction, the longitude of the ☋ for –771 was shorter by about 7° 17′, and hence the same eclipse was total, or nearly total, in Latium. The correction of the longitude of the moon for the same epoch was (p. 429–30) about –2° 30′, i.e. 4h. 43m. Consequently, the ancient reports agree sufficiently with the Table on p. 429–30. Petavius referred to the eclipse in –771, June 24th, 10h. 15m. a.m., ☊ 13° W.; but June 24th never corresponded with Chœak (November), and the obscuration of the sun reached only the 60° of northern latitude, and therefore it was invisible near Rome.

2. Plutarch (Rom. 12) reports that Romulus, being 18 years old, founded Rome in Ol. 6, 3, on the 9th day of Pharmuthi, between the 2d and 3d hours of the day, and that during the same hour of the same day a partial eclipse of the sun took place (σύνοδος ἐκλειπτική), observed also in Teos, Asia Minor. Since Plutarch, as we have seen, counts the Olympiads from − 775, the foundation of Rome on the Parilia belongs to − 752, and Pharmuthi corresponded with March and April. Solinus (Pol. i. 18) says: "Romulus fundamenta murorum jecit, xviii annus natus, xi. Kal. Majas (Apr. 21), hora post secundam ante tertiam plenam"; and he specifies the planetary configuration, previous to this event, which refers again the foundation of Rome to − 752, and to the day of the Roman Parilia. Even Cicero (Divin. ii. 47) certifies to the same eclipse. Since about that time no solar eclipse about the vernal equinox, as Pingré's computations evidence, was possible except that in − 752, May 25, 16h., the beginning of the æra urbis conditæ is incontrovertibly fixed. This eclipse took place nearly 4 hours later (p. 429), which corresponds with the ancient reports, according to which the same eclipse was observed nearly 2h. 30m. after sunrise in Rome. Moreover, the longitude of ☊ 8° E. was shorter by 7° 11′, and hence a small obscuration of the sun was visible both in Rome and Teos. Petavius had reference to the eclipse in − 753, July 5th, 5h.; but Rome was founded on the Parilia and not in July, and in − 753, July 5th, Romulus was 17 and not 18 years old. Moreover, July 5th belonged to Ol. 6, 4, and not to Ol. 6, 3. Besides, July 5th corresponded by no means with Pharmuthi (March and April). In consequence of this blunder Petavius antedated by one year all events of Roman history down to Julius Cæsar, and hence his chronology of the Roman eclipses is in general wrong.

3. Livy (i. 16), in accordance with the Annales Maximi and many other authorities, reports that Romulus, having reigned 37 years, disappeared during a total eclipse of the sun. Cicero (De rep. i. 16) writes: "Defectio solis, quæ Nonis Quinctilibus fuit regnante Romulo, quibus Romulum tenebris natura ad humanum exitum abripuit." Plutarch (Rom. c. 27) represents this eclipse to have been a total one (τοῦ ἡλίου τὸ φῶς ἐκλιπεῖν—νύκτα καταςχεῖν). The same we read in Florus (i. 1), Seneca (Ep. xviii. 5, 31), Dionysius Hal. (ii. 56), Lampridius Com. (Ant. c. 2); and

Livy shows this eclipse to have taken place past noon in Rome. Since the Romans obtained their lunar calendar first by Numa, it is evident that the "Nonæ Quinctiles" referred to the ancient solar year of the Romans, and hence the Nonæ Quinctiles signified, as is well known, June 5th. About that time only one solar eclipse coincided with June 5th, viz. that in -715, June 5th, 21h. 15m. Moreover, from the foundation of Rome in -752 down to -715, as history reports, exactly 37 years, 1 month, and some days elapsed. That eclipse, however, preceded noon in Rome. but, according to our Table (p. 429–30), the conjunction took place 4h. 30m. later, which agrees with Livy. Further, the place of the ☊ was then, according to all Lunar Tables, 2° E. of the sun, and hence Pingré found that, during this eclipse, the shadow of the moon described the following curve: $-14°, +10°, -9°$. Consequently this eclipse was invisible in Rome. According to our Table (p. 429–30) the longitude of the ☊, however, was shorter by nearly 7° 3′; it lay, in accordance with history, 4° W. of the sun. Petavius referred to the eclipse in -714, May 26, 5h. 30m.; but this ecliptic new moon happened not on the "Nonæ Quinctiles," viz. June 5th; it coincided with sunset, and not with noon; it was, moreover, a small one ☊ 6° W. (the curve of the moon's shadow being 25°, 34°, 40°); and from Petavius's date of the foundation of Rome in -753 to this eclipse, not 37 years only, as history says, but 39 elapsed.

4. Livy (vii. 28) reports that, in the course of the consulate of Rutilus and Torquatus, u.c. 409, an eclipse about sunrise and a shower of stones occurred, and that both *prodigia* had likewise happened in the last years of Tullus Hostilius, who died in -638. Indeed, a similar eclipse occurred in -642, Jan. 11, 18h., ☊ 1° E. ($-6°$ 28′); but Livy (i. 31) mentions only the shower of stones, and not the eclipse; therefore the latter is dubious.

5. Pliny (ii. 12, 9) reports that, u.c. 170, and Ol. 48, 4, a total eclipse of the sun, predicted by Thales, occurred in Miletus. Solinus (Pol. c. 15, 16) refers the same eclipse to Ol. 49 [1], and to the 604th year after the destruction of Troy; consequently to -581 ($1184-603=581$). Since Rome was founded in -752, the 1st Julian year p.u.c. commenced in -751, and hence u.c. 170 commenced likewise in -581, to which Pliny refers the Thalesian eclipse. Pliny, moreover, counting the Olympiads from -775,

confirms the date; because Ol. 48, 4, began, according to the Romans, with January in –581. This eclipse, moreover, was a total one in Miletus, and it coincided with sunrise. For Themistius (Or. xxvi. p. 317, Dind.) says: *προεφήτευσαν—Μιλησίοις, ὅτι νὺξ ἔσοιτο ἐν ἡμέρᾳ καὶ δύσετο ἅμα ὁ ἥλιος καὶ ὑποθεύσεται αὐτὸν ἡ σελήνη, ὥστε ἀποτέμνεσθαι τὴν αὐγὴν καὶ τὰς ἀκτῖνας.* Herodotus (i. 74 & 103) combines the eclipses in –581 and –621, as we shall see hereafter, and refers the latter to noon, the former to sunrise (*εἶδον νύκτα ἀντὶ ἡμέρης γινομένην*). Eusebius (Chr. ii. p. 161) likewise refers this Thalesian eclipse to Ol. 48, 3, i.e. —581. About that time only one eclipse coincided with sunrise in Miletus, viz. that in –581, Mar. 27, 17h. 45m.; for, according to our Table (p. 429–30), the obscuration of the sun happened 4h. 9m. later. The longitude of ☊ being 2° E. according to the present theory, the curve of the moon's shadow was –22°, –4°, + 13°; but, according to our Table (p. 429–30), the place of the ☊ was about 4° W. Hence the eclipse was total near Miletus.

6. Cicero (De rep. i. 16)—"Anno CCCL. fere post Romam conditam," says he, "Nonis Junîs soli luna obstitit et nox. Atque hac in re tanta inest ratio atque sollertia, ut ex hac die, quam apud Ennium et in Maximis Annalibus consignatam videmus, superiores solis defectiones reputatæ sint, usque ad illam quæ Nonis Quinctilibus fuit regnante Romulo" (no. 3). These words are uttered by Scipio, who, as Ideler's Chronology states, referred, according to Dionysius, the foundation of Rome to –750. Hence u.c. CCCL is the year –400, and about that time only one eclipse was possible in June or July, viz. that in –400, July 1st, 17h. 45m., ☊ 1° 5′ E., which was invisible in Rome (p. 426). According to our Table (p. 429–30), however, the place of the ☊ was 4° 21′ W., and the conjunction happened 3h. 31m. later. (See the specialities of this eclipse, computed by means of Carlini's and Damoiseau's Tables, in the premises, p. 427). From –407 to —401 and –399 to –398 no similar eclipse was possible.

7. Livy (vii. 28) and Jul. Obsequens (c. 22) report that u c. 490, during Rutilus and Manlius's consulate, extending from the Kal. Quinctiles (July) in –341 to the same in –340, "nox interdiu visa intendi," which words signify a total or great eclipse of the sun at sunrise. Since the lunar months of the Romans, as is well known, sometimes preceded our solar ones by 30, even

by 60 days, and vice versa, it was natural that July corresponded sometimes with our September. About that time only one great eclipse happened near sunrise in Rome, viz. that in −340, Sept. 25th, 18h., ☋ 10° E. This eclipse, however, preceded sunrise in Rome, and, the present theory of the moon being true, it would have been a small one; but, according to our Table (p. 429–30), the obscuration happened 3 hours later, and the longitude of the ☋ was 4° shorter. According to Pingré, the shadow of the moon extended only to 48°, 32°, 12° N. Lat.; hence this eclipse was, according to the present theory, invisible in Rome.

8. Livy (x. 23) narrates that u.c. 457, coss. Claud. Cæcus and Vol. Flamma, "prodigia fuerunt — supplicationes in biduum." Solar eclipses belonging to Roman prodigia, Calvisius referred to the eclipse in −295, Nov. 6, 22h. 30m., ☊., 3° W.; but it was not yet known that the consuls of this time ruled one year later than Petavius had stated, and that the Roman year commenced several months later than the Julian one (Comp. no. 7). Hence the eclipse reported by Livy was that in −293, March 23d, 23h. (+3h. 13m.), ☋ 12° E., or rather 7° E. Without this correction the shadow of the moon would seem to have reached 0°, 29°, 58° only.

9. Polybius (v. 78, p. 383 Sh.) reports that coss. Liv. Salinator and Æmil. Paullus, u.c. 535, a lunar eclipse (ἔκλειψις σελήνης) was observed in Mysia (29° E.) Petavius alluded to the eclipse in −218, March 19th, 14h., because he had antedated the consuls by one year. Polybius's eclipse is that in −217, March 9, 4h., ☋ 3° W. According to our Table (p. 429–30), the said eclipse happened three hours later, and without this correction the eclipse would have preceded sunset in Mysia.

10. Livy (xxii. 1) and Obsequens (c. 31) bear witness that u.c. 536, i.e. −216, a small eclipse of the sun (solis orbem minui visum) happened in Sardinia. This was the eclipse in −216, Feb. 11, 2h. 30m., ☋ 5° E. According to the usual theory of the moon's Nodes, the obscuration of the sun in Sardinia amounted to eight inches, which clearly contradicts Livy; consequently the longitude of the ☋ must have been shorter by about 4° 34′ (p. 430). Since, moreover, in −217, no solar eclipse was possible, this eclipse again demonstrates that Petavius has antedated the consuls down to Cæsar by one year.

11. Zonaras (An. ix. 14, p. 441, ὁ ἥλιος σύμπας ἐξέλιπεν) narrates that, during the battle at Zama, near Carthage (35° 30′ N. Lat.) a total eclipse of the sun happened. Petavius had reference to the eclipse in –201, Oct. 18, 23h. 30m. ☊ 2° W.; which eclipse, however, was very small in Northern Africa, because the curve described by the moon's shadow was 37°, 2°, –18°. The longitude of the ☊ being rather 6° 30′ W. of the sun (p. 430), the obscuration of the latter must have been total, or nearly total, near Carthage. Moreover, Zonaras erroneously referred the same eclipse to the battle at Zama, for Livy puts the latter in –200, coss. Cl. Nero and Serv Pulex. To the same consuls Livy (xxx. 2) refers a phenomenon described as follows—"arcus solem tenui linea amplexus est, circulum deinde ipsum major solis orbis extrinsicus inclusit"; which words Petavius took for a description of an annular eclipse of the sun observed in –202, May 6, 1h. 45m. p.m., ☋ 7° E.; magnitude of the obscuration of the sun 7 inches on the northern part of the sun's disc. This eclipse, however, contradicts the series of the consuls, and the description rather points us, as Struyk in Ruperti's Maazin (i. p. 353) maintained, to a ring, or iris, encompassing the sun's disc.

12. Livy (xxx. 38) and Obsequens (c. 45) report that u.c. 551, coss. Cl. Nero and Serv. Pulex, a small eclipse of the sun was noticed at Cumæ, near Rome (Cumis solis orbis minui visus). The year u.c. 551, extending from the Parilia in –200 to the same in –199, and the consuls reigning since the Idus Martiæ, the said eclipse was that in –199, March 3, 22h., ☋ 13° E. This eclipse, however, was hardly visible at Cumæ, because the shadow of the moon touched only 12°, 32°, 67°; but, according to our Table (p. 429–30) the longitude of the ☋ was shorter by 4° 31′.

13 & 14. Julius Obsequens (c. 48) refers another small eclipse (solis orbis minui visus) to u.c. 555, and to the consuls Flaminius and Paitus, who ruled since the Idus Martiæ in –196. The only eclipse of this year on July 25th, 21h. 45m., ☊ 11° W., was invisible in Italy. The eclipse in –197, Aug. 6, 15h. 30m., happened, according to Calvisius, one hour prior to sunrise, but, according to our Table (p. 430), two hours after sunrise. The ☊ lay 3° W., and the curve described by the central shadow of the moon was 34°, 34°, –5°; but, according to our Table (p. 430), the ☊ lay 7° 30′ west, and hence this eclipse was indeed a small one in Rome.

This eclipse is not mentioned in the original work of Obsequens, but in its later additions, and hence it may be offered in excuse that the late interpolator confounded the consuls Lentulus and Tappulus with Flaminius and Paitus.

15. Livy (xxxvii. 4) certifies that coss. Corn. Scipio and C. Lælius, who ruled since the Idus Martiæ in — 188, u.c. 564, about the Idus Quinctiles (June 13th), and about the time of the Ludi Apollinares (July 5th), and, especially, about noon (interdiu), a partial solar eclipse occurred in Rome (cœlo sereno interdiu obscurata lux est, cum luna sub orbem solis subisset). U.c. 564 being the year — 187, and the said consuls ruling in the same and the following years, we obtain the eclipse in — 187, July 16, 20h., ☋ 4° E., whilst the curve of the central shadow of the moon was 24°, 46°, 19°. According to our Table (p. 429–30), the longitude of the ☋ 4° E. must be shortened by about 4° 27′, and the conjunction happened 2h. 54m. later. In the preceding year, — 188, no solar eclipse was possible; for which reason our chronology of the consuls is confirmed, and that of Petavius refuted.

16. Livy (xxxviii. 36) reports that u.c. 565, i.e. — 186, a short time prior to the coss. Salinator and Messalla of the same year, a great eclipse of the sun occurred between the 3d and 4th hours of the day (luce inter horam tertiam ferme et quartam tenebræ obortæ fuerant). Since the aforesaid consuls ruled from the Idus Martiæ in — 186 to the same in — 185, our eclipse, observed prior to the Idus Martiæ in — 186, was that in — 186, Jan. 20, 23h. 30m., ☊ 3° W.; curve, Southern Egypt, Arabia, Western India, centr. 17°. Accordingly this eclipse would have been very small, or invisible, in Rome, and the obscuration of the sun would not have taken place between the 3d and 4th hours past noon (interdiu). Our Table (p. 429–30), however, brings the ☊ nearly to 7° 30′ west of the sun, and the conjunction to about 2h. 55m. past noon. In — 187 no eclipse was possible in January, February, March, April, and May. Petavius, on the contrary, alluded to the eclipse in — 187, Jul. 16, 20h.; but this is irreconcilable with the Annales Maximi referring to an eclipse in January or February, and not to an eclipse 4h. 40m. after sunrise. Besides, this eclipse again confirms the result that the consuls, down to J. Cæsar, ruled one year later than formerly was believed.

17. Cicero (De rep. i. 15) bears witness that a total eclipse of the moon (serena nocte subito candens et plena luna defecit) was seen near Apollonia (20° 10′ E.) during the year preceding the consulate of Æm. Paullus and Cæpio, which office commenced in —166, Idus Mart., consequently in —167. Plutarch (Æm. Paul. c. 17) confirms the statement that this eclipse was a total one (ἡ σελήνη ἐμελαίνετο καὶ τοῦ φωτὸς ἀπολιπόντος αὐγὴν χροὰς ἐμείψασα παντοδαπὰς ἠφανίσθη). This is the eclipse in —167, June 21st, 7h.45m., ☋ 3° E., which, as Cicero says, Gallus, "anno fere antequam Consul est declaratus, haud dubitavit postridie palam in castris docere, nullum esse prodigium." Gallus being consul in —164–5, he was designatus in —166, and, since that eclipse happened one year prior to Gallus's designation, the former indeed belonged to June in —167. This eclipse has erroneously been confounded by Petavius with the following.

18. Livy (xliv. 37) says: "Gallus pronunciavit, nocte proxima —ab hora secunda ad quartam horam noctis lunam defecturam esse.—Nocte, quam pridie Nonas Septembres insecuta est dies, edita hora luna cum defecisset." Pliny (H. N. ii. 12) writes: "Gallus—tum Tribunus militum—pridie quam Persus superatus a Paulo est, in concionem ab imperatore productus—ad predicandam eclipsin." The same we read in Quinctil. Ins. or. i. 10, 47; Frontin. Stra. i. 12, 8; Justin. H xxxiii. 1.; Plutarch Æm. c. 17. Since the battle near Pydna (20° 10′ E.) belongs to the consulate of Æm. Paullus and Lic. Crassus (—166), and to the first days of the lunar September of the Romans, the partial eclipse was that in — 166, Jun. 10, 13h. 30m., ☋ 5° W. The obscuration of the moon amounted, according to the present theory of the moon, to 11¾ inches, which disagrees with Livy, who reports that this eclipse lasted only two hours. Total eclipses of the moon lasting three hours, and never only two, it is apparent that the longitude of the ☋ must have been shorter, namely, by about 4° 24′, as our Table (p. 429–30) shows. Moreover, *nox*, in its specific sense, designated the time from midnight to morning, whilst *vesper* lasted from sunset to midnight. Hence *mane* comprised the hours from sunrise to noon; *dies* specifically extended, as we have seen, from noon to sunset; accordingly *vespera* was the time from sunset to midnight; wherefore Venus, being visible after sunset, was called Vesper, Hesperus. Now, Livy narrates that the middle of our eclipse was 3 o'clock a.m. local time, whilst the usual Tables refer the middle of the same eclipse to about 2h. 10m. a.m., Pydna time. This result confirms our Table (p. 429–30). Petavius erroneously referred both eclipses (Nos. 17 & 18) to —167, June 21st, 7h. 45m.

19. J. Obsequens (c. 103) reports that u.c. 649, coss. C. Marius and C. Flavius ruling since the Kal. Jan., "hora diei tertia solis defectus lucem obscuravit." This is the great eclipse in — 102, Dec. 2d, 19h., ☋ 15° E.; for on that day the sun rose in Rome about 7h. 30m. a.m. (local time), and hence the third hour of the day in Rome extended from 9h. 0m. to 9h. 45m. a.m. The middle of the said eclipse coincided in Rome with 7h. 42m. a.m., consequently nearly two hours too early. But, according to our Table (p. 429–30), the conjunction happened 2h. 40m. later. Moreover, the moon's shadow touched, according to Pingré, 41°, 22°, 35° only, and hence this eclipse could not *obscurare lucem* in Rome. Our Table (p. 429–30), on the contrary, shortens the longitude of the ☋ by 4° 6′, and makes the eclipse greater.

20. Cicero (De cons. suo, ii. 17) testifies that, in the course of his consulate, and about the Latinæ, held in January of the lunar year, which at that time preceded the Julian January (as we shall see below, Nos. 22 & 23) by some 60 days, and whilst the mountains of Albano were already "covered with snow," a total eclipse of the moon (luna stellanti nocte peremta est) occurred in Rome. The beginning of Cicero's consulate in —62, is, apart from all other evidence, fixed by the Ara Albani (p. 407), the nativity of Augustus; for this emperor was born whilst Cicero delivered the fourth Catilinaria in January, and the said planetary configuration refers to —62, Dec. 23. Moreover, Josephus (An. xiv. 4, 2; B. J. v. 9, 4) reports that, during Cicero's consulate (Ol. 179, 1), Pompeius captured the Jewish temple on the 10th day of Thishri (Hyperberetæus), i.e. Sept. 11 (p. 414), being a Saturday (*Κρόνου ἡμέρα*, Dio 37, 15), and this day was only in —62 a Saturday. Hence Cicero's eclipse was that in —62, Oct. 27th, 7h. 30m., ☊ 5° 37′ E. (—4°). Ideler points us to the eclipse on May 3d, 3h. 30m. a.m., but during May no snow exists in Italy near Rome.

21. Jul. Obsequens (c. 123) reports that, coss. Afranius and Cæcilius, u.c. 693, an eclipse of the sun happened one hour prior to sunset (die toto ante sereno circa horam undecimam nox se intendit, deinde restitus fulgor). The said consuls ruled in — 58 since the Kal. Jan., and u.c. 693 commenced about the same time; but the eclipse in —58, July 31, 9h. 15m., ☋ 14° E., was invisible in Europe, because it commenced after sunset. The same is the case with the eclipse in —58, March 5, 7h. 30m. ☊ 16° W. One year earlier the ecliptic new moon happened on March 16th, 4h. 45m., ☊ 8° W., which likewise followed sunset; wherefore Petavius transferred this eclipse to Spain, and yet it was there also invisible. The only eclipse coinciding with sunset was, at that time, that in —60, March 27th, 4h. 15m. P. T., ☊ 0° W.; curve, —15°, 3°, 24°. Since the ☊, however, lay rather nearly 4° W. of the sun, the obscuration of the latter must have been nearly total in Rome, where the sun set about 6h. 4m. (local time), in Spain 20m. later. During this eclipse, as Pingré's Cometography states, Posidonius discovered a comet, probably near the sun, which would have been impossible without a great obscuration of the sun in Rome. The correction of the secular acceleration of the moon, amounting for —60 to about + 2h. 34m., the retardation of the apsides is to be taken into account. Moreover, since Obsequens lived 400 years A.C., and since at that time the true series of the consuls was already corrupted, as the chronographer of A.D 354 bears witness, it is not to be wondered at that Obsequens referred this eclipse to the consuls Afranius and Cæcilius instead of Silanus and Licinius.

22 & 23. Lucan (i. 535, "Titan involvit orbem tenebris—Phœbe expalluit umbra"), Petronius (Sat. cxxii. 44, "Titan vultus caligine texit"), and Dio (xli. 1, 14, p. 692 St., "ὁ ἥλιος σύμπας ἐξέλιπε"), bear witness that whilst J. Cæsar, having crossed the Rubicon, marched against Rome, two total eclipses were noted within fifteen days, coss. Marcellus and Lentullus, u.c. 704, i.e. —47. The season when these eclipses took place is fixed by Lucan, who certifies that on that day "the Rubicon was covered with ice," and that Cæsar crossed it three weeks after the inauguration of the consuls, Kal. Jan., which at that time coincided with Dec. 8. Cæsar himself (Bell. C. i. 13) says that he, having passed over the Rubicon, took first of all Corfinium on viii. Kal. Mart., i.e. on the 28th of January. The said solar eclipse took place on Jan. 3d, 21h. 30m., ☋ 12° E., curve touching 30°, 22°, 48°; consequently the obscuration of the sun was, in Italy, a very small one. But, according to our Table (p. 429–30), the longitude of the ☋ was 3° 53′ shorter, and the conjunction happened 2h. 30m. later, to the effect that a total, or nearly total, obscuration of the sun took place at that time near Rome. The total lunar eclipse took place on Jan. 18th, 9h. 30m., ☊ 0° W. Since the phenomenon of two eclipses within fifteen days recurs only after long intervals, the first year of Cæsar's reign is fixed with mathematical certainty. Petavius arbitrarily referred Cæsar's first year to —50, but failed to produce two eclipses in January of —50.

24, 25. Ovid (Met. xv. 789) reports that about the day of Cæsar's assassination (Mar. 15) a total eclipse of the moon had taken place (sparsi lunares sanguine currus); and Servius (ad Virgil. Georg. i. 467) says, "Constat occiso Cæsare solis fuisse defectum ab hora sexta usque ad noctem." The sixth hour means, according to the Julian day, sunrise, and the obscuration of the sun from sunrise to sunset involves the opinion that on the occasion of solar eclipses our globe is successively twelve hours under the shadow of the moon. Virgil (Geor. i. 467) mentions only a solar eclipse about the time of Cæsar's death—"sol caput ferrugine texit"; and the same we read in Ovid (Met. xv. 789)—"Lucifer ferrugine textus erat"; and Tibullus (ii. 5, 75)—"solem defectum lumine vidit." Euseb. Chr. ii. 197, refers the same eclipse to Ol. 184, 3, consequently to the same year —41, because he commenced the Olympian years with the preceding local newyears day. Both eclipses, it is true, were invisible in Rome; but the said authors intended only to commemorate the singular phenomenon, that, about the time of Cæsar's assassination, two eclipses occurred, and that the Roman astronomers had, for a long time prior thereto (p. 445), been able to determine in advance the times of eclipses. The eclipses referred to are the following: ☽ —41, March 13th, 1h. 45m. p.m (P. T.), ☊ 7° E., obscuration 8,5 inches. ☉ —41, March 27th, 11h. 45m. p.m., ☊ 7° W. The former being partial, was total, according to our Table (p. 429–30), because the ☊ lay only about 3° 10′ E. of the sun, which agrees with the ancient reports. Since in the preceding year —42 two eclipses within fifteen days was impossible, these eclipses put it beyond question that Cæsar ruled 6 yrs. 3 mos., and not, as Petavius and his adherents believed, 5 yrs. 3 mos.; that, moreover, Cæsar died in —41, and not, as stated in all our Chronological Tables, in —43. This result, *from which the whole of the Greek history depends,* is historically and astronomically confirmed by the following eighteen irrefutable arguments:

The Fasti Capitolini and other authorities attest that Cæsar was six times dictator, but, according to the present chronology, five dictaturæ only came out, and this contradiction our chronologers could not obviate by the hypothesis that the Romans omitted to mention Cæsar's first dictatorship. Even Josephus, Plutarch, Cassiodor, Eusebius, and others, assign to Cæsar a reign of six years and three months, and not of five years and three months.

Further, supposing Cæsar to have governed only five years and three months, then we have to accept the absurdity that during Cæsar's last year two annual magistri equitum existed simultaneously, namely, Antonius and Lepidus. (See Fischer's "Römische Zeittafeln.")

Furthermore, Josephus (Ant. xiv. 4, 2) reports that Pompeius captured Jerusalem, in the course of Cicero's consulate, on Sept. 11th, a Saturday (Dio 37, 15), consequently, as has been shown, in —62. On the same day of the week, during the consulate of Agrippa and Gallus, Herod conquered Jerusalem, but "27 years later." (Joseph. B. J. v. 9, 4; Ant. xiv. 6, 4), i.e. —35. According to Petavius, however, the interval amounts to 26 years only, because he had shortened the ruling-time of Cæsar by one year.

Moreover, the battle at Pharsalus was given, coss. Cæsar II. and Vatia Isauricus (—47), v. Idus Sept. (June 28th) in —46 (B. C. iii. 85), and, two months after, Pompeius died pridie Kal. Oct., i.e. Aug. 18 in —46. From this day to Cæsar's assassination on March 15th, says Plutarch (Cæs. § 267), four years and some months elapsed. Consequently Cæsar died in —41, and not in —43. The nativity of Cæsarion (p. 407) mathematically confirms that Pompeius died in —46, and Cæsar took Alexandria in December —46.

Add to this that Augustus, being born in January, —61, as we have seen (p. 407), testifies himself to have been 19 Roman years old (Mon. Anc. 1, "annos undeviginti natus") subsequent to Cæsar's death. The latter, accordingly, belongs to —41, and not to —43. The same results from the ages of Cæsar, Varro, Virgil, Horace (p. 432, 434), Cleopatra, and other notables of that period, who would otherwise have been made to live one year less than history reports.

To these the following astronomical certainties may be appended. Macrobius (Sat. i. 14) reports that the first day of the Julian Calendar commenced, consistently with the preceding Roman months, with a new moon, the same day on which the crescent appeared in Rome, and this was the case only in —41, Jan. 1st. Even the Julian coins, struck at the same time, and for the purpose of perpetuating the introduction of the tropic year, represent the crescent as visible on the first day of the first solar January of the Romans, as will be seen in Eckhel's "Doctrina Numorum." According to Petavius, who referred the introduction of the Julian year to —41, the crescent appeared 22 days prior to the first day of January.

The last lunar year, the so-called *annus confusionis* of the Romans, contained, as the ancients report, and as every historian knows, 445 days; in other words, fifteen lunar months; wherefore that lunar year must have commenced on Oct. 13th, being the 445th day prior to the 1st day of January of the first Julian year. The Romans being in the habit of beginning their lunar months and years with the appearance of the crescent, the *annus confusionis* must have begun in —43, Oct. 13, because on that day only the crescent became visible, 445 days prior to the beginning of the first Julian year. According to Petavius, who referred the beginning of the *annus confusionis* to Oct. 13 in —45, the Romans had been in the habit of commencing their lunar months 22 days previous to the new moons. Is not this nonsense?

Many ancient authors (Plutarch, Cæs. 63; Sueton, Cæs. 81; Dio, 44, 17; Obsequens, c. 127) recount that in the night preceding Cæsar's assassination i.e. on March 14th, Calpurnia, Cæsar's wife was awakened by the light of the full moon (lunæ splendore, καταλαμπούσης τῆς σελήνης). The latter happened, as we have seen (p. 448), on March 13th in —41; consequently the still full-orbed moon (σελήνη, p. 414) rose in Rome, on the 14th day of March, about 8 o'clock p.m., and so it could, about midnight of the same day, awaken Calpurnia. In —43, on the contrary, to which Petavius refers Cæsar's assassination, the moon rose about daybreak, and, being crescent-shaped, could not awaken anybody "at midnight" on March 14th.

The fact that Cæsar died in —41, and not in —43, is, finally, confirmed by the Olympian games, celebrated, as Cicero testifies in several places (Epis. ad Att. 15, 5 & 24; 16, 7), three months after Cæsar's decease; for the planetary configuration of —777 (p. 404) demonstrates that the Olympian games were celebrated in all years before Christ which, being divided by 4, give the remainder 1, and of this character is the year —41.

Further, Livy (27, 35; 18, 7) and Polybius (11, 5) narrate that during the consulate of Nero and Salinator, i.e., as we have seen (p. 432), in —205, u.c. 547, the Olympian games were celebrated. Petavius, on the contrary, refers the same consuls to —206, and Cæsar's death to —43, to the effect that the Olympian games were once celebrated every third year. In —205, even a Roman embassy, as Livy in the Annales Maximi found, had been sent to attend the same Olympian games.

Further, Cramer's An. P. p. 151, and Euseb. Arm. ad. Ol. 199, 1, bear witness that during the third year of Tiberius, who reigned since the death of Augustus on Aug. 19th, A.D 16, the emperor's quadriga conquered during the Olympian games, consequently A.D. 18. According to Petavius, who refers the third year of Tiberius to A.D. 17, and the consuls Nero and Salinator to —206, the Olympian games again would once have been enacted every three years.

Moreover, Plutarch (Anton. p. 942) relates that Antonius and Cleopatra assisted at the Olympian games in the course of Ahenobarbus and Sosius's consulate, i.e. in —29 (p. 435), which agrees with the Olympian games in —41, and those in the years +19, —205, and —777. According to Petavius, the said consuls ruled in —31 (p. 435), and accordingly the Olympian games were repeated every three years, if Petavius was right.

Again, Josephus (B. J. i. 21, 8; Ant. xvi. 5, 3) recounts that Herod I. participated in the Olympian games during the 25th year of his reign. This Herod obtained the crown of Palestine within the consular year of Calvinus and Pollio in —37, and in the same year the Olympian games took place (Joseph. An. xiv. 14, 4; xv. 10, 1; B. J. i. 9, 13). After that time Herod is said to have reigned 37 years (Joseph. Ant. xvii. 8, 1). The same king, assisted by Sosius, conquered Jerusalem during the consulate of Pulcher and Flaccus in —35 (p. 434), and after this year he is said to have actually reigned 35 years (Joseph. Ant. xiv. 16, 2; xv. 1, 2). The year —35 is confirmed by the report (Dio xlix. 22) that Herod took Jerusalem on the 10th day of Hyperberetæus (Sept. 14th), "being a Saturday"; for in —35 only the 11th day of September was a Saturday. The year being fixed in which Herod's reign of 35 years commenced, it is evident that Herod must have died in the year called 0, that is to say, during the first year of our original Dionysian era, and subsequent to Christ's birth, as the Evangelists and the Fathers of the Church bear witness. These results are astronomically confirmed by the total lunar eclipse in the year 0, January 9, 11h. 30m., ☊ 3° E., which preceded Herod's death by about three months (Joseph. An. xvii. 6, 4). According to Josephus, Herod died a short time prior to Easter, celebrated always on March 20th (p. 414), and nearly three months after the eclipse on Jan. 9th of the year 0. Since, then, Herod visited the Olympian games in the 25th year of his reign, he assisted at them in —9; and this, again, is a year which, being divided by 4, gives the remainder 1, like —41.

Besides these epochs, thirteen years of later times are mentioned by Roman and Greek authors, during which the same Olympian games have been held. We specify the following: Philostratus (V. A. iv. 24, 17; 18, 34) relates that the Olympian games took place seven years prior to Nero's departure for Greece, namely, A.D. 59, Ol. 209, 1. Nero himself, coss. Telesinus and Paulinus, went over to Greece for the purpose of attending the Olympian games to be held in the same year, A.D. 67. as Philostratus (v. 7, 11), Pausanias (x. 36, 4), Eusebius (ad Ol. 211, 1), Cramer (Anecd. ii. p, 151), Dio (63, 8), Suetonius (Nero 19), Josephus (B. J. ii. 20, 1), Suetonius (Vesp. 4), recount.

Further, Philostratus (V. A. viii. 14) relates that, coss. Domitian XVII. and Clemens, i.e. A.D. 95, Ol. 218, 1, the Olympian games were celebrated.—Again, Pausanias (Perieg. v. 21, 6) reports that the Olympian games were re-enacted A.D 127, Ol. 226, 1.) The same author (x. 34, 2) writes that during the third year of Aurelius and Verus, Ol. 235, 1, the same games were repeated.

Furthermore, Gellius (N. A. viii. 3), Lucian (viii. 297), Ammian (xxix. 1, 39), Hieronymus (a. A. 2181), and others, report that the philosopher Peregrinus burned himself to ashes while the Olympian games were taking place in Ol. 236, 1, A.D. 167.— Still further, Malala (xii. 372) recites that the same games were transferred to Antiochia in the 260th year of the Antiochian era, which commenced in —48, consequently A.D. 211, Ol. 247, 1.— Besides, Censorin (c. 18, 21) says that u.c. 991, Ær. Act. 267, Ær. Jul. 283, in the first year of Gordian, i.e. A.D. 239, Ol. 254, 1, the Olympian games were repeated.—Again, Libanius (i. p. 91, 94) tells us that, subsequent to Julian's death, Ol. 285, 1, the same festival took place A.D. 263.

Finally, Cedrenus (p. 325) bears witness that the Olympian games, interdicted by Theodosius during his 16th year, came to an end A.D. 395, Ol. 293, 1; u.c. 1146. All these epochs of the Olympian games concur in putting beyond question that, after Christ, these games were repeated in such years which, being divided by 4, leave the remainder 3. And these arguments will suffice to convince every unbiased man that Cæsar died in —41, and not in —43; that, consequently, the following consuls and emperors, down to A.D. 47, ruled two years later than Petavius brought out. (See the Chronological Table p. 433-37.) In one word, the Olympian games are the basis of the whole Greek and Roman histories.

We return now to the chronology of the Roman eclipses observed after J. Cæsar's death in —41.

26. Dio Cassius (L. xlv. to the coss. Hirtius and Pansa, u.c. 711) reports that a small eclipse of the sun occurred in —40 (*τό τε φῶς τοῦ ἡλίου ἐλαττοῦσθαί τε καὶ σβέννυσθαι*), and the Chronicon Paschale refers the same eclipse to the second year of Augustus, Ol. 184, 4; consequently to —40, viz. to Aug. 10, 16h. 15m., ☋ 14° E. According to the present theory of the moon, however, the obscuration of the sun reached only 60°, 71°, 39°, and finished prior to sunrise in Rome. Our Table (p. 429) puts ☋ 10° only E. of the sun, and the conjunction 2h. 29m. later. In —41 and —42 no solar eclipses were visible in Rome, as Pingré's computations evince.

27. Dio Cassius (xlvii. 40, p. 519, Reim.) mentions a great eclipse of the sun (*ὁ ἥλιος τότε ἐλαττοῦτο καί ἐλάχιστος ἐγένετο*), u.c. 712, coss. Lepidus II. and Plancus. This is the eclipse in

—39, July 30, 18h. 15m., ☋ 6° E., curve 44°, 55°, 29°. The conjunction happened (p. 429) about 2h. 29m. later, and the longitude of the ☋ was shorter by about 3° 50′. Hence this eclipse was a great one in Rome.

28. Eusebius (Chron. ii. 197, ad coss. Censorinus and Calv. Sabinus, Ol. 185, 2) mentions a solar eclipse which belongs to the year — 37, because in —36, the real consulate of Censorinus and Sabinus, no eclipse of the sun was possible in Europe. This eclipse, —37, Jan. 13th, 21h. 30m., ☊ 9° W., curve 47°, 33°, 51°, was nearly total in Rome, owing to the longitude of the ☊ being shorter by about 3° 50′.

29. The Fasti Siculi (p. 190) report that coss. L. Gellius Poplicula and M. Coccejus Nerva, u.c. 719, Ol. 185, 4, *ἔκλειψις ἡλίου ἐγένετο.* The said consuls ruled in —33, but no eclipse was visible in that year; wherefore we have again to recur to the preceding year, viz. to the eclipse in —34, Oct. 31st, 22h., ☊ 7° W., curve 62°, 28°, 14°, which was greater in Rome by reason of the ☊ lying about 10° west of the sun.

30. The Fasti Siculi (p. 190) mention another eclipse (*ἔκλειψις ἡλίου ἐγένετο*), coss. Augustus III. and Corvilius (read Corvinus), u.c. 722, Ol. 187, 4 (?). The aforesaid consuls officiated in —28, during which the eclipse of Jan. 4, 19h. (+2h. 26m.), ☋ 10° E. (—3° 48′), curve 24°, 13°, 42°, occurred. In the preceding year both ecliptic new moons happened after sunset. In —30, which Petavius referred to the same consuls, the only eclipse, on Aug. 20, 7h. 15m. p.m., was likewise invisible in Italy. The eclipse in —27, June 18, 15h. 45m., curve 20°, 38°, 10°, ☊ 1° W., was greater, but it does not agree with the consuls, and preceded sunrise in Rome.

31. Josephus (A. xvii. 6, 4) reports that an eclipse of the moon happened in Jerusalem on the night preceding the fast-day solemnized in commemoration of the siege of Jerusalem by Nebuchadnezzar, which siege commenced on the 10th day of the tenth month (Tebeth) of the civil year, as proven by 2 Kings, xxv. 1. *Ὁ Ματθίας*, Josephus says, *ἱερόμενος ἐν νυκτί τῇ φερούσῃ εἰς ἡμέραν, ᾗ ἡ νηστεία ἐνίστατο, ἔδοξεν—καὶ ἡ σελήνη δὲ τῇ αὐτῇ νύκτὶ ἐξέλιπεν.* This eclipse, moreover, occurred, according to Josephus, nearly three months before Easter (March 20th) and the death of Herod. About that time only one lunar eclipse was

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visible in Jerusalem during January (Tebeth, p. 414), viz. that in the year 0 of our era, Jan. 9, 11h. 30m. p.m., ☊ 0° W. On this occasion it may be seen that the Hebrews never used lunar months; for on the 9th or 10th day of any lunar month no lunar eclipse could take place, consequently Tebeth and all other Hebrew months must have been solar ones. (See p. 414.) It is evident, by the way, that, prior to the Babylonian captivity, Tebeth 1st must have coincided with Jan. 1st, as has been demonstrated in the author's Chronologia Sacra, p. 40. Petavius and Ideler referred this eclipse to —3, March 12th, 13h., ☊ 8° W., obscuration 4¼ inches; but this eclipse was invisible because the ☊ lay 11° 42′ W., and it is refuted by Josephus's History, which specifies that two months intervened between the eclipse and Herod's death, previous to March 20th.

32. Dio (lv. 22, p. 390 St.) asseverates that in the 28th year of Augustus, which year extended from Mar. 15, A.D. 6, to Mar. 15, A.D. 7, U.C. 759, coss. Messalla and Cinna, a small eclipse of the sun was seen in Rome (*τοῦ ἡλίου τι ἐκλιπὲς ἐγένετο*). This is the eclipse A.D. 7, Feb. 5, 23h., ☋ 15° E., which was, however, invisible in Rome, as Pingré states, provided the longitude of the ☋ was not shorter by 3° 23′ (p. 429). In A.D. 5, to which Petavius refers the 28th year of Augustus and the aforesaid consuls, no such eclipse was possible at all.

33. Eusebius Armen. mentions a solar eclipse about the time of Augustus's death (Aug. 19) in Ol. 198 (*καθ' ὃν χρόνον ἔκλειψις ἡλίου ἐγένετο*). The same is reported by Hieronymus (Chron. ii. p. 157), and by Dio (lvi. 29, p. 472 St.) Since Jerome commences the Olympiads, in accordance with the Roman year, on Jan. 1st, his Ol. 198, 1, coincided with A.D. 16. Indeed, during this year, only one day after Augustus's death, an ecliptic new moon occurred, viz. on Aug. 20th, 17h., ☋ 2° E., curve 27°–30°, 15°, 12°. Since the sun rose on that day about 5 o'clock (Rom. T.), the conjunction took place, in consequence of the parallax, prior to sunrise, but, according to our Table (p. 429), 2h. 23m. later. The longitude of the ☋ being shorter by 3° 37′, it is probable that the eclipse was perceived in Thebes, Egypt, 25° N., or in Ethiopia. Petavius referred, in accordance with Ptolemy, the death of Augustus to A.D. 14, and, no solar eclipse being possible in A.D. 14, he metamorphosed even this eclipse into a "supernatural phenomenon."

34. Tacitus (Ann. i. 28) and Dio Cass. (lvii. 4, p. 522 St.) report that nearly six months after Augustus's death a total eclipse of the moon happened, soon after sunset, in Laybach, Tyrol, which eclipse terminated the rebellion of the legions there stationed. This is the eclipse of A.D. 17, January 30th, 8h., ☊ 8° E., obscuration 6 inches. Since, however, the ☊ lay only 4° 20′ E. of the sun (p. 429), the eclipse was indeed a total one. Petavius had in his mind the eclipse of A.D. 14, Sept. 26ht, 18h. (Laybach T.); but this eclipse commenced a short time before sunrise, and not soon after sunset. It is, moreover, opposed to all the facts; for, from Augustus's death (Aug. 19) to Sept. 26th, only 38 days elapsed, and this short time was insufficient to accomplish what, according to Tacitus's detailed description, was done during this period. This eclipse apparently confirms the fact that the consuls after Cæsar ruled two years later.

35. Phlegon and Thallus (Euseb. Chr. i. 77 & ii. 202; Paulus Diac. Hist. Misc. 7, p. 253; Syncel. p. 256, Ven.; Fasti. Sic. p. 222, Par.), bear witness that in the 19th year of Tiberius, Ol. 202, 4, a total eclipse of the sun (ἔκλειψις ἡλίου τελεία) had been seen about noon (ὥρῃ ἕκτῃ τῆς ἡμέρας) in Nicæa, Bithynia (40° 30′ N., 27° 30′ E.) The years of the emperors commenced in Egypt and other Roman provinces, it is well known, with the local newyears day preceding the epoch of their actual reign; and hence the 19th year of Tiberius began in the autumn of A.D. 32. Accordingly, the attested eclipse must have been that in A.D. 33, Sept. 11, 22h. 30m. P. T., ☋ 8° E., curve 78°, 63°, 33°. According to our Table (p. 429–30), the conjunction happened 2h. 18m. later, and the ☋ lay 4° 25′ only east of the sun, and hence the eclipse was total in Nicæa, and, as history reports, there it coincided with noon.

36. Dio Cass. (ix. 26, p. 776) relates that "on Claudius's birthday" (Aug. 1st) an eclipse of the sun occurred in Rome (ὁ ἥλιος ἐκλείψειν ἔμελλεν). The only eclipse, about that time, coinciding with the 1st day of August was that of A.D. 45, July 31, 22h. ☊ 0° W., curve 22°, 19°, —14°. The longitude of the ☊ being shorter by 3° 30′, the obscuration of the sun in Rome must have been nearly total. Dio refers the same eclipse to the 5th, and not to the 3d year of Claudius, probably because Γ (3) was taken for E (5), being very similar to the former.

37. The decree of Claudius, and the inscriptions referring to his consulates, demonstrate, as we have seen (p. 422), that Claudius reigned not 14, but 13 years only, and that the consuls Asiaticus and Silanus were extraordinarii, and not ordinarii. Hence all consuls subsequent to A.D. 47 must have ruled only one year later than Petavius stated. This result is mathematically confirmed by the transit of Venus referred to the 6th year of Claudius (p. 415) and to the 800th year urbis conditæ; for, since Rome was founded in the spring of —752, the 800th year u.c. was annus Domini 48 (800—752=48. or 799—751=48). According to the same rule, the third centennial jubilee of Rome was, as Livy narrates, celebrated, coss. Valer. Maximus and Vir. Tricostus in —452 (p. 431), i.e. u.c. 300; consequently, the lunar eclipse referred to the 5th year of Claudius belongs to A.D. 47. Seneca (Q N. ii. 26) mentions an eclipse of the moon observed in the course of the consulate of Valerius Asiaticus, i.e. A.D. 47; but the same eclipse Cassiodor refers both to the consuls Vinicius Quartinus and Corvinus and to the 5th year of Claudius. Consequently Valerius Asiaticus must have been a consul suffectus in the 5th year of Claudius, A.D. 47, whilst Quartinus and Corvinus were ordinarii. Hence the said eclipse was that in A.D. 47, June 25, 15h. 30m. (+2h. 15m.), ☊ 1° E. (—3° 29′) Petavius referred the same consuls to A.D. 45, during which year, however, no lunar eclipse whatsoever occurred.

38. Aurelius Victor (Claud. iv. 12) reports that in the same year in the course of which the jubilee of Rome was celebrated, and Phœnix appeared, i.e. in the 6th year of Claudius, an eclipse of the moon happened. The same eclipse Dio Cassius (lx. 29) refers to the coss. Claudius IV. and L. Vitellius III., i.e. A.D. 48. Hence the attested eclipse was that in A.D. 48, June 14, 6h. p.m., ☊ 7° W. During the same eclipse, however, the island of Thera emerged from the Ægean sea, which Seneca (Q. N. ii. 26) refers to the preceding year, in accordance with Eusebius (Chron. ii. p. 204, ad Ol. 205, 4).

39. Pliny (H. N. ii. 70=72) says: "Solis defectum Vipstano et Fontejo consulibus, qui fuere ante paucos annos, factum......

Campania hora diei vii et viii. sensit, Corbulo, dux in Armenia, inter horam diei x. et xi. prodidit visum." As the same author (ii. 79) informs us that the Roman priests only counted the hours from midnight ("a media nocte ad mediam"), and since the same eclipse was seen in Armenia between 10 and 11 o'clock after sunrise (diei), it is evident that this eclipse coincided (p. 429) with about noon, R. T. Tacitus (xiv. 12) simply reports, "sol repente obscuratus"; but according to Dio (lxi. 16, p. 36 St.) this eclipse was total (*σύμπας ἐξέλιπεν*) in Calabria. This is the eclipse A.D. 60, Oct. 12th, 19h. (+2h. 11m.), ☊ 6° W., curve 58°, 32°, 22°. According to our Table (p. 430), the longitude of the ☊ was shorter by 3° 27′, and hence the eclipse must have been nearly total in Campania. Petavius, of course, alluded to the eclipse A.D. 59, April 30, 23h., ☋ 3° E., obscuration 9 inches; but this eclipse stands in direct opposition to Pliny and Dio, and to Roman chronology. The date "pridie Kal. Majas" is, in the present editions of Pliny, apparently altered to suit Petavius's erratic chronology.

40. Philostratus (V. A. 45, 8 & 11, p. 180 & 184 St.) testifies that during the consulate of Telesinus an eclipse of the sun took place (*γενομένης ἐκλείψεως τοῦ ἡλίου*), consequently A.D. 67, May 31st, 3h., ☊ 3° W., curve 40°–28°; Europe, Africa, S.W. Asia. Since the longitude of the ☊ was shorter by about 3° 25′, the obscuration was great in Rome. The year of the eclipse is fixed by the Olympian games, for in the course of the same consulate, and the 12th year of Nero, the emperor went over to Greece to assist at the festival; the games, however, were postponed for one year, and were celebrated A.D. 68 (Philost. V. A. iv. 17, 18, 24; Sueton. Nero, 19; Pausan. x. 36, 4, etc.) Petavius referred the same consuls Telesinus and Paulinus to A.D. 66, but alas! during this year no eclipse at all was possible.

But it came to light on occasion of the observance of the jubilee of the martyrdom of St. Peter and St. Paul, notoriously solemnized in Rome A.D. 1867, that it was enacted too early by one year. For, in the first place, the Father of Church History, Eusebius, insists that both Apostles died during the 13th year of Nero's reign, and this 13th year extended from Oct. 13, A.D. 67 to the same day A.D. 68, and not, as Petavius, in consequence of insufficient chronological resources and erroneous conclusions, stated, from A.D. 66 to 67. (See p. 437.) This result is mathematically confirmed by the solar eclipse A.D. 67 (No. 40), and by the epochs of all Olympian games mentioned in Roman and Greek histories (p. 449). Petavius erroneously referred the usual celebration of the Olympian games to A.D. 65, accordingly their postponed celebration to A.D 67, and then the nonsense comes out that πέρυσι (the preceding year) signified two years earlier. As, then, both Apostles died in the "13th year of Nero, June 29, they must have been martyred A.D. 68, and not A.D. 67. Hieronymus, it is true, twice refers the same martyrdom to the following, the 14th year of Nero, A.D. 69, and not, as Eusebius does, to his era 2083, but 2084 after Abraham. Yet the statements of Eusebius and Hieronymus do not contradict each other; for Hieronymus commences the same era, as every historian knows, one year earlier than Eusebius does, and hence his 14th year of Nero was the same that Eusebius calls the 13th; in the second place, Clemens Romanus, who lived at the same time in Rome, testifies (Ad Corinth. i. 5) that both Apostles were put to death in that year in which Nero attended the postponed Olympian games, A.D. 68. This testimony is confirmed, furthermore, by the explicit reports in the "Martyrologium Pauli" (A.D. 396); for it relates that the Apostles suffered death on the "iii. Kal. Jul." (June 29th) "in the 69th year after Christ's birth." Christ, as we have seen, was born seven days previous to the year 0, the first of the original Christian era, two weeks prior to Herod's eclipse of the moon (No. 31, p. 454), observed in the same year 0; consequently the Apostles were put to death A.D. 68, this year being the 69th after Christ's birth. Further, the same Martyrologium adds that the martyrdom of the Apostles occurred "in the 36th year after the crucifixion of the Lord," which, as we have seen, took place A.D. 33, Mar. 19th, being the 14th day of Nisan (p. 414); therefore the Apostles died A.D. 68, this year being the 36th after the crucifixion. This result, finally, is positively confirmed by the eclipse A.D. 67, May 31, observed in the course of the consulate of Telesinus A.D. 67, because the Apostles died during the following consulate, the 13th year of Nero, and in A.D. 66 no solar eclipse was possible. These historical and astronomical certainties put it beyond any question that the jubilee of St. Paul's and St. Peter's martyrdom ought to have been solemnized A.D. 1868, and not in 1867. Perhaps, however, the next jubilee will take place A.D. 1968.

41 & 42. Dio Cass. (lxv. 8, p. 180; lxv. 11, p. 184 St.) and Zonaras (An. xi. 16, p. 574 D.) relate that, coss. Sulp. Galba and T. Vin Rufinus, two total eclipses of the moon (αἱματώδης καὶ μέλαινη) occurred, of which one happened "on Oct. 29th." These are the eclipses in A.D. 68, May 6, 12h., ☊ 2° E. (—3° 25′), and Oct. 29, 6h. 30m. a.m., ☋ 2° W. (—3° 25′); for in A.D. 70, to which the said consuls belong, no lunar eclipse at all occurred, and in A.D. 69, to which Petavius referred, both lunar eclipses were partial, and neither coincided with Oct. 29th. The consuls of A.D. 68, Capito and Rufus, have been confounded with the consuls A.D. 70, Galba and Rufinus, because Rufus and Rufinus were very similar names.

43 & 44. Pliny (H. N. ii. 13 & 10) narrates, "ut quindecim diebus utrumque sidus quereretur, Vespasianis patre et filio consulibus." The rare phenomenon of two great eclipses within fifteen days occurred, about that time, only A.D. 71, Mar. 4, 8h., ☋ 8° E., obscuration 4¼ inches; and March 19, 21h. 30m., ☊ 7° W., curve 16°, 39°, 66°. Since the longitude of the nodes was then 3° 25′ shorter, both eclipses were great ones, as Pliny says. The words "Vespasianis patre et filio consulibus" obviously mean that year in which Emperor Vespasian was, for the first time, associated with Titus in exercising the consulate. Accordingly, these consuls must of necessity be referred to A.D. 71, and not, as Petavius calculated, to A.D. 70; for Pliny was an eye-witness. Since these eclipses were irreconcilable with Petavius's chronology, according to which the consuls Vespasian II. and Titus I. ruled A.D. 70, Petavius would have us to read, "Vespasiano III. filio iterum consulibus"; and hence our philologers have been so kind as to transform Pliny's genuine words, as they read in old manuscripts, into "Vespasiano III., filio iterum consulibus"; or even into, "Vespasiano IV. filio iterum consulibus. But Vespasian cos. III. was the colleague of Nerva, and not of Titus. The other so-called emendation of Pliny, "Vespasiano IV. Tito II. coss." is the worst of all, because these consuls ruled, according to Petavius himself, A.D. 72, during which year Pliny's eclipses were quite impossible. These two eclipses incontrovertibly demonstrate that all consuls and emperors from Claudius to Titus reigned one year later than hitherto has been believed; hence Jerusalem was destroyed A.D. 71, and not, as Petavius conjectured, A.D. 70.

Since the consuls Commodus Verus and Novius Priscus, as we have seen (p. 423), were A.D. 78 extraordinarii, and not ordinarii, to whom Petavius erroneously attributed an entire year, and since Vespasian reigned not ten but nine years only, it is natural to conclude that Titus commenced to reign in the same year (A.D. 80) which Petavius assigned him, and that all consuls and all Roman eclipses after Titus belong to the very same years during which the latter were observed, according to Petavius's Doctrina Temporum.

Roman Eclipses from Titus to Constantinus Magnus.

45. ⊙ +73, July 22, 22h., Bœotia. Plutarch (De facie in orbe lunæ, c. 13; vol. ix. p. 680 Reis) asserts that he really witnessed a total eclipse of the sun in Chæronea (38° 30′ N. Lat., 20° 46′ Long.), which commenced with noon—(*ταύτης ἔναγχος τῆς συνόδου, μνεσθέντες, ἣ πολλὰ μὲν ἄστρα πολλαχόδεν τοῦ οὐρανοῦ διέφηνεν, εὐθὺς ἐκ μεσημβρίας ἀρξαμένη κρᾶσιν δὲ, οἷον τὸ λυκαυγὲς, τῷ ἀέρι παρέσχεν.*) The computation will be found further on, Greek Eclipses, No. 26.

46. ⊙ +95, May 21st, 15h. 30m., ☋ 5° E., Ephesus (38° N., 25° 15′ E.), curve 16°, 47°, 50°. Philostratus (V. A. viii. 23, p. 365) narrates as follows: *τὸν τοῦ ἡλίου κύκλον περιελθὼν στέφανος, ἐοικὼς Ἴριδι, τὴν ἀκτῖνα ἠμαύρου,*—which phenomenon happened in the 14th year of Domitian, A.D. 95. Eusebius (Chr. ii. 203, ad Ol. 218, 4) mentions only *διοσημεῖαι πολλαί.* Lambert and Struyk took the phenomenon for an eclipse of the sun in Ephesus, A.D. 95, May 21, 15h. 30m., ☋ 5° E., curve 16°, 47°, 50°, obscuration 1 inch. The report, however, is very doubtful, and it probably means an iris around the sun; otherwise the eclipse agrees with our Table (p. 429).

47. ⊙ +98, March 21st, 3h, ☋ 10° E., Rome, curve 71°–73°, Europe. Aurel. Victor (Ep. xii. 12) reports that "eo die, quo (Nerva) obiit (Jan. 25th), solis defectio facta est." Since Nerva died on Jan. 25, and no eclipse, about that time, occurred on that day, it is evident that Aurelius Victor referred the death of Nerva to a wrong date. According to Petavius and his adherents, Nerva died A.D. 98, and hence they recurred to the aforesaid eclipse. It is, however, still a question whether Nerva died A.D. 98 or 99; for Petavius himself (Doct. Tem. xi. 20, p. 182) concedes that the year 98 is in conflict with Domitian's coins, and that some authors put the death of Domitian in 97 A.D. Since, then, Nerva died 16 months after Domitian, Nerva's death belongs to A.D. 99; and, in this case, the eclipse under consideration was that of A.D. 99, Sept. 2d, 22h. (+2h. 9m.) P. T., ☊ 2° 20′ E., curve 11°, 0°, ✱. According to our Table (p. 429), the ☊ lay 1° west of the sun, and hence the eclipse was plainly visible in Rome.

48. ⊙ +118, Sept. 2, 22h. 30m., ☊ 6° W., Rome, curve 53°, 42°, 14°. The Chronographer of A.D. 354 (Anonymus Norisii), published by the R. Saxon Society of Science, Leips., 1850, p. 660, relates that "his consulibus" (Hadrian II. and Tib. Claud. Tusc. Salinator A.D. 118) "sol eclipsin passus est." The longitude of the ☊ was (p. 430) shorter by 3° 15′, and the conjunction happened later, about 2 p.m.

49. ⊙ +200, Mar. 31, 21h. 30m., ☊ 4° E., Utica (Carthage), curve ✱, — 15°, 7°. Tertullian (Apol. ad Scop. c. 3, p. 70) narrates: "nam et sol illo in conventu Uticensi extincto pæne lumine adeo portentum fuit, ut non potuerit ex ordinario deliquio hoc pati, potius in suo hypsomate et domicilio," etc. The Council of Utica took place in the course of the eight-year reign of Emperor Severus, A.D. 200; for which reason Petavius recurred to the aforesaid eclipse, although it was invisible in 36° 51′ N. Lat. According to our Table (p. 429), however, the ☊ lay only 1° E. of the sun. The eclipse in +199, Oct. 7th, 5h. 30m., happened after sunset, and that of A.D. 201, March 21st, 12h., was visible only in Eastern Asia.

50. ⊙ +219, April 1st, 20h., ☊ 3° W., Rome, curve 1°, 26°, 48°. The eye-witness Dio Cassius (78, 30, p. 769) reports that U.C. 971, A.D. 219, *ἡλίου ἔκλειψις περιφανεστάτη ὑπὸ τὰς ἡμέρας ἐκείνας ἐγένετο.* This small eclipse, however, was nearly a total one in Rome, because the ☊ lay about 6° W. of the sun, and the conjunction happened nearly two hours later (p. 429).

51. ⊙ +237, April 12, 3h. 30m., ☊ 2° W., Rome, curve 44°–48°. Julius Capitolinus (Gord. iii. 23; vol. ii. p. 13, ed. Lugd.) bears witness that, within the first year of Gordian (A.D. 237), a total eclipse of the sun was observed ("eclipsis solis facta est, ut nox crederetur, neque sine luminibus accensis quidquam agi [legi?] posset.") Scaliger. Petavius, and Calvisius, computed the aforementioned eclipse, but the obscuration of the sun in Rome amounted, on the southern part of the sun, to 10 inches only. Since the ☊, however, lay (p. 429) nearly 4° 38′ west of the sun, the eclipse must have been total, or nearly total, in Rome. This is a clear confirmation of the Table, p. 429. Struyk (Ruperti's Magaz. i. 353) maintains that, two years later, a similar eclipse occurred, which, according to ancient reports, happened a short time after the Olympian games, A.D, 239. Hence the latter eclipse would have been that of A.D. 239, Aug. 16, 2h., ☋ 12°, or rather 9° 20′ E., which eclipse was a partial one.

52. ☉ +291, May 15th, 2h. 30m. ☊ 0° W., Rome, curve 30°–24°. Idatius (Scaliger's Thesaur. p. 30) reports that in the 7th year of Diocletian, coss. Tiberianus and Cassius Dio, A.D. 291, "tenebræ fuerunt inter diem." According to Petavius, the obscuration of the sun amounted to 8 inches south. The longitude of ☊ being shorter by 2° 39′, the obscuration was greater in Rome.

53. ☽ +303, Sept. 11th, 7h. 30m., ☊ 5° E., Rome. Scaliger (Emend. temp., Proleg. xviii. ed. Col. 1629) cites a Martyrologium, according to which Bishop Felix suffered martyrdom in the 19th year of Diocletian, which was, according to Eusebius (Chron. ad 2319), A.D. 303; and in the following night a total eclipse of the moon happened (et ductus est ad passionis locum, cum etiam ipsa luna in sanguinem conversa est, die iii. Kal. Septembres). Calvisius had reference to the eclipse A.D. 304, Aug. 31, 9h., ☊ 3° W. (—2° 37′), which was not total, but nearly coincided with iii. Kal. Sept. Scaliger alluded to the eclipse A.D. 301, Nov. 3d, 2h. 45m. a.m., ☊ 9° W.; but the moon scarcely touched the shadow of the earth, and iii. Kal. Sept. is not Nov. 3d. The eclipse A.D. 303, Sept. 11, 7h. 30m., ☊ 5° E., amounted, according to the present theory of the moon, to 11¾ inches only, and hence the lunar orb could not "assume the hue of blood"; but, according to our Table (p. 429), the ☊ stood distant from the centre of the shadow of the earth by two degrees only. Accordingly, this eclipse was total indeed. For the rest, instead of "iii. Kal. Sept.," read iii. Id. Sept.

54. ☉ +316, Dec. 30th, 19h. 30m., ☊ 2° W., Constantinople, curve 13°, —2°, 25°. Aurel. Victor (Cæs. xli. 1) reports that a short time after Diocletian's death, which happened A.D. 316, Dec. 3d, a partial eclipse of the sun occurred in Constantinople (quod—defectu solis fœdato iisdem mensibus die patefactum est.) Calvisius had reference to the eclipse A.D. 316, July 5th, 17h., ☋ 0° E., curve 20°–36°, 35°, 27°; obscuration in Constantinople 5 inches. This eclipse, however, did not "follow," but preceded, Diocletian's death, and, since the ☋ lay (p. 429) 2° 36′ west of the sun, no solar eclipse was visible in Constantinople. On occasion of the aforesaid eclipse, A.D. 316, Dec. 30, the corrected place of the ☊ was 4° 26′ W., and the obscuration of the sun, visible in Constantinople, was very great.

55. ☉ +317, Dec. 20th, 1h., ☊ 11° W., Constantinople, curve Eur., Afr. Idatius (in Scaliger's Thes. p. 30, ed. Roncall. p. 10) reports that, coss. Val. Licinian. Licinus Aug. V. and Fl. Jul. Crispus Cæsar, A.D. 317, a solar eclipse occurred three hours prior to sunset in Constantinople (tenebræ fuerunt hora nona). Petavius referred this eclipse to A.D. 317, Dec. 20th, 1h. ☊ 11° west, obscuration 7 inches. Indeed, this eclipse happened in Constantinople three hours past noon (p. 429), but the obscuration was greater, the longitude of the ☊ being shorter by about 2° 36′. This result agrees with Idatius's words, "tenebræ fuerunt," and confirms our Table, p. 429.

56. ☉ +324, August 6th, 2h., ☊ 4° W., Campania, curve 40–20°, Eur., Afr. Cedrenus (p. 285 Par.) recounts that, coss. Crispus and Constantinus III.., A.D. 324, a total eclipse of the sun occurred in the afternoon (*ἡλίου ἔκλειψις τοιαύτη, ὡς ἀστέρας φανῆναι ἐν ἡμέρᾳ*). Calvisius, agreeably to the present lunar theory, stated this eclipse to have been partial, for the reason that only seven inches of the southern limb of the sun were covered. According to our Table, p. 429, however, the longitude of the ☊ was shorter by about 2° 36′, and hence the obscuration of the sun must have been a total one in Campania about 4 o'clock p.m.

57. ☉ +334, July 16th, 23h. 30m., ☋ 2° E., Rome(?), curve 42°, 40°, 5°. Firmicus (Astron. i. 2) informs us that "sol medio diei tempore—cuncta mortalibus fulgida splendoris sui denegat lumina, quod Optati et Paulini consulatu—mathematicorum sagax prædixit intentio." Petavius computed the eclipse A.D. 334, July 16th, 45m. past noon, Roman time, obscuration 11 inches. Since the longitude of the ☋ was shorter by 2° 35′, the eclipse was smaller in Rome, and total in Africa only. The eclipse in the preceding year, July 27. 21h. P. T., or, according to the Table on p. 429, 23h. 22m. in Rome, likewise took place about noon; but, the ☋ being 11°, i.e. about 8° E., the obscuration was great only in Northern Italy.

58. ⊙ +346, June 5th, 17h. 30m., ☊ 7° W., Constantinople, curve 30°, 65°, 64°. Theophanes (p. 31 ed. Goar) reports that, within the 10th year of Constantius (A.D. 346), on the 6th day of Dæsius (June 6th), and within the 3d hour of the day, a total, or nearly total, eclipse of the sun took place in Constantinople (*τῷ δ'αὐτῷ ἔτει· ἔκλειψις ἡλίου ἐγένετο, ὥστε καὶ ἀστέρας φανῆναι ἐν τῷ οὐρανῷ, ἐν ὥρᾳ γ′ τῆς ἡμέρας, μηνὶ Δαισίου ϛ′*). The same we read in Cedrenus. Eusebius and Hieronymus (Chr. ii. p. 183) refer the same eclipse to the same 10th year of Constantius. The position of Constantinople presumed to be 31° 10′ Long. and 41° N. Lat., the sun rose there on June 5th about 4h. 40m.; consequently the 3d hour of the day commenced about 6h. 26m. local time, and, in consequence of the parallax, the eclipse began nearly two hours earlier. According to our Table (p. 429), however, the conjunction was nearly 1h. 34m. later, which agrees with Theophanes and Cedrenus. Even Petavius found that, according to his Lunar Tables, the eclipse happened one hour too early. This eclipse, moreover, was partial in Constantinople. (See No. 59.)

59. ⊙ +347, Oct. 20th, 3h., ☋ 14° E., Constantinople. Theophanes reports that in the course of the 11th year of Constantius, "on a Monday," a partial eclipse of the sun occurred (Theoph. p. 32 ed. Goar: *ὁ ἥλιος πάλιν αὐχμηρότερος γέγονεν ἐν ὥρᾳ β′ τῆς κυριακῆς ἡμέρας*). The obscuration amounted, as Petavius calculated, to 7 inches; but, according to our Table (p. 429), the longitude of ☋ was shorter by 2° 33′, and hence the obscuration was greater. But this eclipse happened on a Tuesday, and not on a Sunday. The only eclipse coinciding about that time with a Sunday was that of A.D. 345, June 16th, 1h., ☊ 1° E., or rather (p. 429) 1° 34′ W., which must have been a nearly total one in Constantinople; for the central shadow of the moon traversed at noon, as Pingré states, the 16th degree of north latitude, but, according to our correction, the former really touched the latitude of Constantinople. Hence it is evident that three consecutive eclipses, observed in Constantinople, have been confounded with each other: the first was that of A.D. 345, June 16th, 1h., on a Sunday; the following happened A.D. 346, June 6th, two hours after sunrise, on a Friday; and the third, A.D. 347, Oct. 20th, 3h. p.m., on a Tuesday. The first was the total one witnessed by Theophanes and Cedrenus.

60. ⊙ +348, Oct. 8, 20h., ☋ 5° 17′ E., Constantinople, curve 52°, 23°, 1–2°. Hieronymus, as Petavius found in some manuscripts, mentions an eclipse (solis facta defectio) observed in the 12th year of Constantius, and Ol. 282, 1. The latter points to A.D. 347 (No. 59), the former to A.D. 348. The same eclipse Cassiodor (p. 220, ed. Rom.) refers to the consuls Fl. Philippus and Fl. Salius (his coss. solis facta defectio), and to the 12th year of Constantius. The obscuration amounted, according to Petavius, to 8 inches in Constantinople; but it must have been smaller, the ☋ lying 2° 33′ nearer to the sun.

61. ⊙ +360, Aug. 27th, 16h., ☊ 3° W., Mesopotamia, curve 34–37°, 26°, 21°. Amm. Marcellinus (xx. 3, p. 203 Wagn.) says: "eodem tempore per Eoos tractus cœlum subtectum caligine cernebatur obscura, et a primo auroræ exortu ad usque meridiem intermitabant jugiter stellæ—primo attenuatum in lunæ corniculantis effigiem, deinde in speciem semestrem, postea in integrum restitutum." This eclipse Ammian refers to the coss. Constantius X. and Julianus III., A.D. 360. According to Petavius, 11 inches of the sun's disc were covered in Mesopotamia, but, according to a careful computation of the same eclipse, performed by means of Damoiseau's Tables and Airy's corrections of the latter (Zech's Preischriften über die wichtigeren Finsternisse der Griechen und Römer, 1855), the eclipse was over prior to sunrise in Mesopotamia. Both contradictions are removed by our Table, p. 429, for the ☊ lay 2° 32′ more west of the sun, and the conjunction happened nearly 1h. 32m. later. The report that the eclipse lasted six hours, means that the total shadow of the moon traversed different places of Asia successively. (See p. 448.)

62. ⊙ +364, June 16, 1h., ☊ 6° W., Alexandria, Egypt; curve *, 60° (noon), 38°. Theon (Can. L vi. p. 277 & 282 ed. Bas., p. 161 Hal.) testifies that on the 23d day of Thoth (June 16), Ær.

Dioclet. 80 (A.D. 364), a solar eclipse occurred (γενομένης συνόδου—ἐκλιπτικῆς τυγχανούσης). The longitude of the ☊ was shorter by 2° 32′, and the obscuration commenced about 1h. 32m. later.

63. ☽ +364, Nov. 25th, 14h., ☋ 6° E., Alexandria. Theon (Can. vi. p. 90. 162) records that on the 6th day of Phamenoth (Nov. 25th), Ær. Dioclet. 81 (A.D. 364), a "total eclipse of the moon" was there seen. According to our Table, p. 429, the ☊ lay 2° 32′ nearer to the centre of the earth's shadow, and hence that "total" eclipse was a total one indeed.

64. ☉ +374, Nov. 19, 22h, 30m., ☊ 2° W., Alexandria, curve 7°, —9°, *. Theon (Can. vi. p. 74 Hal.) reports that a solar eclipse happened in Alexandria during Phamenoth, Ær. Diocl. 90, and in the 3d hour of the day (p.m.) Theon's eclipse, however, was invisible in Alexandria, provided the ☊ lay not farther from the sun by about 2° 32′ (p. 429). The eclipse commenced 1h. 32m. later (p. 429), which agrees with Theon.

65. ☉+378, Sept. 7th, 23h. 30m. ☊ 2° W., Alexandria, curve 25°, 19°, *. Theon (Can. vi. p. 74 H.) mentions a solar eclipse belonging to the year 94, Ær. Diocl. Since the ☊ lay farther from the sun by 2° 32′, the obscuration was greater in Alexandria (31° 13′ N.)

66. ☉ +393, Nov. 19, 23h. (+1h. 29m.), ☊ 10° W. (—2° 29′) Rome or Constantinople, curve 53°, 40°, 37–45°. Zosimus (Hist. iv. 58, 3) narrates that coss. Theodosius III. and Abundantius, A.D. 393, during the battle of Theodosius against Eugenius, a great eclipse of the sun occurred (ἡλίου ἔκλειψιν ἐν αὐτῷ τῷ καίρῳ τῆς μάχης συνέβη γενέσθαι τοιαύτην ὥστε νύκτα εἶναι μᾶλλον ἐπὶ πλείονα νομίζεσθαι χρόνον). Marcellin (Scal. p. 36; ed. Ronc. 271) says: "tunc quippe hora diei tertia tenebræ factæ sunt." The same is reported by Prosper Aquit. Chron. i. 672. The obscuration in Rome amounted, according to Petavius, to 10 inches on the northern side of the solar disc. Since the ☊, however, lay nearly 2° 29′ farther from the sun, the eclipse was smaller in Rome. The "third" hour probably means the third hour after noon, because the Romans were in the habit, in later times, of counting the hours both from midnight and from noon. (Comp. No. 64.) Hieronymus refers, perhaps, the same eclipse to Pentecost (June), and such a one occurred A.D. 392, June 6th, 18h , ☋ 1° E., curve 3°, 28°, 29°; which was also a small one because the ☋ lay 1° W. of the sun. (Comp. Calvisius's Opus Chron.)

The Actual History of the Greeks.

The history of the Greeks and the chronology of their eclipses depending, of course, upon the Olympiads, it is to be borne in mind that the Olympian games were celebrated two years later than hitherto was universally believed, namely, in all years B.C. which, being divided by 4, gives the remainder 1, and in all years of the Christian era which, being divided by 4, leaves the remainder 3. The proofs have been discussed in the premises (p. 437.) For the planetary configuration referring to the celebration of the first Olympian games, as well as all those mentioned in Roman history, place this statement beyond question, as the following 18 examples clearly show:

1. —777, March 29, Plan. Conf. Schol. Pind. Ol. v. 10; Pausan. v. 14.
2. —205, Coss. Cl. Nero and Salinator II. Livy, 27, 34.
3. — 41, Subsequent to Julius Cæsar's death. Cic. Epis. ad Att. 16, 7; 15, 5; 24.
4. — 37, Coss. Calvinus and Pollio. Joseph. Ant. xiv. 14,4; xv. 10, 1; B. J. i. 19, 13. (See p. 433.)
5. — 29, Coss. Ahenobarbus and Sosius, 13th year of Augustus. Plut. Ant. p. 942.
6. — 9, Coss. Messalla and Appianus, 33d year of Augustus. Joseph. B. J. i. 21, 8. 12.
7. + 19, Coss. Rufus and Flaccus, 3d year of Tiberius. Cramer An. p. 151.
8. + 43, Coss. Caligula IV. and Saturninus, 1st year of Claudius. Malala x. 320.
9. + 59, Coss. Nero III. & Messalla. Philostr. V. A. iv. 24, 17. 18. 34.
10. + 67, Coss. Telesinus & Paulinus, 12th year of Nero. Pausan. x. 36, 4.
11. + 95, Coss. Domitian XVII. and Clemens, 15th year of Domitian. Philostr. viii. 14-18.
12. +127, in the 8th year of Hadrian. Pausan. Perieg. v. 21, 6; Ol. 226, 1.
13. +163, in the 3d year of Aurelius and Verus. Pausan. x. 24, 2; Ol. 135, 1.
14. +167, death of Peregrinus. Amm. xxix. 1, 39; Gell. N. A. viii. 3; Hieron. A. A. 2181.
15. +211, Æræ Ant. 260. Malala xii. 372.
16. +239, u.c. 991; Æræ Act. 267; Æræ Jul. 283; Censorin c. 5.
17. +327, 331, 335, 363. Liban. iii. 123, 110.
18. +395, interdicted in the 16th year of Theodosius. Cedren. p. 325 C.

Accordingly, the following epochs of Grecian history are incontrovertibly fixed in advance:

1. —777, the first year of Archon Æshylus. Euseb. Chron. ii. 318.
2. —477, battle near Thermopylæ. Her. vii. 206; Arch. Calliades. Par. Marb. Ep. 52; Her. viii. 51.
3. —425, the 4th year of Peloponn. war, Arch. Diotimus. Thuc. iii. 8.
4. —417, the 12th year of Pelopon. war, Arch. Astyphilus. Thuc. v. 40.
5. —405, the 24th year of Pelop. war, Arch. Antigenes. Xen. H. i. 3, 1.
6. —401, the 28th yr. of Pelop. war, Arch. Pythodor II. Xen. H. ii. 3, 1.
7. —361, the 2d year of the Arcadian war, Arch. Arimnestus; battle near Olympia. Xen. H. vii. 4, 29.
8. —353, Alexander the Great born, Arch. Elpines. Plut. Al. 3.
9. —345, the 13th year of Philippus. Æsh. F. L. p. 29. Arch. Theophilus.
10. —321, Arch. Hegesias; Alexander the Great dies in the following year. Arr. vii. 28, 1.

All these epochs refer to the years in which the Olympian games were actually held, and by means of them it will be an easy matter to determine the true dates of all the eclipses mentioned in the history of the Greeks.

The following Chronological Table summarily shows the difference between Petavius's Greek history and that of the author, and, at the same time, it includes the true dates of all Greek eclipses. We add the epochs of the Persian kings, fixed by classic authorities. In the next place, the Olympian games confirm the results (p. 409). that the Peloponnesian war lasted, from the first naval expedition of the Athenians to the destruction of the Piræus, 28 full years, as Thucydides and Xenophon testify, and not, as Petavius "*post ingentem laborem*" made out, 27 years only; that, moreover, the history of the 21st year of the Peloponnesian war and the first chapters of Xenophon's Hellenica have been lost. For, the Olympian games being held in the 4th and 12th years of the Peloponnesian war (Thuc. iii. 8; v. 49), i.e. —425 and —409, do not agree with the Olympian games celebrated during the 23d and 27th years of the same war (Xen. Hell. ii. 3, 1, and i. 2, 1), i.e., according to Petavius, in —406 and —402, because the games would have been repeated after an interval of 3 years. Besides, the years —406 and —402 disagree with the epochs of all Olympian games mentioned in Roman history (p. 451). The aforesaid loss of one year of the Peloponnesian war is, moreover, placed beyond question by the Parian Marble, which, for the same period, counts one archon more than Petavius did; further, by the 28 ephori who ruled during the Peloponnesian war (Xen. Hell. ii. 3, 10, and Thuc. viii. 6), by the history of the kings of Bosporus (Diod. xii. 31, 36), by the ages of Sophocles, Plato, Socrates, Isocrates, and others, who, the missing year between Thucydides and Xenophon not being expleted, would have lived one year less than history reports. Add to this the celebration of the Isthmia, and Pythia, mentioned in ancient history. The Pythia, repeated, like Olympian games, every four years, were celebrated during the autumn in —419, the 10th year of the Peloponnesian war, Arch. Alcæus (Thuc. v. 1); consequently in a year which, being divided by 4, gives the remainder 3. Since the same Pythia were held (Xen. Hell. iv. 13, 14; v. 2, 29, etc.) in —391, —379, —371, —367, —327, —287, it is evident that one year must be inserted between Thucydides and Xenophon. Furthermore, Thucydides (viii. 10) tells us that the Isthmia æstiva were celebrated during the 20th year of the Peloponnesian war, i.e. 409, a year which, being divided by 4, gives the remainder 1. Since the same Isthmia took place in —389, Arch. Philocles, as Xenophon (Hel. iv. 5, 1) testifies, Petavius must have omitted one year intervening between Thucydides and Xenophon. This is confirmed by the same Isthmia celebrated in —385 (Xen. Hel. v. 1, 29), and those in —193, Coss. Purpurio and Marcellus ruling from the Idus Mart. 194 to the same in 193 (Livy 33, 32), as well as by the Isthmia hiberna A.D. 68, during the 12th year of Nero (Sueton. Nero 24; Philost. V. A. v. 41). The simple logical deduction, therefore, is that Petavius has antedated all events of Greek history down to Xenophon by one year; the following, however, by two years.

Olymp.	Seyf.	Petav.	Historical Events and Eclipses.
0	—777	—779	The first Olympian games. Pind. Ol. v. 10, x. 59; Paus. v. 14.
1, 1	773	775	The 2d year of Archon Æshylus.
38, 4	622	623	July 2. The 6th year of the Medo-Lydian war. Herod. i. 74.
	621	622	(1) ⊙ T. on the Halys, May 17, 20h. 15m. Herod. i. 74, 103.
48, 4	582	583	July 2. Archon Damasias.

Olymp.	Seyf.	Petav.	Historical Events and Eclipses.
	—581	—582	(2) ⊙ T., Miletus, March 27th, 17h. 45m. Pliny ii. 12, 9; u.c. 170, Ol. 48, 4.
59, 4	538	539	(3) ⊙ Nov. 22d, 19h. Greece. Fasti Sic. p. 144, ad Ol. 59, 4.
61, 1	533	534	July 2. Daniel (ix. 24) predicts the birth of Christ in —1.
	532	533	(4) ⊙ T., Nineveh (Mosul), Jan. 26, 15h. 45m. Xen. An. iii. 4, 7.
	526	527	Cyrus dies, Cambyses reigns, 7 yrs. after Nineveh's fall.
65, 1	517	518	July 2. Darius Hystaspes reigns. Par. Marb. Ep. 44.
	516	517	(5) ⊙ Mar. 28th, 2h. 45m. Fast. Sic. p. 146, ad Ol. [65, 1.
74, 3	479	480	July 2. Archon Themistocles.
	478	479	(6) ⊙ T., Sardes, Feb. 27, 15h. 30m. Her. vii. 3, 7; viii. 51; Arist. Or. 46, p. 241 D., and his Schol. p. 232 From.
75, 1	477	479	Olympian games during the battle at Thermopylæ. Her. vii. 206.
75, 2	476	477	(7) ⊙ Corinth, Aug. 1, 1h. 30m. Herod. ix. 10.
76, 4	470	471	July 2. Archon Chares. Par. Marb. Ep. 55.
	469	470	(8) ⊙ T., Thebes. Bœot., Mar. 20, 1h. 30m. Pindar, in Dion. Hal. p. 167 Syl.
78, 4	463	464	July 2. Archon Lysistratus.
			(9) ⊙ T., Athens, —465, Dec. 25th, 20h. Fasti Sic. Ol. 78, 4; Pliny H. N ii. 22.
79, 4	458	459	July 2. Archon Phrasiclides.
			(10) ⊙, Greece, —460, March 9, 23h. 30m. Euseb. Arm. ad Ol. 79, 4.
86, 4	430	431	July 2. Archon Pythodor. I. The Pelopon. war begins, according to Xenophon.
	429	430	(11) ⊙, μηνοειδής, Athens, Jan. 26th, 22h. Thuc. ii. 28; Cic. R. P. i. 16.
88, 1	425	427	The Olympian games celebrated. Thuc. iii. 8.
88, 4	422	423	July 2. Archon Stratocles.
			Artaxerxes Long. obit in χειμών. Xerxes II. & Sogdian reign. Thuc. iv. 50.
89, 1	421	422	(12) ☽, Athens, Aug. 8, 15h. Schol. Aristoph. Nubes v. 580.
			Darius Nothus reigns. Thuc. viii. 58. Arch. Isarchus.
	420	421	(13) ⊙, small in Athens, Jan. 18th, 2h. Aristoph. Nub. 580, and Scholiast.
			(14) ☽ T., Feb. 2, 6h., Athens. Aristoph. Nub. 580.
90, 1	417	419	The Olympian games celebrated. Thuc. v. 49.
91, 4	410	411	July 2. Archon Callias. Diodor. xiii. 34.
			(15) ☽ T., Sicily, July 8, 7h. 45m. Thuc. vii. 50.
92, 1	409	411	Isthmia celebrated in θέρος. Thuc. viii. 10.
			The 13th year of Darius Nothus. Thucyd. viii. 58. (See the year —421)
92, 2	408	409	July 2. Archon Glaucippus. (See p. 411.)
93, 2	404	406	July 2. Archon Callias II. Diod. xiii. 103; Xenoph. Hel. i. 6, 1.
	403	405	(16) ☽, Athens, Feb. 23, 6h. 30m. Xen. Hel. i. 6, 1.
93, 4	402	404	July 2. Archon Pythador II., during the last (28th) year of the Peloponnesian war.

Olymp.	Seyf.	Petav.	Historical Events and Eclipses.
	401	403	(17) ☉, Athens, Jan. 17, 21h. 30m. Xen. ii. 3. 4.
			Darius Nothus obit. Diod. xii. 104. Artaxerxes Mnemon reigns.
			June—The Olympian games celebrated, Xen. ii. 3, 1.
96. 2	392	394	July 2. Archon Eubulides.
	391	393	(18) ☉ (μηνοειδής), Bœotia, Jan. 26, 22h. 30m. Xen. H. iv. 3, 10.
104, 1	361	363	July 2. Archon. Timocrates.
			(19) ☉ T., Thebes, Bœotia, May 12, 3h. 15m. Plut. Pel. 31, p. 389.
105, 1	357	359	July 2. Archon Callimedes.
	356	358	(20) ☉, Syracuse, Feb. 28th, 23h. 15m. Plut. Dion, c. 19, p. 286 R.
105, 2	356	358	July 2. Archon Agathocles.
			(21) ☽ T., Sicily, Aug. 9th, 6h. 45m. Plut. Nic. 22; Dion. 24.
111, 4	330	332	July 2. Archon. Nicocrates.
			(22) ☽ T., Sept. 10th, 7h. 30m., referred to Arbela. Ptolemy Geog. i. 4.
112, 2	328	330	July 2. Archon Aristophanes. The last year of Darius Codomannus, succeeded by Alexander the Great.
			(23) ☽ P., near Arbela, Aug. 29, 12 h. Cic. Div. i. 55; Arr. iii. 7, 6.
116, 4	312	314	Archon Nicodorus. (See p. 412.)
117, 3	307	309	July 2. Archon Hieromnemon.
	306	308	(24) ☉ T., near Syracuse, June 13, 21h. 25m. Justin. xxii. 6; Diod. xx. 5.
162, 4	126	128	(25) ☽, Athens, Oct. 14th, 13h. 30m. Diog. Laërt. iv. 9, 64.
212, 3	+73	+ 70	(26) ☉ T., Chæronea, Bœotia, July 22d, 22h. Plut. vol. ix. p. 680 R.

It is true, the succession of the archons in general is less reliable than the epochs of the Olympian, Isthmian, and Pythian games mentioned in history. For, e.g. the scholiast of Aristophanes Aves, 997, places Apseudes before Pythador I. in —430; the scholiast of Æshines (p. 15 St., p. 740 R.) makes Nicephorus the predecessor of Themistocles; the Parian Marble counts, from Diphilus to Pythador I. one archon more than Diodorus does; the latter puts all archons mentioned by Xenophon later by one year, whilst the Parian Marble, in accordance with Xenophon, puts the archons Antigenes, Micon, and Laches, one year earlier; the same Diodor puts Apseudes before Pythador I., and he (xiii. 7) inserts between Chabrias and Cleocritus, in —411, Pisander, an archon not elsewhere mentioned; Pausanias names archon Charon four years prior to Diodor's date, and the latter postdates archon Phœnippus and the battle at Marathon by one year; Diodor, comparing his archons with the Roman consuls, was compelled to repeat five consular magistracies twice in order to harmonize his Greek history with the Roman. Besides this, in the present editions of Xenophon (Hell. i. 2, 1) the Olympian games are referred to the year —406, instead of —405, which is obviously a repeated blunder of an ancient copier of Xenophon. Furthermore, Thucydides calls the year preceding the first expedition of the Athenians against Sparta, namely, the year in which the Spartans destroyed Potidæa, the first year of the Peloponnesian war. This is evident from the fact that Thucydides (ii. 56) makes Pericles to have been a participant in the naval expeditions of the Athenians first "in the 2d year of the Peloponnesian war," and this year is astronomically fixed by the nearly total eclipse No. 11 of the Table. Xenophon, on the contrary, counted the years of the Peloponnesian war from that January during which the Attic fleet first started against the Peloponnesus. Hence it came to pass that both the last year of Thucydides and the first (now lost) year of Xenophon were originally termed the 23d year of the war. Nevertheless, none of these incongruities, as we shall see, affect at all the aforesaid dates of the eclipses observed during the Peloponnesian war. The inscriptions (p. 411) demonstrate that, in general, all archons down to —409 ruled one year (the following, two years) later.

Examination of the Eclipses of the Peloponnesian War.

11. The eye-witness Thucydides (ii. 28) reports that 3 yrs. and 5 mos. prior to the Olympian games in —425 (p. 471), consequently in the year —429, in the beginning of *θέρος*, therefore during January of —429, at noon (*μετὰ μεσημβρίαν*), a nearly total (*μενοειδής*) eclipse of the sun happened in Athens. Ὁ ἥλιος (says he) *ἐξέλιπε μετὰ μεσημβρίαν, καὶ πάλιν ἀναπληρώθη γενόμενος μηνοειδής, καὶ ἀστέρων τινῶν ἐκφανέντων.* Cicero (R. P. i. 16): adds, "cum tota se luna sub orbem solis subjecisset"; and Quinctilian (In. Or. i. 10. 47), "cum Pericles Athenienses solis obscuratione territos, redditis ejus causis, metu liberavit." Plutarch (Per. c. 35, p. 661 R.) narrates the same. The words *μετὰ* μεσημβρίαν signify, analagously to μεθ' ἡμέραν (during day), during noon. About that time, as Pingré's computations show, only one eclipse was possible in the early spring, and about noon, viz. that in —429, Jan. 26, 22h, P. T., ☊ 1° E., curve —15°, —30°, —3°. According to our Table, p. 429, however, the longitude of the ☊ was shorter by about 5° 35′, and hence the obscuration amounted to about 11 inches in Athens. The conjunction took place nearly 3h. 37m. later, which agrees with Thucydides. Petavius, of course, had reference to the eclipse in —430, Aug. 3d, 5h. local time, obscuration 11.20 inches; but, unfortunately, this eclipse belonged to χειμών (p. 418), and not, as Thucydides testifies, to the early θέρος. Moreover, this eclipse was too late in the afternoon, and it happened, according to Petavius's chronology, one year too late; for Petavius referred the Olympian games to —427, and since the said eclipse, as reported by Thucydides, occurred 3 yrs. 5 mos. prior to the Olympian games, Petavius ought to have recurred to a similar eclipse in —431, during which year no similar eclipse was possible. Finally, it is well known that Buerg's Lunar Tables were, in spite of the Almagest, based upon 3200 Greenwich observations, and that the former, on occasion of the total eclipse in 1851, proved more correct than Burckhardt's and Damoiseau's Tables. By the aid of Buerg's Tables, Prof. Heiss (Ueber die Finsternisse des Peloponnesischen Kriegs, Köln, 1834) computed the same eclipse, but the obscuration of the sun was only 7.9 inches. In this case, as Thucydides testifies, nobody would have seen fixed stars.

12. Thucydides (iv. 2.) narrates that, in the course of the 9th year of the Peloponnesian war, the tanner Cleon was extraordinarily elected strategus; and the scholiast of Aristophanes (Nub. 581) says that about that time, Arch. Stratocles, a lunar eclipse occurred in August (ἐπιδὴ ἔκλειψις σελήνης τῷ προτέρῳ ἔτει ἐπὶ Στρατοκλέους Βοηδρομιῶνι, i.e. August; p. 23). This is the eclipse in —421, Aug. 8, 0h. 15m., ☋ 10° E., which confirms our Table, p. 429; for the longitude of the moon was 1° 57′ shorter (3h. 35m. later), and without this correction the said full moon would not have been visible in Athens. The obscuration of the moon must have coincided with sunset. Petavius recurred to the eclipse in —424, Oct. 23d, 6h. 30m., which clearly is contradicted by the consecution of the archons and the years of the

Peloponnesian war, as the two following eclipses evidence. The later scholiast perhaps viewed the eclipse during the rule of Arch. Isarchus, in —420.

13 & 14. The eye-witness Aristophanes (Nubes, 581) testifies that in the early spring of the following year, and a short time previous to Cleon's first orderly election as strategus, a very small eclipse of the sun and a total one of the moon were perceived in Athens. *Εἶτα* (says Aristophanes) *τὸν Παφλαγόνα ἡνίχ' ᾑρεῖσθαι στρατηγόν, τὰς ὀφρῦς ξυνήγογεν κἀποιοῦμεν δεινά—ἡ σελήνη δ' ἐχλέλοιπε τὰς ὁδοὺς* (Herod. vii. 37), *ὁ δ' ἥλιος, τὴν ϑριαλλίδ' εἰς ἑαυτὸν εὐθέως ξυνελκύσας, οὐ φανεῖν ἔφασκεν ὑμῖν, εἰ στρατηγήσει Κλεων· ἀλλ' ὁμῶς εἵλεσθαι τοῦτον.* The scholiast in Scaliger's "Synage" informs us that the same solar eclipse took place on the 16th day of Anthesterion (*Ἀνθεστηριῶνος ἕκτῃ ἐπὶ δέκα*), that is, as we have seen (p. 408), on Jan. 18th. About that time there is but one year to be found within which a solar and lunar eclipse occurred during spring, at which time the strategi were elected; and it was only in —420, Jan. 18, 2h., ☋ 17° E., that a small eclipse of the sun happened on the 18th day of January. The total eclipse of the moon took place on Feb. 2d, 6h. p.m., ☊ 2° E. (—5° 32′). These two eclipses, then, mathematically demonstrate that Arch. Isarchus ruled during *ϑέρος* in —420, and his predecessor Stratocles in —421; that, moreover, Thucydides' reports (iv. 119) concern the year —420. It is strange, however, that Thucydides (iv. 52) refers the same solar eclipse to the preceding year, to —421, in which neither solar nor lunar eclipses were visible in Athens during the spring; for the solar eclipse on Feb. 26th, 23h., ☋ 5° W., described the curve —56°, —44°, —5°, and hence it was invisible in Athens. The lunar eclipse in —421, Feb. 12th, 22h. 30m. coincided with noon. It is, therefore, probable that Thucydides (iv. 52: *τοῦ ἐπιγιγνομένου ϑέρους εὐθὺς τοῦ ἡλίου ἐχλιπής τι ἐγένετο*), whilst writing his work twenty years later, confounded with each other, concerning the eclipse, the 9th and 10th years of the Peloponnesian war. Besides, the ascertained solar eclipse in —420, Jan. 18th, confirms our Table, p. 429; for the ☋ lay not 17°, but 12° only east of the sun, and without this correction no eclipse would have been possible at all. Petavius, as was natural, had recourse to the eclipse in —423, March 20th, 20h., ☋ 10° E., which contradicts the eye-witness

Aristophanes, because in —423, about Mar. 20, no lunar eclipse was possible. Moreover, supposing the Athenians had elected their strategi after March 20th, the warlike exploits of the Greeks would have commenced prior to the orderly elections of the strategi; for Thucydides and Xenophon narrate that usually all belligerent expeditions set out in January.

15. Thucydides (vii. 50, ἡ σελήνη ἐκλείπει), Plutarch (Nic. 33, p. 393 R.), Diodorus (xiii. 12, p. 551 S.), Polybius (Exc. ix. 19, τῆς σελήνης ἐκλιπούσης δεισιδαιμονήσας ὥς τι δεινὸν προσημαινούσης ἐπέσχε τὴν ἀναζυγήν), report that at the end of θέρος a total elipse of the moon, soon after sunset, happened in Sicily, which caused the ruin of the Attic army in Sicily, viz. during the 20th year of the Peloponnesian war, Archon Callias, two years prior to the archonship of Glaucippus in —408, which year is mathematically fixed by the calendrical inscription, p. 411. Thucydides specifies 21 days from the eclipse to the capture of the army (see Clinton's F. H. to this event), and the latter Plutarch (Nic. 33, p. 393 R.) refers to "the 27th day of the Spartan month Carneius," consequently to the 29th of Metageitnion, the 31st day of July, Julian style (p. 408). Since, then, the eclipse happened 21 days prior to July 31, the same must have taken place on July 8th in —410, soon, as Thucydides says, after sunset. Indeed, in —410 only, July 8th, 7h. 45m., a total eclipse of the moon happened soon after sunset. No lunar eclipse coinciding twice, during a period of 19 years, with July 8th, the epoch of this eclipse is fixed with mathematical certainty. The obscuration of the moon, however, amounted, according to the present lunar theory, to 6.5 inches only, because the ☋ lay 7° E. of the centre of the earth's shadow; but, according to our Table, p. 429, the longitude of the ☋ was shorter by 5° 29′, and hence this eclipse was, as the authors report, a total one after sunset. Petavius, according to his erratic chronology, recurred to the eclipse in —412, Aug. 27, 10h. 15m., ☋ 4° W.; yet this eclipse belonged to χειμών, and it did not precede, but followed, the fall of Nicias, on July 31st, by 27 days.

16. Xenophon (Hell. i. 6, 1) reports that during the 26th year of the Peloponnesian war, Archon Callias, namely, a short time after the beginning of θέρος, and soon after sunset (ἡ σελήνη ἐξέλιπεν ἑσπέρας), a lunar eclipse occurred in Athens. This is apparently the eclipse in −403, Feb. 23d, 6h. 30m. ☋ 9° E., i.e. 4° E. (p. 429). Agreeably to the present lunar theory, 3¼ inches only were obscured. Petavius computed, as was to be expected, the eclipse in −405, April 15th, 10h. local time; but this eclipse is irreconcilable with Xenophon, who refers that eclipse to the first and not to the last months of ϑέρος, and, especially, to evening (ἑσπέρα). (Comp. p. 445.) The present editions of Xenophon put this eclipse in the 25th year of the Peloponnesian war, beginning, as Xenophon affirms, with the first expedition of the Athenians against Sparta in −429 (p. 471); but the particular passage (παραληλυϑότος ἤδη τοῦ χρόνου καὶ τῷ πολέμῳ τεττάρων), according to the Petavian chronology, contains an alteration of the original text, perpetrated by some ancient transcriber or modern editor.

17. Xenophon (Hell. ii. 3, 4) attests that in the spring, during ϑέρος of the last (28th) year of the Peloponnesian war, Archon Pythodor II., an eclipse of the sun occurred in Athens (κατὰ τοῦτον τὸν καιρὸν περὶ ἡλίου ἔκλειψιν). This is the eclipse in −401, Jan. 17th, 21h. 30m., ☋ 10° E. (−5° 26′), curve 37°, 32°, 60°, which commenced 3h. 31m. later (p. 429). Petavius mistook Xenophon's eclipse for that in −403, Sept. 2d, 21h. 30m., ☊ 6° W., curve 57°, 38°, 2°; but, unhappily for him, this eclipse belonged to χειμών, and not to ϑέρος. The same eclipse, moreover, confirms Thucydides and Xenophon, who unanimously bear witness that the Peloponnesian war lasted fully 28 years; for, from the eclipse in −429 (No. 11) to this eclipse in −401, the last year of the Peloponnesian war, 28 years really transpired. Petavius, on the contrary, referring the first eclipse of the war to −430 and the last to −403, made out, of course, that the war lasted only 27 years, and that "bonus Xenophon erravit."

18. Xenophon (iv. 3, 10) narrates that within ϑέρος of the 1st year of the Corinthian war, Arch. Eubulides (−392 to 391), a great eclipse of the sun (μηνοειδής) was seen on the northern bounds of Bœotia (38° 40′ N. Lat.) The same we read in Plutarch (Ages. 17, vol. viii. p. 654 R) On the occasion of the ecliptic new moon in −391, Jan. 26th, 22h. 30m., ☊ 9° W., the shadow of the moon touched only 33°, 23°, 47°. Since, however, the longitude of the ☊ was shorter by 5° 24′ (p. 429), the obscuration must have been very great in Bœotia. Petavius, of course, computed the two-years earlier eclipse in —393, Aug. 13th, 23h., ☊ 2° east, curve 24°, 29°, 0°; but, alas! this eclipse happened in χειμών and not in θέρος, and it was, moreover, too small (nine inches according to La Hire's Tables) and not μηνοειδής.

19. Xenophon (Hell. vii. 4, 29–32) reports that the battle near Olympia was fought during both the Olympian games and the archonship of Timocrates, accordingly in the month of June of —361. Nearly 10 months after this battle, consequently in —360, during the spring, Pelopidas died in the city of Thebes, Bœotia, as Plutarch (Pel. 31, p. 389 R.) narrates, whilst a great eclipse of the sun took place (σκότος ἐν ἡμέρᾳ τὴν πόλιν ἔσχεν). The same we read in Diodor (xv. 81, p. 65 W.) This is obviously the eclipse in —360, May 12th, 3h. 15m., ☊ 1° W., curve 2°, 28°, 21°; but the obscuration was very small in Bœotia (38° 20′ N., 21° 5′ E.), according to the prevalent lunar theory. Thus, our correction (p. 429), according to which the ☊ lay nearly 6° west of the sun, and the conjunction happened 3h. 25m. later, is confirmed. In —351 no solar eclipse was visible in Greece. Petavius, who put the Olympian games two years earlier, and the archons of this time earlier by three years, recurred to the eclipse in —363, July 12, 22h. 15m., ☊ 6° W., curve 41°, 54°, 17°; but the obscuration of the sun amounted to 4 inches only, and it is contradicted by Plutarch, by the epochs of the Olympian games, and by the succession of the archons.

20. Plutarch (Dion 19, p. 286 R.) narrates that during Plato's third sojourn in Sicily (Ol. 105, 3), a remarkable eclipse of the sun, predicted by Helicon, occurred in Syracuse (37° 2′ N., 12° 56° E.) Petavius, commencing the Olympiad 105, 3, with July in —357, computed the eclipse in —356, Feb. 28, 23h. 13m., ☊ 4° W., curve 11°, 26°, 41°; but the obscuration amounted in Sicily to 4 inches only. Other authors refer the same eclipse to Ol. 104, 3, commencing with July 2d in —361, and in this case Helicon's eclipse would have been the same mentioned by Xenophon (No. 19). But Petavius correctly demonstrates that Plato's third visit to Sicily belongs to —357. According to our Table, p. 429, the longitude of the ☊ was shorter by 5° 14′ in —356, Feb. 28th, 23h. 30m. P. T., to the effect that the sun was probably totally obscured in Syracuse.

21. Diodor (xvi. 9), Plutarch (Nic. 23, p. 394 R., *καθ' ὃν χρόνον ἤμελλεν ἄρας ἐκ Ζακύνθου πλεῖν ἐπὶ Διονύσιον ἐκλιπούσης τῆς σελήνης. οὐδὲν διαταραχθεὶς ἀνήχθη*), Plutarch (Dion. 24, *μετὰ τὰς σπονδὰς καὶ τὰς νενομισμένας κατευχὰς ἐξέλιπεν ἡ σελήνη*), and Quinctilian (In. or. i. 10, 48, "Dion cum ad destruendam Dionysii tyrannidem venit, not est tali casu deterritus), report that in Ol. 105, 4, in the course of the archonship of Agathocles, a total eclipse of the moon happened in Sicily, soon after sunset. This eclipse must have been total, because it is paralleled with that of Nicias (No. 15) and called a "terrible" one. About that time only one lunar eclipse was possible soon after sunset, viz. that in —356, Aug, 9, 6h. 45m. P. T., ☋ 10° E., obscuration 2½ inches (Pingré), or 4 inches (Calvisius). Since, however, the longitude of the ☋ was nearly 5° 14′ (p. 429) shorter, this eclipse was total indeed. Diodor refers the eclipse to the same year, because Ol. 105, 4, commenced, according to his Olympiads, with July 2d in —356. Even Calvisius recurred to the same eclipse, because all the following eclipses disagreed with the ancient reports. Agathocles ruled two years later; but in —354 no similar eclipse occurred, as Pingré demonstrates.

22 & 23. The ancient authors erroneously refer two different eclipses of the moon to the same battle near Arbela, for they refer those eclipses to different Greek months and hours. Cicero and Arian, who are the most reliable authorities, place the battle in —328, Sept. 10, and the eclipse preceding the latter in Aug. 29, 12h. P. T.; for Cicero (Div. i. 53) says, "si luna paullo ante solis ortum defecisset et in signo Leonis, fore ut armis Darius et Persæ, prælio vincerentur." Arrian (Exp. Al. iii. 7, 6, & 15, 7) reports that the battle, 11 days after the eclipse, was fought both in the month of Pyanepsion, i.e. in September (p. 408), and during the archonship of Aristophanes; moreover, that the eclipse was a partial one (*τῆς σελήνης τὸ πολὺ ἐκλιπὲς ἐγένετο*). About that time only one lunar eclipse coincided with sunrise near Arbela (41° 40′ E.), viz. that in —328, Aug. 29, 12h., i.e. 3h. 44m. after midnight, Arbela time, and, according to our Table (p. 429–30), about three hours later, 6h. 50m. local time. In consequence of the parallax, the obscuration became visible at Arbela nearly two hours earlier. Since Plutarch (Al. 31), however, reports that the Persian army, 11 days after the eclipse, came in sight of Alexander, and since he may have marched eastwardly, during these 11 days, 40 geographical miles, the difference of time is to be diminished by about 3 hours. The ☊ lay 9°, according to our Table nearly (p. 429) 14° W. This eclipse agrees with the reporters. First, it was indeed a small one; it occurred a short time before sunrise; the sun stood then among the eastern stars of Leo, near Virgo; the eclipse happened during Boëdromion (Aug., p. 408), as Arrian teaches. Further, since the archons of this time ruled, as we have seen, two years later, Aristophanes belongs to —328. Finally, a few months prior to the battle near Arbela, Alexandria was founded, and this event Solinus (32 & 42) refers to the consuls Luc. Pap. Cursor and C. Pœtilius in —328, and to Ol. 112, [1,] that is again to —328 (p. 432).

The two-years earlier eclipse in —330, Sept. 20th, erroneously referred to the battle at Arbela, is mentioned by the following authors: Pliny (H. N. ii. 70=72) says, "nobili apud Arbelam Magni Alexandri victoria luna defecisse noctis secunda hora prodita est, eaque in Sicilia oriens." Ptolemy (Geogr. i. 4) puts the same eclipse in the 5th hour (ἐν Ἀρβήλοις πέμπτης ὥρας φανῆναι, ἐν δὲ Καρχηδόνι δευτέρας). Plutarch (Alex. 31) refers this eclipse to Boëdromion, and to the beginning of the *mysteria* in Athens (περὶ τὴν τῶν μυστερίων τῶν Ἀθήνησιν ἀρχήν). Curtius (Hist. Alex. iv. 10) reports: "prima fere vigilia luna deficiens primum nitorem sideris sui condidit, deinde sanguinis colore suffusum lumen omne fœdavit." Plutarch, referring the eclipse to the beginning of the mysteria celebrated according to lunar months, it is evident that Boëdromion means the lunar month which corresponded with September, because in the course of the preceding year a lunar month had been intercalated. This total eclipse of the moon, then, belongs to —330, Sept. 20th, 7h. 30m., ☊ 4° E.; according to Petavius, 5h. 47m., or 6h. 31m. The sun then rising in Arbela 3h. 40m. P. T., this eclipse agrees with our Table, p. 429–30: for the opposition took place 3h. 19m. later; consequently the eclipse was perceived a short time prior to midnight in Arbela, which agrees with Ptolemy. Besides it is easily explained how it came to pass that the later authors antedated by two years the battle near Arbela; for in later times, as we have seen, the practice of counting the Olympiads from 775, instead of 773, prevailed.

24. Justinus (xxii. 6) and Diodor (xx. 5, p. 409 S.) relate that during θέρος of Ol. 117, 3, consequently in the summer of —306, the 7th year of King Agathocles of Sicily, whilst Hieromnemon was archon in Athens, a total eclipse of the sun occurred between Syracuse and Carthage. This eclipse happening one day after the fleet left Syracuse, the locality and the time of this really total eclipse of the sun are sufficiently fixed (τῇ δ' ὑστεραίᾳ τηλικαύτην ἔκλιψιν ἡλίου συνέβη γενέσθαι, ὥστε ὁλοχερῶς φανῆναι νύκτα, θεωρουμένων τῶν ἀστέρων πανταχοῦ). Petavius recurred, of course, to the eclipse in —309, Aug. 14th, 20h. 15m., ☊ 4° W., curve 42°, 35°, —4°, obscuration 10 inches; but, alas! this eclipse occurred in χειμών and not in θέρος, and the archons of this time ruled two, even three, years later than Petavius believed. About that time, viz. in θέρος, only one total eclipse of the sun was possible near Syracuse, i.e. that in —306, June 13, 22h, 45m., ☊ 0° 43′ E., curve 0°, 21°, —3°. According to our Table, p. 429, the ☊ lay 4° W. of the sun, and hence the obscuration of the sun was total near Syracuse. The calendrical inscription (p. 412) referring to Archon Nicodorus mathematically demonstrates that the archons of this time ruled two, even three, years later than Petavius made out. Compare No. 19, p. 478, and the eclipses, discussed further on, referring to —197 and —196 (Babylonian eclipses Nos. 11, 12, 13).

25. Diogenes Laërt. (iv. 9, 64) reports that, according to Apollodor (Ol. 162, 4), the death of Carneades was followed by an eclipse of the moon (ἔκλιψις σελήνης) in Athens. On occasion of the ecliptic full moon in —126, Oct. 14, 13h. 30m., ☊ 9° W., which the Olympiads point to, 6 inches were obscured. This eclipse, however, being too small, and Apollonius living in later times, we may presume the Olympiads to have been counted from —775, and in this case Apollonius and Diogenes Laërtius would have had in view the two-years earlier eclipse in 128, Nov. 5th, 13h. 30m., ☊ 7° E., which was, according to our Table (p. 429,) a total one, because the longitude of the ☊ was 4° 12′ shorter.

26. Plutarch (De fac. i. o. l. chap. 13, vol. ix., p. 680 R.; see the passage p. 461), being born A.D. 45 (Clinton F. R. p. 85) in Chæronea, Bœotia, became when nearly 20 years old a pupil of Ammonius in Athens (Plutarch De εἰ, p. 385), and returned, several years after, to Chæronea (38° 30′ N.), where he had the good fortune to see a total eclipse of the sun, of which an exact description is to be found in the before-mentioned passage (p. 461). "During the eclipse," he says, "which I lately observed, many stars in all directions of the sky became visible, and while it (the eclipse) commenced exactly at noon (ἐκ μεσημβρίας ἀρξαμένη), the air assumed a hue like that of twilight." Really total eclipses of the sun, it is well known, return to the same places of our globe only after centuries, and it happens very seldom that solar eclipses commence with noon; wherefore Plutarch, during his life-time, could not have seen in Bœotia two such obscurations of the sun as he describes. Pingré's computations of ancient eclipses show that, about that time, only the following eclipses coincided with noon in Bœotia whilst Plutarch lived there:

A.D. 71, March 19, 21h. 30m., ☊ 8° W., curve 16°, 39°, 66°.
" 73, July 22, 22h., ☋ 4° E., curve 63–64°, 61°, 24°.
" 75, Jan. 5, 1h. 30m., ☊ 6° W., curve 16–42°.
" 76, May 21, noon, ☋ 12° E., curve Northern Europe & Northern Asia.
" 78, April 29, 22th. 30m, ☋ 2° W., Southern India.
" 81, Feb, 27, noon, ☋ 2° E., curve ✱, 20°, S.W. Asia.

None of these eclipses could have been really total in Bœotia, or other regions of Greece except that in A.D. 73, July 22, 22h. P.T., Plutarch at that time being aged 26 years. Prof. Hind ("Nature," New York, July 25th, 1872) computed, by means of Hansen's Tables, all the eclipses visible during the last half of the first century of our era and during the first part of the second century, but none of them corresponded with Plutarch; and this fact alone will suffice to convince every astronomer that the present theory of the moon's motions is incorrect. According to our Table, p. 429, the ☋ lay, A.D. 73, July 22, about 3° 24′ nearer to the sun, and hence the central shadow of the moon traversed, about noon, nearly the 38th degree of N. Lat., and not, as Pingré found, the 61st degree. The conjunction happened 2h. 12m. later, which agrees with Plutarch, who testifies that the eclipse "commenced at noon." All these 26 Greek eclipses confirm the result, obtained by the Roman eclipses, that the longitudes of the moon and her Nodes were, in earlier times, shorter than our Lunar Tables, based upon the Almagest, induce.

Some earlier Solar and Lunar Eclipses of the Greeks.

2. This eclipse, predicted to the Milesians by Thales, and referred to sunrise by Herodotus (i. 74), has already been alluded to (p. 440). It is not the same which Herodotus refers to the battle on the Halys; for the latter, likewise predicted by Thales, happened several hours later, as we now shall see.

1 & 4. The dates of these two eclipses depend on the stages of Cyrus's life. Herodotus (iii. 27) reports that Cambyses, the son of Cyrus, conquered Egypt in the course of the 5th year of his reign, and that in the following year a new Apis period of 25 years commenced. These periods began, as we have seen (p. 405), together with the Canicular periods, in —2780, —1320, and A.D. 140; and the renewals of Apis periods occurred, subsequent to —1320, in all years which, being divided by 25, give the remainder 20, e.g. in —520, —495, —320, and so on. Since, then, an Apis period recommenced in the 6th year after Cyrus's death, viz. in — 520, it is apparent that Cyrus must have died in —526, and not, as Ptolemy's Historical Canon erroneously presumed, in —528. Even Eusebius refers the death of Cyrus to Ol. 62, 3, that is, to —526. This result is confirmed by Daniel, Cyrus's contemporary, and by the "turnus" of the Hebrew priests down to the birth of John the Baptist and that of Christ. Xenophon (Cyr. vii. 4, 16) bears witness that Cyrus, subsequent to the capture of Babylon, reigned nine years, and Herodotus (viii. 7) reports that Cyrus, seven years prior to his death, destroyed Nineveh and the Median supremacy in Asia, in consequence of which Cyrus permitted the Hebrews to return to Palestine and to rebuild the temple. Daniel (ix. 25) says: "Know therefore and understand, that from the going forth of the commandment to restore and to build Jerusalem unto Messiah the Prince, shall be seven weeks and threescore and two weeks," etc. Likewise, Daniel reckons 33 years from the birth of Christ to the crucifixion. The seventy weeks of Daniel have been explained in the author's Chronologia Sacra (p. 107, 112), and in "Gettysburg Review," 1861, p. 341. Daniel, in one word, reckons 532 years from the destruction of Nineveh to Christ's birth, which happened, as we have seen (p. 454), seven days prior to the beginning of the year 0, the first of the original Dionysian era. Moreover, the same year —532 results from the "turnus" of the 24 classes of the

Hebrew priests, as has been demonstrated in the author's Chronologia Sacra, 1846, p. 97. John the Baptist having been born six months before Christ, and on the longest day (June 24), had been announced to Zacharias, a priest of the 8th class (course), viz. that of Abia, whilst the same was in the temple about September 20th, in the year — 2. Now, the Hebrews, having returned to Jerusalem, inaugurated the new altar on the day of the autumnal equinox (Ezra iii. 8), and during this week the first of the reorganized 24 classes of the priests had to serve the sanctuary. Hence, an easy computation establishes the fact that from the inauguration of the new altar down to the annunciation of the Baptist neither more nor less than 1151 turnus and 7 weeks transpired. Consequently the end of the Babylonian captivity, and the destruction of Nineveh, with which the reign of Darius Medus alias Cyaxares II. expired, and the monarchy of Cyrus commenced, belongs to — 532; and this year was, as we have seen, the seventh prior to Cyrus's death in — 526. To this very year, then, the solar eclipse belongs which preceded the conquest of Nineveh, called Laryssa (Heb. *rasas*, the ruins), the present Mosul (36° 31′ N., 43° 30′ E.), as Xenophon (Anab. iii. 4, 7) testifies. He reports that the king of Persia (Cyrus), when taking the supremacy from the Medians, besieged Nineveh (Laryssa) for a long time, but in vain, till, one day, the sun disappeared (*ἥλιον νεφέλη προσκαλύψουσα ἠφάνισκε μέχρι ἐξέλιπον οἱ ἄνθρωποι καὶ οὕτως ἑάλω*). This is, then, the eclipse in — 532, Jan. 26th, 15h. 45m. P. T., ☋ 20° E.; curve, touched by the shadow of the moon, 34°, 36°, 64°. According to our Table (p. 429), the longitude of the ☋ was shorter by nearly 6° 5′, and hence the eclipse must have been a large one in Nineveh. The conjunction happened, according to our Table, nearly one hour before noon, local time. On this occasion it comes to light that Layard referred the destruction of Nineveh too early by 74 years. Prof. Airy referred Xenophon's eclipse to — 558, May 19, 2h. 15m. Paris time; but in this case the Babylonian captivity would have lasted 47 years only, and not, as the Hebrew chroniclers testify, 70 full years.

We come now to the famous total eclipse (No. 4) observed on the Halys in the course of the battle between the Medians and Lydians. Herodotus (i. 74) reports that in the sixth year of the war between the Medians and Lydians, during the battle on the southern Halys near Lydia, that is, about 39° N. Lat., 36° E., a total eclipse of the sun occurred (συνεστεώσης τῆς μάχης τὴν ἡμέραν ἐξαπίνης νύκτα γενέσθαι). In consequence of this unexpected phenomenon the battle immediately ceased, and the kings Cyaxares and Alyattes resolved to intermarry their adult children. From this marriage Mandane, the mother of Cyrus, originated in the following year. In another place Herodotus (i. 103) repeats that this eclipse was a total one, and that it took place in the course of the battle (ὅτε νύξ ἡ ἡμέρη ἐγένετο σφι μαχομένοισι), and that Thales had predicted it. The same is reported by Clemens Alex. (Strom. i. 130, 5), Cicero (De div. i. 50), Themistius (Orat. xxvi., p. 317 Dind.), even in the Shanameh, as Hammer (Wiener Jahrbücher ix. p. 13) vouches. This eclipse has been very often confounded with that mentioned by Pliny, Eudemus, Eusebius, Hieronymus, and referred by the same authors to Ol. 48, 4, and u.c. 170, i.e. to —581 (No. 2), because both eclipses had been predicted by Thales. Oltmanns, however, correctly distinguished two eclipses mentioned by Herodotus, and he referred the older one, that on the Halys, to —609, Sept. 30; but this eclipse is inconsistent with history, as we shall see directly. The date of the eclipse on the Halys is fixed by the following data:—Cyrus died, as we have seen, in —526, six years prior to the renewal of the Apis period in —520, viz.. as Cicero (De div. i. 33) avers, "70 years old"; consequently Cyrus was born in —596. Further, he destroyed Nineveh, as Xenophon (Cyrop. viii. 7, 1), the seventy weeks of Daniel, and the "turnus" of the Hebrew priests, corroborate, seven years prior to his death, i.e. in —532; and two years earlier, i.e. nine years prior to his departure (Cyrop. vii. 4, 16), he took Babylon, and at that time, as Daniel (vi. 1), Cyrus's contemporary, testifies, he was 62 years old. Consequently Cyrus was really born in —596. Now, Herodotus (i. 107) narrates that Mandane, the daughter of Astyages, the son of Alyattes, at the time of marrying the father of Cyrus, was a marriageable virgin (ἐοῦσα ἤδη ἀνδρὸς ὡραίη); and in that heroic age, mirrored in the monuments of Nineveh, being at present 2470 years old, no girl could be called a marriageable virgin before reaching her 20th year. Accordingly, the eclipse during the battle on the Halys must have taken place twenty or more years prior to Cyrus's birth in —596, that is, about the year

—621. About that time only one total eclipse was possible near the southern Halys, viz. that in –621, May 17th, 20h. 15m., ☊ 2° 46′ E., curve –25°, 2°, –6°. According to our Table (p. 429), however, the ☊ lay 4° W. of the sun, and hence the obscuration of the sun near the 39° N. Lat. must have been total. The eclipse happened on the Halys, as Pingré states, about noon, but, according to our Table (p. 429–30), 4h. 14m. later, which agrees with Herodotus, who reports the eclipse to have taken place in the course of the battle (*σφι μαχομενοῖσι*). Oltmanns, it is true, had reference to the eclipse in –609, Sept. 29th, 21h. P. T., ☊ 4° W., curve +, 55°, 22°; but this eclipse was not total on the Halys, and the battle-field would have been nearly the middle of the Black Sea. Moreover, according to this eclipse, Mandane would have been born in –608, and, since Cyrus was born in –596, Mandane, aged eleven years, would have been married to Cyrus's father. Who is able to believe that, 2470 years ago, girls of 11 years were "marriageable virgins"? Prof. Hind, however, as well as Prof. Airy, took the eclipse in –583, May 17th, 20h., for that on the Halys witnessed by Herodotus; but this eclipse was total only between Sardes, Iconium, Tarsus, Issus, Ancyra, and not on the Halys. By the way, since Mandane, according to this eclipse, was born in –582, whilst Cyrus was born in –596, the wonderful discovery is made that Cyrus was born fourteen years prior to his mother.

3. The Fasti Siculi (Chronicon Pashale, p. 144 Par.) refer a solar eclipse, probably observed in Greece, to Ol. 59, 4 (*ἡλίου ἔκλειψις ἐγένετο*). In the year —538, Nov. 22d, 19h., ☊ 8° W., curve 12°, –26°, –25°, the longitude of the ☊ was shorter by 6° 6′ (p. 429); consequently the obscuration of the sun must have been great in Greece. Two years earlier no solar eclipse occurred.

5. The same Fasti (p.146) refer a solar eclipse (*ἔκλειψις ἡλίου ἐγένετο*) to Ol. 65, 1. In —519, Nov. 22d, 17h, ☊ 7° W., curve 52°, 17°, 16°; the conjunction happened 3h. 54m. later, and the longitude of the ☊ was shorter by about 6° (p. 429), and hence this obscuration of the sun in Greece must have been great. Two years earlier no eclipse of the sun occurred in Greece. Hence it is evident that the Fasti Siculi counted the Olympiads from –773 correctly.

6. We proceed now to the famous total eclipse of Xerxes, observed near Sardes (Smyrna), about sunrise, in the early spring. Herodotus (vii. 37), who was born about the same time, reports as follows: *ὁ ἥλιος ἐκλιπὼν τῆν ἐκ τοῦ οὐρανοῦ ἕδραν ἀφανὴς ἦν, οὐτ' ἐπινεφελέων ἐόντων, αἰθρίης τε τὰ μάλιστα· Ἀντὶ ἡμέρας τε νὺξ ἐγένετο.* Aristides (Or. 46, p. 241 Din.) calls the same eclipse a total one (*ἡ τοῦ ἡλίου συμπάσα ἔκλιψις*), and the scholiast to Aristides (p. 222 Fr.) likewise refers the obscuration to sunrise (*ἐξ ἀνατολῆς*), and to the vicinity of the Hellespont. Herodotus (viii 51), the Parian Marble (Ep. 52), Dionysius (ix. 17, 38), and Diodor (xi. 1) put the eclipse a few months prior to Archon Calliades (June, 478), and Thucydides (i. 18) counts ten years from the battle at Marathon, of which the date (—488, Aug. 6) is fixed with mathematical certainty (p. 408–410) down to the eclipse of Xerxes. Moreover, a short time after this eclipse the battle at Salamis (according to Plutarch) on the 16th day of Munychion (March 19) was fought in —477; and from Xerxes' eclipse to the Olympian games in —477, during which the battle at Thermopylæ took place, Herodotus (vii. 206) counts about 18 months. Even Plutarch (Ages. ii. 1) and Nepos (Ages. 4) also specify 1 year 6 months from Xerxes' passage over the Hellespont to the Olympian games. Thus the date of the eclipse near Smyrna is both mathematically and historically ascertained. About that time, during spring, only one eclipse coincided with sunrise in Smyrna (29° 26′ E., 38° 28′ N.), viz. that in —478, Feb. 27th, 15h. 30m. P. T., ☋ 17° E., curve touching 39°, 57°, 1°. According to our Table (p. 429), however, the conjunction occurred 3h. 46m. later, i.e. about 7h. 10m. a.m. Paris time, i.e. 9h. Smyrna time. The parallax makes the obscuration of the sun nearly two hours earlier (7h. local time), and about the same moment the sun rose on Feb. 27th in Smyrna. The ☋, moreover, lay (p. 429) 5° 49′ nearer to the sun, i.e. 12° E., and hence the eclipse must have been total near Smyrna. Finally, in the preceding and following years no other eclipse coincided, as Pingré's computations demonstrate, with sunrise. Petavius being unable to produce, about that time, a total eclipse of the sun coinciding with sunrise, did not hesitate, in spite of all ancient authorities, to declare this eclipse to have been "a supernatural phenomenon." Hind recurred to the eclipse in —477, Feb. 17th, 11h. 10m. a.m., modern historians maintained it to be improbable in the extreme that the Chinese recorded the names and years of their sovereigns and dynasties as early as 3332, or, at least, 2598 B.C. This argument, however, falls short; for the Egyptians have likewise recorded, from Menes (2780 B C.) down to the Roman emperors, the years of their kings and dynasties, and this chronology in general has been confirmed by a great number of planetary configurations. (See the author's "Astronomia Ægypt." and "Berichtigungen," p. 137.) Similar historical traditions, moreover, going back to the dispersion of the nations, to the deluge (3446 B.C.) and even to the antediluvian patriarchs, are to be found among other nations of antiquity, especially the Chaldeans, Parsees, Indians, Phœnicians, etc. Hence it is not at all improbable that the Chinese dynasties are in general as reliable as the Egyptian.

The second and more important objection is that the Chinese dynasties do not agree with the chronology in the present Hebrew text of the Old Testament. The question, however, is whether the present Masoretic Hebrew Testament, or else the Hebrew Bibles of the Israelites in Ethiopia, examined by Bishop Gobat in Jerusalem, which reckon 6,000 and not 4,000 years from the creation to Christ, and if the texts of the Septuagint interpreters who translated, about 280 B.C. the Hebrew Testaments obtained from Jerusalem, which manuscripts likewise once reckoned 6,000 years from Adam to Christ, contain the true chronology. This question has, from the Fathers of the Church down to this day, been ventilated by numberless savans, and they all, with few exceptions, arrived at the result that the Septuagint, apart from some clerical errors, has preserved the true chronology. Apart from Josephus, a faithful and orthodox Israelite, and all Fathers of the Church, we mention the following vindicators of the Septuagint:—Julian of Toledo (A.D. 680), P. Burgensis, Pagninus, Bredambachius, Porchetus, Hieronymus a S. Fide, Galatius, F. de Escatante, Leo a Castro, Huntæus, Alf. Samero, Gretserus, Dieghus, Peyva ab Andrada, Bellarminus, Baronius, Fr. Vatablus, Joh. Lorinus, Gilb. Genebrardus, Joh. Isaac, Adam a Conzen, Sim. de Muis, Joh. d'Espeires, Huetius, Phil. Quadagnolus, Calvinus, Drusius, Casaubonus, Junius, Am. Polanus, Mencerus, Andr. Rivetus, Chamierus, Sixt. Amama, Buxtorf, Hottinger, Pokok, Walton, Bochart, Perizonius, Fréret. Many other learned men of the same conviction will be found mentioned in Fabricii "Bibliotheca Antiquaria." Even P. Mailla, the author of the "Histoire de la Chine" (vol. i. p. clix.), wrote to Fréret, one of the most distinguished chronologers and members of the French Academy at that time, as follows: "Vous êtes pleinement convaincu, me dites vous, qu'il faut préférer la chronologie des Septante; il est en effait evident qu'aucun des quarante et plus sentiments des chronologists Hebraisants ne seroit s'accorder avec la chronologie des Chinois, sans parler de celle des autres peuples." The chronology of the Septuagint, moreover, has been verified by many new methods, especially by planetary configurations and other astronomical observations going back to — 1951, — 1578, — 3446, — 3725, — 5870. (See the author's "Summary of Recent Discoveries," etc., New York, 1857, pp. 114–60; "Die wahre Zeitrechnung des Alten Testaments," St. Louis, Mo. 1857, pp. 22–69.) Hence, seeing that the corrected Greek text of the Old Testament refers the deluge to — 3446, it would be preposterous to condemn Chinese history which commences with the year 3332, and refers the first emperor of the first dynasty to — 2598, whilst the Egyptians, according to numerous astronomical monuments, put their first king, Menes (Thinites), residing at This, i.e. Tanis (Heb. *Koan*), in — 2780, viz. in the 666th year after the Deluge, and in Peleg's days.

Now, the aforesaid Chinese history and chronology—which is, by the way, excepting a few ciphers corrupted by transcribers, the same in all Chinese annals—state that King Tchouen-Hio, the 2d regent of the 1st Chinese dynasty, reigned 78 years, namely, from — 2513 to — 2435. And in the Universal History of China we find a passage, translated by P. de Mailla (Histoire de la Chine, Paris, 1777, vol. i. p. cliv.), as follows: "*Imperator Tchouen-Hio fecit Calendarium, ut principio veris Luna esset prima. Hoc anno primæ Lunæ primâ die processerat ver. Quinque planetæ convenere in cœlo, transmissa constellatione* '*Che*' *(Aquarii).*" "Lorsque l'empereur Tchouen-Hio fit le Calendrier, il établit le commencement de l'anné au commencement du primtemps. Cette année le premier jour de la première Lune etait entrée dans le primtemps. Cinque planètes s'assemblèrent au ciel, on avait passé la constellation 'Che' (Aquarius)." The sun standing, in our age, on the day of the vernal equinox, near the middle of the constellation Pisces, the sun was, one month prior to the vernal equinox, as the Chinese say, in conjunction with the first stars of Pisces about the year —2460; and, indeed, in this very year the conjunction of five planets, referred to, actually took place, as the following approximate computation, according to Lalande's Tables, demonstrates:

Longitudes of the Planets in —2460, March 12, Julian style.

	Heliocentrically.	Geocentrically.
The Sun		$10^s\ 30^\circ = 11^s\ 0^\circ$
The Moon		10 30 = 11 0
Mercury	$5^s\ 27^\circ\ 33'$	10 24
Venus	11 27 28	10 29
Jupiter	11 16 13	10 24
[Mars	$7^s\ 26^\circ\ 42' = 9^s\ 1^\circ$]	
[Saturn	6 22 15 = 6 26]	

On the vernal equinoctial day in —2460, April 11th, 7h. 36m. Pekin time, the longitude of the sun was $11^s\ 29^\circ\ 23'$, and that of the moon $0^s\ 17^\circ\ 7'$; hence the crescent (luna prima) was visible on the vernal equinoctial day, which Thouen-Hio made the beginning of the year, after sunset, namely, 30 days subsequent to that conjunction of five planets.

Such a conjunction of the said planets, whilst the sun stood between the constellations Aquarius and Pisces, occurs but once within 164,000 years: for the sun stands again in conjunction with the moon, on the corresponding day of the year, first after 19 years, with Mercury not until after 26 years, with Venus not until after 4 years, with Jupiter not until after 83 years; consequently a similar conjunction of the same planets returns after $19 \times 26 \times 4 \times 83$ years. This observation, then, of the year —2460 proves that the history of China, according to which Tchuen-Hio, the 2d regent of the 1st Chinese dynasty reigned from —2513 to —2435, cannot be very wrong, because the planetary configuration referred by the Chinese annals to this king occurred in —2460 only.

The same annals report that Emperor Thong-khang, the 4th of the 2d dynasty, reigned from 2158 to 2145 B.C., and about that time a total eclipse of the sun occurred in Pekin, as the Chou-king, a compilation of the oldest religious books of the Chinese, testifies. This Chou-king contains the following passage, according to P. Gaubil's translation in "Histoire de l'Astronomie Chinoise," Par., 1732, vol. ii. p. 140: "Thong-khang venoit de monter sur le thrône. Au premier jour de la dernière lune d'Autumne le soleil et la lune dans leur conjunction n'étant pas d'accord" (i.e. they conflicted with each other) "dans Fang; l'Aveucle a frappé le tambour, les Mandarins sont montés à cheval, et le peuple s'accourut," etc. The first day of autumn was the autumnal equinoctial day, as we have seen; for Tchouen-Hio ordered the spring and the year to commence with the vernal equinox, thirty days after the aforesaid planetary configuration. The date of the eclipse under consideration is confirmed by the Chou-king itself, because the latter reports that on that day the sun and the moon stood in "Fong," which is, as Gaubil states, the second star south of the bright star in the front of Scorpio, and about the year —2200 the sun stood, on the day of the autumal equinox, near the first stars of Scorpio. Moreover, the Chinese astronomers themselves, as well as Gaubil, referred that eclipse to the day of the autumnal equinox. It is natural, however, that neither the Chinese, with their imperfect theory of the moon, nor the European astronomers in China, being destitute of a correct Lunar Table, succeeded in authenticating this very important eclipse, which must have been a total one, because it caused such great excitement in the capital. The Chinese astronomers, from A.D. 620 to 908, who, moreover, were not yet acquainted with the secular acceleration of the moon, nor even with the *Anomalia media*, came to the conclusion that the eclipse of the Chou-king referred to —2127, which decision is obviously wrong; for the year does not agree at all with the Chinese annals, according to which Thong-khang reigned from —2158 to —2145, or some years earlier. Besides this, on the autumnal equinoctial day of —2127 the ☊ lay 13° west of the sun, and hence the obscuration was not visible in Pekin. Gaubil insisted upon it, that the same eclipse was that in —2154, Oct. 12th, 6h. 57m. a.m. Pekin time; ☉ and ☽ in Libra 0° 23′ 19″, ☊ in Virgo 25° 24′ 27″, i.e. 4° 58′ 52″ west of the sun, latitude of the moon 26′ 10″, obscuration in Pekin 51′. Unfortunately, however, this eclipse preceded sunrise in Pekin (114° 7′ E. of Paris, 39° 54′ N.), and even in Gan-y-hein, where the sun rose 20 minutes earlier; for Gaubil states that the Tartarian translation of the Chou-king narrates that the same eclipse took place in Gan-y-hein at 8 o'clock;

that is, obviously, after sunrise, and not 8 hours after midnight. He accordingly computed the said eclipse by means of eight different Tables, those of La Hire, Riccoli, Longomontanus, Wing, Philolaus, Rudolph, Carlin, Flamstedt; and yet the eclipse preceded sunrise, according to all Tables, both in Pekin and Gan-y-hein. And why? Because all these Tables were based on Ptolemy's eclipses in the Almagest. Since, then, during a period of 912 years only one total eclipse of the sun happened on the same day in Pekin, we come to the eclipse in —2192, Oct. 10th, 18h. Par. T., i.e. 1h. 36m. p.m., in Pekin (114° 7′ E. of Paris); for all nations of antiquity commenced the civil day with sunrise, for which reason the "8th hour" commenced about 2 o'clock p.m. Since, moreover, the eclipse of the Chou-king happened in Gan-y-hein, where the sun rose 20 minutes earlier than in Pekin, it follows that the said eclipse was seen in Pekin about two hours past noon, on Oct. 11th, in —2192, which day was, as the Chinese report, the day of the autumnal equinox. On that day (—2192, Oct. 10, 18h. Par. T.) the longitude of the sun was nearly $6^s\ 0^\circ\ 46'$, that of the moon $6^s\ 6^\circ\ 41'$, that of the ☊ $6^s\ 10^\circ\ 22^\circ$. According to our Table, p. 429, the longitude of the ☊ was shorter by 17° 15′, and hence the ☊ lay 7° 40′, likely 6° only, west of the sun. According to the same Table, the moon's longitude was $6^s 0^\circ\ 35'$, because it must be shortened by 6° 6′; but a more exact computation of the moon's place will decide the question how far our coëfficients concerning the secular accelerations of the moon, and her Apsides and Nodes, are to be diminished. Besides, this eclipse is well qualified to determine more exactly the secular acceleration of our globe. Finally, it is not to be wondered at that our eclipse happened 34 years earlier than the present chronology of the Chinese emperors requires; for, in copying ancient manuscripts, the ciphers were most of all subject to corruptions, as the copies of Manetho's dynasties in Eusebius, Africanus, and in the Armenian translation of Eusebius, demonstrate. (See the author's "Theologische Schriften der alten Æg.," 1855, p. 104.) By the way, all students of Chinese literature maintain that Fohi, "during whose life the columns of the heavens broke down," is the Chinese Noah; and the Chinese annals, as we have seen, refer him to —3332, whilst the deluge, according to the corrected Septuagint and the planetary configuration represented in the Noachian alphabet, ended in —3446. The date of our eclipse, it is evident, harmonizes Chinese chronology with the Septuagint, apart from 80 years (3446 = 3332 + 34 + 80). The time will come when all other Chinese eclipses will be once more chronologically fixed, as well as the Greek and Roman ones.

28. The oldest eclipse mentioned in history is, no doubt, that observed in Tanis about the time of Menes' arrival in Egypt, and about the beginning of the first Canicular and the first Apis periods, namely, the lunar eclipse in — 2780, May 23, 15h. P. T., ☊ 12° E. It is known that the Apis-bull was a living emblem of the moon, as Jablonskius (Panth. Æg. iv. 4) and my Astronomia Ægyptiaca (p. 100) have demonstrated. Horapollo (ii. 4, i. 10), and the fact that the so-called Isis Table, the Nativity of Trajan, signifies the sun by the Mnevis-bull and the moon by the Apis-bull, put beyond question that Apis represented the moon. This is, moreover, confirmed by the Apis periods of 25 Egyptian years; for the longitude of the moon is exactly the same after 25 vague years of 365 days, i.e. after 25 Julian years, minus 6 days and 6 hours; and this singular phenomenon gave rise to the Apis periods, i.e. Moon periods of 25 Egyptian years, so often mentioned in Egyptian, Greek, and Roman histories. For instance, in — 2780, July 19, which was the first newyears day of the Egyptians, on the 1st day of the month called Thoth, the moon and the sun stood distant from each other 180°; it was the day of the full moon: and 25 years later, viz. in — 2755, the full of the moon again coincided with the 1st day of Thoth, the newyears day. The Apis, however, signified the moon not in general only, but also especially an obscuration of the moon, a lunar eclipse about the beginning of the first Apis period in — 2780; for the Apis must always be a black bull with a white crescent-like figure on his side, as archæology reports. It would have been absurd to represent the candent full moon by a black bull with a crescent-shaped white sickle. Even the myth that Menes was devoured by a crocodile, points us to a lunar eclipse about the time of Menes' arrival in Egypt; for Menes simply signifies the moon, like the Greek *Μην*, the Hebrew *meni*, the Arabic *mana*, our moon, *men-sis*, and the like; and even the Coptic name of poppy, viz. *ne-man*, signifies the flower of the moon, the German *Mohn*.

Hence the name of King Menes was syllabically expressed by the figure of the crescent, e.g. in Burton's "Excerpta hieroglyphica," Pl. XV. representing the planetary configuration of Menes in —2780, July 16, the day of the summer solstice. The mythical description of a lunar eclipse by the devouring of the moon by a crocodile, is analogous to that of the Chinese, according to whom, the sun and the moon, at the time of eclipse, are being devoured by a dragon.

The time of Menes' eclipse is astronomically fixed; for the Vetus Chronicon, preserved by Syncellus, expressly reports that Menes and the following dynasties reigned "since the beginning of the Canicular period" in —2780, and, accordingly, that the first fifteen dynasties ruled simultaneously in different provinces of Egypt. The Table of Abydos, furthermore, specifies 38 kings successively reigning from Menes to the last king of the 18th dynasty, viz. Ramses, who ruled, according to the nativities of several kings of the same dynasty, about —1647 B.C., and Eratosthenes counted from Menes down to the end of the same dynasty 1076 (+57) years. The Table of Karnak, in accordance therewith, enumerates in juxtaposition, on one side, the kings from Menes to the middle of the 18th dynasty who ruled suecessively, and on the other side it specifies the coëtaneous dynasties. All these ancient reports concur in demonstrating that Menes settled in Egypt in the course of the year —2780. The matter has been discussed *in extenso* in the author's "Theologische Schriften der alten Ægypter," Leipzig, 1855, p. 102. This result, finally, is mathematically confirmed by the so often mentioned astronomical monuments referring Menes' arrival in Egypt to the 16th day of July, in the year —2780. According to these monuments, one of which is represented in the author's "Berichtigungen," Pl. I., and another in Burton's "Excerpta hieroglyphica," Pl. XV., the places of the planets were on that day as follows: the sun in Cancer 0°, Saturn in Sagittarius 4°, Mars in Sagittarius 6°, Jupiter in Aries 13°, Venus in Cancer 15°, Mercury in Cancer 2°, the moon in Scorpio 3°. The said day was, therefore, the day of the summer solstice, and it is odd only that Menes commenced the Egyptian year not with the summer solstice, a cardinal day, but, three or four days later, with the 19th or 20th day of July. The prevailing opinion is that the Canicular periods commenced with July

20th, but Ideler's Chronology, vol. ii. p. 594, cites Hephæstion, according to whom that period commenced one day earlier, and the latter has been confirmed by the Tanis stone. How, then, came it to pass that Menes made July 19th the first day of the year? The reason is, no doubt, that he intended to commence the year and the Canicular period with the beginning of a new week; for July 19th in −2780 was a Saturday, and this day was the first day of the original week, as the series of the ancient planets, arranged according to their apparent velocities, evidences. Take of the natural series of the planets always the fourth, and you will have the succession of the days of the week as follows:

♄	♃	♂	☉	♀	☿	☽
1	6	4	2	7	5	3

Moreover, our week was already known to Menes, because the sacred records of the Egyptians, written, as history reports, in Menes' days, mention the week. (See the author's "Summary of Recent Discoveries," p. 65.)

The conclusion, therefore, is that the image of the Apis-bull, his black skin and white sickle, signified a partial obscuration of the moon about the time of Menes' arrival in Egypt, and this is the lunar eclipse in −2780, May 23, 15h. P. T. Long. ☉ $1^s 8° 4'$; Long. ☽ $7^s 15° 18'$, i.e., according to our Table, p. 429, nearly $7^s 6° 18'$; Long. ☊ $1^s 20° 48'$, i.e. according to p. 429, ☋ $9° 9'$ W. of the centre of the earth's shadow, which caused a partial obscuration of the moon.

The actual History of the Persians, Medians, and Babylonians.

No man will gainsay that the Babylonian and other lunar eclipses specified in the Almagest have taken place in the same years of the kings and archons to whom they were linked in the catalogue of eclipses which, as Ptolemy says, came once into his hands. The question, however, is whether Ptolemy, in whose days (140 A.D.) chronology was in its very infancy, and who was still destitute of correct Lunar and Chronological Tables, correctly referred his first 15 lunar eclipses to their real dates, or not. This question principally depends on the question whether Ptolemy's Historical Canon, by means of which he determined the dates of the said eclipses, is true or erroneous. Since many of the Babylonian

Median, Persian, Greek, Egyptian, and Roman regents, successively enumerated in Ptolemy's Historical Canon, occur in Biblical, Greek, and Roman histories; and since the principal epochs of the latter have been, in the premises, fixed by a multitude of infallible historical and astronomical certainties, it will be an easy matter to decide the question to what years the 19 eclipses in the Almagest really belong. In advance we put in juxtaposition the successive regents according both to Ptolemy's Historical Canon and his Nabonassarian Era on one side, and on the other their epochs according to other chronological resources, especially the Greek ones (pp. 470–72). It is to be remembered that the basis of Ptolemy's Narbonassian Era is the vague year of the Egyptians, which every four years commences earlier by one Julian day, and that Ptolemy in general referred, as Ideler says, the last months of a king subsequent to the newyears day to the reigning-time of his successor. Fréret, on the contrary, maintained that Ptolemy referred the first months of the Babylonian kings preceding the newyears day to the reigning-years of his predecessor.

REGENTS.	Ptolemy.	Correctly.
Nabonassar	— 746 Feb. 26	— 743
Mardokempad's first year	— 720 Feb. 20	— 717
(1) ☽ Ec. March 19.		
Mardokempad's 2d year	— 719	— 716
(2) ☽ Ec. March 8.		
(3) ☽ Ec. September 1.		
Nabopolassar	— 624 Jan. 27	— 621
(4) ☽ Ec. during his 5th year.		
Nabokolassar (Nebuchadnezzar)	— 603 Jan. 21	— 601
Babylonian captivity, 2 Kings xxv. 1.		
Nabonnad (Belshazzar)	— 554 Jan. 9	— 552
Cyrus	— 537 Jan. 5	— 535
Cambyses	— 528 Jan. 3	— 526
(5) ☽ Ec. during his 7th year.		
Darius (Hystaspes)	— 520 Jan. 1	— 518
(6) ☽ Ec. during his 20th year.		
(7) ☽ Ec. during his 31st year.		
Xerxes	— 485 Dec. 23	— 484
Artaxerxes (Longimanus)	— 464 Dec. 17	— 462
Darius (Nothus)	— 423 Dec. 7	— 422
Artaxerxes (Mnemon)	— 404 Dec. 2	— 402
(8) ☽ Ec. 381, December 12.		
(9) ☽ Ec. 380, June 6.		
(10) ☽ Ec. 380, December 1.		
Ochus	— 358 Nov. 21	— 356
Arses (Arogus)	— 337 Nov. 16	— 335

REGENTS.	Ptolemy.		Correctly.
Darius (Codomannus)	— 335	Nov. 15	— 332
Alexander (Magnus)	— 331	Nov. 14	— 328
Philippus Aridæus (Ptolemæus Soter I.)	— 323	Nov. 12	— 320
Alexander II. (Cassandra)	— 316	Nov. 10	— 314
Ptolemæus Lagi (Soter I.)	— 304	Nov. 7	— 303
Pt. Philadelphus	— 284	Nov. 2	— 283
Pt. Evergeta I.	— 246	Oct. 24	— 245
Pt. Philopator	— 221	Oct. 18	— 220
Pt. Epiphanes	— 204	Oct. 13	— 203
(11) ☽ Ec. —199, September 12.			
(12) ☽ Ec. —197, July 23.			
(13) ☽ Ec. —196, January 16.			
Pt. Philometor	— 180	Oct. 7	— 179
(14) ☽ Ec. during 7th year of Philometor.			
Pt. Evergeta II.	— 145	Sept. 29	— 144
(15) ☽ Ec. —138, June 1.			
Pt. Soter II.	— 116	Sept. 21	— 115
Pt. Dionysus (Alexander II.)	— 80	Sept. 12	— 79
Cleopatra	— 51	Sept. 5	— 50
Augustus	— 31	Aug. 29	— 28
Tiberius	+ 14	Aug. 20	+ 16
Caligula	+ 36	Aug. 14	+ 38
Claudius	+ 40	Aug. 13	+ 42
Nero	+ 54	Aug. 10	+ 55
Vespasian	+ 68	Aug. 6	+ 69
Titus	+ 78	Aug. 4	+ 78
Domitian	+ 81	Aug. 3	+ 81
Nerva	+ 96	July 30	+ 96
Trajan	+ 97	July 30	+ 97
Hadrian	+ 116	July 25	+ 116
(16) ☽ Ec. +125, November 5.			
(17) ☽ Ec. +133, May 6.			
(18) ☽ Ec. +134, October 20.			
(19) ☽ Ec. +136, March 5.			
Antoninus Pius	+ 138	July 20	+ 139

Examination of Ptolemy's Historical Canon.

1. History reports that Antonine, in the 2d year of his reign, visited Egypt, and that, at the same time, a new Canicular period commenced. Since Hadrian died, according to Spartianus, A.D. 139, July 10, Antonine's 2d year commenced with the same day A.D. 140, and consequently the new Canicular period belongs to A.D. 140, the 2d year of Antoninus Pius. Ideler, on the contrary, referring the beginning of the Canicular period to the preceding year (A.D. 139), had recourse to the hypothesis that Antonine obtained his first Tribunitia potestas one year prior to Hadrian's death. Yet no emperor was invested with that office previous to the day of his predecessor's death, and the planetary configurations, the Apis periods, the Triacontaëteris, mentioned in the premises (p. 405), place beyond question that the Canicular periods commenced in −2780, −1320, +140, and not, as Ptolemy, Petavius, and Ideler imagined, one year earlier.

2. Ptolemy counts from Caligula to Titus two years too much, because he knew not that the consuls A.D. 47 and 78 were Extraordinarii, as the inscriptions and coins have demonstrated (p. 422).

3. In consequence of this gross blunder, Ptolemy referred the death of Augustus to A.D. 14 instead of 16, for the latter year is confirmed by many infallible arguments, both historical and astronomical (p. 455). Hence the battle near Actium, fought during the 14th year of Augustus, happened in −28, and not, as Ptolemy stated, in −30, which result is confirmed by Josephus (An. xv. 5, 2); for he refers this battle to the 7th year of Herod, who reigned from Sept. 11th in −35, as we have seen (p. 454), accordingly to −28. Moreover, since the reign of Augustus commenced with Cæsar's death, three months prior to the Olympian games, i.e., as a multitude of historical and mathematical developments have brought to light (p. 448), in −41, and not in −43, it is evident that Ptolemy must have likewise antedated by two years all events of Greek history connected with the Olympiads; for the Olympian games were celebrated, and the archons, closely connected with the Olympiads, ruled two years later than Ptolemy conjectured. Hence Ptolemy's eclipses, referred to the archons Phanostratus and Evander, and to the Calippian periods, must of necessity be postdated by two years.

4. According to the Olympian games, the Apis periods, and several eclipses, Alexander the Great died in −320, and yet Ptolemy's Historical Canon refers this event to −323, three years too early. On this occasion even Ideler, the boldest advocate of Ptolemy's chronology, concedes Ptolemy's statements to be sometimes wrong. Alexander's death in −320 is, in the first place, confirmed by the renewal of an Apis period in −320; for Diodor (i. 76, 84 p. 25 B.) and the Almagest itself (Ideler Chron. i. 182) report that one year after Alexander's death, i.e. in the first year of Ptolemæus Lagi (Soter I.) the Apis period was renewed, which was also the case in −320 (p. 405). Further, Diodor (xviii. 8) and Dinarch (Dem. 81, p. 108, 28) bear witness that Alexander died in the year subsequent to the Olympian games which were repeated in –321 (p. 451). Consequently Alexander died three years later than Ptolemy supposed.

5. Ptolemy's Canon refers the foundation of Alexandria and the battle near Arbela, fought some months prior to Darius Codomannus's death, to –331 and –330, respectively. But Curtius (iv. 5, 11) testifies that Alexandria was founded some months after the Isthmia æstiva which were celebrated in –329 (p. 432, ad u.c. 558; comp. p. 471, Ol. 92. The eclipse, eleven days prior to the battle at Arbela, observed in –328 (p. 472), demonstrates that Darius Codomannus died in –328, and not, as Ptolemy made out, in –331, or –330.

6. Darius Nothus died, as Diodor (xiii. 104, 108) recounts, a short time (μικρόν) after the end of the Peloponnesian war, viz. in χειμών, –401 (p. 471); Ptolemy, on the contrary, refers the death of the same king to –403, again two years too early.

7. Since Darius Nothus reigned, according to Ptolemy's Historical Canon, 19 years, he should have commenced to reign in –422, as Ptolemy states. But, alas! Thucydides (viii. 58) narrates that the Lacedæmonians, about the end of the 20th year of the Peloponnesian war, namely, "80 days after the summer solstice," confederated with the same king, viz. in the course of his 13th year (τρίτῳ καὶ δεκάτῳ ἔτει Δαρίου βασιλεύοντος). Consequently, since the 20th year of the Peloponnesian war commenced (p. 471) with the year –409, Darius Nothus must have reigned in –422, and not, as Ptolemy misjudged, in 423. Moreover. Clinton's Fasti Hellenici (p. 315) affirm that Ptolemy omits the reigns of Xerxes II. and Sogdianus; that, consequently, Darius Nothus ruled one year less, or that he ruled simultaneously with both the said kings.

8. Thucydides (iv. 50) informs us that Artaxerxes Longimanus died during χειμών of the 7th year of the Peloponnesian war, commencing with the year 422 (p. 471), and yet Ptolemy's Canon refers the death of the same king to –423; that is also one year too early.

9. Herodotus (vii. 1, 3, 4) reports that Darius Hystaspes died four years after the battle at Marathon, and that in the same year Xerxes commenced to reign. The battle at Marathon in –488, Aug. 6th, being incontrovertibly fixed by the Solar Calendar of the Greeks, and by the astronomical full moon four days prior to the 6th day of August (pp. 408, 409, 487), it is apparent that Xerxes must have reigned since –484 and after August 6th. The Parian Marble (Ep. 48), which counts the years down to Archon Diognetus (July in –261), refers the battle at Marathon to the same year (227 + 261 = 488). Hence the same authentic monument counts 223 years from Xerxes to Diognetus, to the effect again that Xerxes reigned not in –485, but in –484. (See Boeckh's Corpus Inscript., vol. ii., p. 294.) Ptolemy's Historical Canon, on the contrary, refers Darius Hystaspes' death and Xerxes' reign to –485. Eusebius likewise puts Xerxes in Ol. 73, 4, i.e. in –484; the Armenian translation reads "Ol. 74, 1," which makes no difference.

10. Herodotus (vii. 20), who was born about the same time in —481, and "53 years prior to the beginning of the Peloponnesian war" (Gell. xv. 23, Suidas ad v.), reports that Xerxes marched out against Greece in the 5th year of his reign, consequently in —479. His arrival at Sardes in —478 is mathematically fixed by the total eclipse of the sun in the early spring of —478, Feb. 27 (p. 487). In the following year, —477, the battle near Thermopylæ was fought during the Olympian games (Her. vii. 206), and the Parian Marble (Ep. 51) counts from this event to Diognetus 216 years; wherefore the said battle belongs to —477. This date is confirmed by Thucydides (i. 18), who counts ten years from the battle at Marathon (Aug. 6, —488) to Xerxes' arrival in Greece, and by Eratosthenes, who counts 48 years from Xerxes' crossing the Hellespont to the beginning of the Peloponnesian war in —429. Ptolemy's Canon, on the contrary, refers the first year of Xerxes to —485, i.e. again one year too early.

11. It is universally known that the Babylonian captivity commenced in the 11th year of King Zedekiah, the 1st of Nebuchadnezzar (Jer. xxxix. 1; 2 Kings xxv. 8; 2 Chron. xxxvi. 17); that the same bondage lasted full 70 years (2 Chron. xxxvi. 22, "fulfil threescore and ten years"), and ended in the 1st year of Cyrus (2 Chron. xxxvi. 22; Ezra i. 1). Now, Ptolemy's Canon puts Nebuchadnezzar in —603 and Cyrus in —537; accordingly, the captivity lasted 66 years only. Indeed, this is the chronology of Petavius, who made out that the Babylonian captivity "lasted only 66" and not 70 years with some months. But, who is capable of believing that the Prophets, who lived at the same time, and the sacred chroniclers were mistaken? The simple conclusion, therefore, is that Ptolemy's Canon is again erroneous. Cyrus, as we have seen (p. 483), permitted the Jews to rebuild Jerusalem subsequent to the conquest of Nineveh in —532, seven years prior to his death, and that year is incontrovertibly determined by the solar eclipse in —532, by the Apis periods, by the *turnus* of the priests, and by the seventy weeks of Daniel. Consequently, Ptolemy ought to have put Cyrus's monarchy in —532, and not in —537, i.e. he put it at least four years too early. How came it to pass that Ptolemy committed this error of four years? First, Ptolemy antedated in general, as we have seen, all his kings by two years, and, moreover, he forgot to mention Cyrus's predecessor, viz. Darius Medus, whom Josephus expressly inserts with two years between Cyrus and Nabonad (Belshazzar).

12. Ptolemy (Almagest iii. 6, p. 204 Hal.) says expressly that from the 1st year of Nabonassar (—746, Feb. 26) down to Alexander's death (—323, Nov. 12th) 424 vague years elapsed. The account is true, for —746, Feb. 26, —424, Nov. 12, gives 322 Julian years and 8 months. But the question, however, is whether Alexander actually died in —323, or not. Alexander's death in —320, on the 6th day of Thargelion (April 6th), is, as we have seen, placed beyond any question, because he died one year after the celebration of the Olympian games (Diodor xviii. 8; Dinarch, in Dem. 81, p. 100, 28; Joseph. c. A. I. 22; Ant. viii. 28; Ælian. V. A. ii. 25; Laërt. vi. 79), and in the 1st year of an Apis period (Diod. i. 84, p. 25 Bip.), and eight years subsequent to the lunar eclipse preceding the battle at Arbela in —328, and, as history says, 33 years after his birth, which happened during the Olympian games in —353 (Plut. Alex. 3; Cic. de Div. i. 23; Euseb. ad Ol. 106, 1). The natural inference, therefore, is that Ptolemy likewise antedated the reigning-time of Mardokempad by three years, because he had antedated Alexander by three years.

13. Finally, so far as Ptolemy's chronology of the Lagides is concerned, at least three errors are apparent. First, Alexander having died in —320, and not —323, Ptolemæus Lagi must have reigned three years later than formerly was believed. Further, Cicero (Legg. agr. ii. 17) testifies that King Ptolemæus Alexander died during his own consulate, and, since the latter's reign began in —62, and not in —63 (p. 432), the said king reigned one year later than Ptolemy stated. Consequently, Soter II., the predecessor of Dionysus, who ruled 30 years, must have died in —79, and not in —80. Finally, since Cleopatra assisted at the Olympian games in —29 (p. 451), and not in —31, and since the battle near Actium happened in —28, and not in —30, it is evident that Cleopatra's history must likewise be postdated by nearly two years.

These arguments, apart from many others, remaining *in petto*, will suffice to convince every sane historian that the whole of Ptolemy's Historical Canon down to Titus is wrong; that, therefore, all eclipses linked to Ptolemy's kings and other historical epochs must of necessity be referred to later dates. It is singular, indeed, that a world-renowned astronomer like Ptolemy should commit such gross errors as his Chronological Canon contains; but *errare humanum est*, and it is known that he does not at all belong to the class of reliable ancient authors; for Biot, a savant whose wisdom and discernment inspired, a long time since, all the scientific world with veneration, pronounced as follows:— "Since I examined Ptolemy's catalogue of fixed stars, I have lost the last remainder of respect concerning the Alexandrian astronomer."

It is, moreover, not the first time that Ptolemy's Historical Canon has appeared to be erroneous, for many historians before us have likewise given judgment against Ptolemy. We mention only the following authors: *Masson's* Histoire critique de la république des lettres, Paris, 1714, T. i. p. 22; *another author*, in the same work, T. v., p. 114; *Lobbi's* Introductio Chron. ii. 38; the *anonymous author* of the Dissertatio de Canone astronomico, p. 149; *Koch's* Entsiegelter Daniel, p. 131; *Drumel's* Proben einer verbesserten Harmonie der heiligen und Profanscriebenten, Frankfurt, 1746; *Johan Kepler's* Eclogæ Chronologicæ; *Megerlin's* Commentar. Chronolog. in Tabulam mathematico-hist.; *Conring's* Canonem mathem. Nabonassaræum non mereri fidem (Grævii Syntagmata); *Harduin's* Chronologia V. T. Amstel., 1709; *Artopœus's* Dissertatio de summis imperiis; *Ravius's* Chronologia infallibilis; *Wagner's* Institutiones historicæ; Ideler's Chronol. ii. 122, etc. Others maintained Ptolemy's Canon to be incorrect, except the years to which eclipses are linked, and to this class of opponents *Scaliger*, *Dodwell*, *Des Vignoles*, and *Fréret*, who put Ptolemy's newyears days in the 1st year of each Babylonian king, are to be numbered.

The real dates of the Eclipses in the Almagest.

11, 12 & 13. Ptolemy (Alm. iv. 10, p. 279) says that in the 54th and 55th years of the second Calippian period three lunar eclipses happened, of which the latter two belonged to the same Calippian year (τῷ αὐτῷ νε′ ἔτει). These eclipses the Almagest refers to — 200, Sept. 22d; to — 199, Mar. 19th, and to — 199, Sept. 12th. Since, however, the Greek year commenced with the 1st day of Hecatombæon, June 2d (p. 408), it is evident that Ptolemy's eclipses in — 199, March 19th, and in — 199, Sept. 12th, refer to two different Calippian years, and not to the same Greek year (αὐτὸν ἔτος): for the eclipse in — 199, March 19th, belonged to the 53d year of the Calippian period, whilst the eclipse on Sept. 12th in — 199 appertained to the 54th year of the same period. Moreover, the first Calippian period was, as is well known, a continuation of Meton's lunar cyclus of 19 years, which had commenced not in — 429, as Ptolemy imagined, but in — 428, with May 15th (p. 408), viz. during Apseudes' archonship; and the archons of the times of Calippus ruled, in consequence of the lost year between Thucydides and Xenophon (p. 469), two years later than Ptolemy calculated. Consequently, the aforesaid 3 eclipses are to be postdated as follows:—The first of them happened in — 199, Sept. 12, 13h. 30m., ☋ 3° W., which year belonged to the 54th year of the Calippian period; the 2d refers to — 197, July 23d, 11h. 45m., ☊ 11° E.; the 3d to — 196, Jan. 16th, 5h. 30m., ☋ 5° E. According to our Table (p. 429), the longitudes of the Nodes were at that time nearly 4° 31′ shorter, and the oppositions happened about 3 hours later. Indeed, the latter eclipses appertained, as Ptolemy's ancient historiographer said, to the same Greek year, because July 23d in — 197 and Jan. 16th in — 196 belonged to the very same Calippian year. Hence these three eclipses alone demonstrate that Ptolemy's chronology of ancient eclipses is wrong.

15. Ptolemy (Alm. vi. 5, p. 390) mentions a lunar eclipse observed in Rhodus during the 37th year of the third Calippian period, which he referred to — 140, Jan. 27th, 9h., ☋ 11° W., obscuration 2¾ inches. But, the Calippian years and archons coming down by two years, that eclipse happened in —138, June 1st, 10h. 15m., ☋ 2° W., i.e., according to our Table (p. 429), 6° W.

8, 9 & 10. Ptolemy (Alm. iv. 10, pp. 275, 277, 278) cites three eclipses observed during the archonships of Phanostratus and Evander, and he referred to Phanostratus both the eclipses in —382, Dec. 23, 5h. 45m., ☋ 10° E., obscuration 2 inches, and in —381, June 18th, 6h. 45m., ☊ 7° 45′ E., obscuration 5½ inches; but to Evander the eclipse in —381, Dec. 12, 9h. 30m., ☋ 2° E., as Pingré states. Inasmuch, however, as the Olympiads and archons of this time come down by two years (p. 470), the latter eclipse should be referred to —379, May 27th, 12h.. ☊ 8° W., obscuration 4 inches, but the ☊ lay too far west of the centre of the earth's shadow (p. 429). On the other hand, it is apparent that Ptolemy's eclipse in —382, Dec. 23, 17h. 45m. P. T., which he referred to the archonship of Phanostratus, is erroneous, because it happened, as we shall see below, subsequent to sunrise both in Babylon and Greece. Ptolemy's eclipses occurred one year later than he imagined, viz. in —381, Dec. 12th, 9h. 30m. P. T., ☋ 2° E., or rather 3° W. (p. 429); further, in —380, June 6th, 7h. 45m. P. T., ☊ 3° E., i.e. 2° W.; and in —380, Dec. 1st, 0h. 30m., that is, 4h. 25m. in Babylon, and, according to our Table, p. 429, nearly at 8 o'clock p.m. local time; ☋ 1° W.

5. The years in which Cambyses and Darius Hystaspes commenced to reign are, as we have seen, historically and mathematically fixed (p. 483); for Cyrus died in —526, seven years after the conquest of Nineveh, and this epoch is put beyond the reach of controversy by the eclipse in —532, by the *turnus* of the Hebrew priests, and by Daniel. Accordingly, Cambyses must have reigned since —526, and this year is confirmed by the renewal of an Apis period in the 6th year of his reign, viz. in —520 (p. 405). Consequently, the lunar eclipse observed in the 7th year of Cambyses (Alm. v. 14, p. 341) belongs to the year —519, Nov. 8th, 2h. p.m. P. T., ☋ 8° E., obscuration 2½ inches. Babylon time being earlier by 2h. 45m., and the opposition beginning later by nearly 4 hours, this eclipse happened in Babylon about 8 o'clock p.m. The ☋ lying 6° nearer to the sun (p. 429), this eclipse was a total one in Babylon. Ptolemy, on the contrary, not knowing in what years the Apis periods were renewed, referred the eclipse observed in the course of the 7th year of Cambyses to —522, July 16th, 9h. 30m. P. T., ☋ 6° E., obscuration 6 inches. Consequently, Ptolemy again antedated Cambyses by two years, or three, the day of Cyrus's death being unknown.

6. Ptolemy (Alm. iv. 8, p. 269) refers the lunar eclipse observed within the 20th year of Darius Hystaspes to —501, Nov. 19, 10h. P. T. Since, however, Cambyses died two years later than Ptolemy supposed, Darius Hystaspes must likewise have reigned and deceased later by two years. This is confirmed by Herodotus, who narrates that Darius died four years after the battle at Marathon, viz. in —484, as we have seen (p. 410). Therefore, the real eclipse of the 20th year of Darius belongs to —499, May 4th, 10h. 15m. p.m. P.T., ☊ 4° W. According to our Table, p. 429, the opposition took place about 4h. 50m. after midnight in Babylon, and the ☊ lay 10° W.

7. Ptolemy (Alm. iv. 5, p. 267) reports that in the course of the 31st year of the same Darius a lunar eclipse was observed, which he refers to —490, April 25, 8h. 45. P. T., ☊ 11° E., obscuration 1 inch. But, Darius reigning two years later (p. 501), his 31st year commenced in —488, or, since the day of Cambyses' death is unknown, in —489. Besides, in —488 no lunar eclipse was visible in Babylon. Hence, the eclipse referred to happened in —489, Oct. 8th, 4h. 30m. p.m. P. T., ☋ 1° W., i.e., according to our Table, p. 429, 1 hour before midnight in Babylon, ☋ 7° W. of the centre of the earth's shadow.

4. Ptolemy having learned that in the 5th year of Nabopolassar a lunar eclipse happened in Babylon, referred it (Alm. v. 14, p. 340) to —620, April 21st, 14h. 45m. P. T., ☊ 9° E., obscuration 1¾ inches. But, since all the following kings reigned two years later, Nabopolassar must likewise have ruled in —622, and not in —624; and hence the eclipse of the 5th year of Nabopolassar belongs to —619, Oct. 6th, 12h. ☋ 0° E., and, according to our Table, p. 429, this total obscuration of the moon took place in Babylonia about one hour prior to sunrise. The eclipse in —620, April 21st, 14h. 15m., happened after sunrise in Babylon.

1. Since Alexander the Great died in —320, three years later than Ptolemy states, and since the latter (Almag. vi. 6, p. 204) expressly reports that from the 1st year of Nabonassar down to

Alexander's death "424 years elapsed," we have to presume that the first kings of the Nabonassarian era reigned not two years, as was the case with Nabopolassar and his successors, but even three years later. Hence the lunar eclipse in the first year of Mardokempad which Ptolemy refers to —720, March 19th, 8h. P.T., ☋ 1° W., is to be referred to —717, Jan. 16th, 4h. P. T., ☋ 5° E. The longitude of the Nodes was (p. 429) shorter by 7° 3′, and this total obscuration of the moon took place about 3 hours 30 minutes after midnight in Babylon.

2. The following eclipse in the 2d year of Mardokempad, which Ptolemy refers to —719, March 8th, 10h. P. T., ☋ 9° W., obscuration ¾ of an inch, belongs to —716, Nov. 26th, 0h. P. T., for the longitude of the ☉ was 7^s 28°, that of the moon 1^s 28°, that of the ☋ 8^s 19°. The opposition in Babylon took place 2h. 46m., and according to our Table, p. 429, nearly 4 hours 33 minutes later, i.e. about 7 o'clock p.m. local time. The longitude of the ☋ was shorter (p. 429) by about 7° 3′. Since, however, the Lunar Ecliptic Limit is commonly 12°, and not 13°, this eclipse is to be computed more exactly. The longitude of the Apsides was then, according to our Table, p. 429, shorter by about 3° 21′.

3. The other ecliptic full moon of the 2d year of Mardokempad, which Ptolemy refers to —719, Sept. 1st, 7h. 30m. P. T., ☋ 9° W., obscuration 5 inches; happened in —715, May 21, 5h. P. T.; long. ☉ 1^s 21° 49′; Long ☽ 7^s 21° 43′; long. ☊ 2^s 9° 59′. The opposition in Babylon took place a few minutes after midnight, and the ☊ lay (p. 429) 10° 41′ E. of the sun.

14. Ptolemy (Alm. vi. 5, p. 389) mentions a lunar eclipse observed in the 7th year of Ptolemæus Philometor, which eclipse he refers to —173, April 30th, 12h. 45m, P. T., ☊ 9° W., obscuration 7 inches. Since Ptolemæus Soter, however, ruled three years later than Ptolemy's Historical Canon states, and since Ptolemæus Alexander and Cleopatra died, respectively, one and two years later, as we have seen (p. 503), it is probable that Philometor likewise reigned one or two years later, and that, accordingly, Ptolemy's eclipse happened in —171, Sept. 2d, 11h. 30m. P. T., ☋ 5° 59′ E., obscuration 11½ inches. The full moon in —173, April 30th, was not ecliptic (p. 429), and in —172 no lunar eclipse occurred.

16–19. Finally, we come to the four Alexandrian eclipses observed by Ptolemy himself, provided the latter really lived at the same time. These ecliptic full moons he referred to A.D. 125, April 5th, 7h., ☊ 10° E., obscuration 1¼ inches; to 133, May 6th, 9h. 15m., ☋ 5° E., obscuration 12¾ inches; to 134, Oct. 20th, 9h. 15m., ☊ 5° W., obscuration 10 inches; to 136, March 5th, 14h., ☋ 9° E., obscuration 5 inches. The times and magnitudes of these four eclipses, however, disagree very much with Hansen's Tables, as will be seen further on. Moreover, since Ptolemy observed these eclipses with the naked eye, and without chronometers and micrometers, it is evident that his statements concerning the times and magnitudes of said eclipses are not sufficiently accurate for establishing a correct theory of the secular accelerations of our satellite, her Nodes and Apsides. According to our Table, p. 429, the eclipses A.D. 125, 133, and 136, were some inches greater than Ptolemy states. The eclipse of A.D. 134, ☊ 5° W., obscuration 10 inches, was smaller, because the ☊ lay 3° 11′ farther from the centre of the earth's shadow. Besides, none of these eclipses took place, according to our corrections, after sunrise.

Corollaries.

The following are, in short, the results of the preceding researches:

1. By means of new historical and astronomical methods, both totally unknown to Petavius and his adherents down to Clinton and Fischer, it came to light that all the dates of ancient history down to 80 A.D. are to be postdated, and the respective eclipses mentioned by Roman, Greek, and Babylonian authorities have been observed later, respectively, by one or two, and even three years than was formerly believed correct. Some of them greatly differ from those that have been determined by the instrumentality of modern Lunar Tables.

2. If we compute the hundred and some ancient eclipses, fixed in the premises, by means of our Lunar Tables, the majority of the former remain irreconcilable with the reports of ancient eyewitnesses; for all total eclipses of the sun mentioned in Greek, Roman, and other histories would have been partial, sometimes even invisible ones. The same is the case with several total eclipses of the moon witnessed by ancient authors. Moreover, all the eclipses which, according to history, coincided with sunrise, or with fixed hours of the civil day, preceded sunrise and the attested hours according to the usual Lunar Tables. Finally, a number of very small eclipses of the sun and the moon, witnessed by coëval authors, would have been fictitious ones if our Lunar Tables were right.

3. If we, on the other hand, compute the hundred and some ancient eclipses by applying the approximate corrections specified on pp. 429–30, none of the ancient eclipses drop out of existence; all total eclipses were total ones, all partial ones agree with the statements of the Tables, and the alleged hours of ancient eclipses prove more or less true.

4. The secular accelerations of the moon, her Nodes, her Apsides, and probably other elements of the moon's motions, are to be determined by means of the classic eclipses, and not by those in Ptolemy's Almagest. It is impossible to bring both classes of eclipses into harmony; either the former or the latter must be given up. *Tertium non datur.*

Objections concerning the New Theory of the Moon's secular accelerations.

The proposition that the principal motions of the moon are to be fixed by means of the classic eclipses, and not by those in the Almagest, is so paradoxical that there is no doubt all kinds of objections will be raised, which deserve to be canvassed in advance.

1. The nineteen eclipses of the moon, of which the times and magnitudes are so nicely specified in the Almagest, it will be objected, agree with all Lunar Tables constructed since Ptolemy; consequently, it will be said, the eclipses cited in the Almagest must have been the same which the ancients observed, and hence any other theory of the moon's motions differing from the Almagest is nonsense. But, alas and alack! this logical deduction is a gross *conclusio in circulo*, a vicious circle, and I cannot altogether conceive how it came to pass that so many learned astronomers and historians were blinded by the chimera. To state the case clearly: the Babylonian eclipses in the Almagest are the principal basis, the *terminus a* *quo*, of all former Lunar Tables; the latter were artificially harmonized with the former; Hansen, and all other authors of Lunar Tables, put, for the epoch —800, the moon, the Apogee, and the Nodes of the moon in the same degrees of the Zodiac presumed by Ptolemy, in order to obtain the same eclipses specified in the Almagest. Therefore, whenever we recalculate Ptolemy's eclipses, our Tables will necessarily, as a matter of course, represent them in conformity to the Almagest. Instead of referring the Babylonian eclipses to the same years, days, and hours, as Ptolemy did, the previous question ought to have been answered, whether or not Ptolemy's Historical Canon is true, and whether or not all other, or at least the most decisive Greek and Roman eclipses, agree with Ptolemy's theory of the moon's motions. Let us take an example for illustration. The learned Kircher, 200 years ago, translated entire Obelisks containing the usual 600 hieroglyphs: he assigned, distinctively, to each figure a word, ideologically expressed, and thus the first Hieroglyphic Dictionary was produced. Many years after, another Egyptologist translates the same inscriptions by the aid of Kircher's Dictionary, when lo! the same words and contents come out. Consequently, says he, Kircher's Hieroglyphic System must be right. In this vicious circle the entire evidence is involved. The previous critical inquiry ought to have been whether other hieroglyphic inscriptions, being interpreted by the said theory, yield a logical sense, or not. And thus, in reference to the moon's theory, adopted in all lunar Tables, the principal inquiry ought to have been, whether the theory of our satellite, derived from the Almagest, agrees with the Greek and Roman eclipses, or not. This *conclusio in circulo*, then, proves nothing.

2. It is a matter of indifference to what years, and days, and hours, the Greek, Roman, and other ancient historians referred their eclipses: the dates of ancient eclipses are to be determined *a priori* by means of the Lunar Tables alone. This was the position of Ptolemy and of his numberless followers, and yet it will meet with the approbation of no scrupulous historian. *History has its inviolable rights.* The historians of the Greeks, Romans, Hebrews, etc., were honest and intelligent men, being both able and willing to tell the truth; and hence their reports, founded on actual personal observation, or on the testimony of

earlier eye-witnesses, must be respected so long as their traditions are not clearly demonstrated to be impossibilities. This is and will be the stand-point of all present and future historians, especially in reference to the chronology of the ecliptic new and full moons witnessed by Greek and Roman authors. History is not to be constructed *a priori*.

3. The correctness of the Babylonian eclipses, as described in Ptolemy's Almagest, is placed beyond question by careful calcutions according to the most perfect Lunar Tables in existence, viz. those of Hansen. This assertion, however, is likewise untenable. The Almagest specifies, as is known, in nearly all instances in what hours and minutes each of its 19 eclipses commenced, in what time they reached the middle, and came to a close; moreover, how many inches and minutes the moon's disc was obscured. Supposing these minute measurements to have been the result of Babylonian observations, those astronomers must, at least since the year --720, have been in the possession of instruments capable of measuring the minutes of hours, and parts of inches of the moon's diameter. In this case, of course, the specified times and magnitudes of the Babylonian eclipses would agree with each other. Now, if we compute the Babylonian eclipses by means of the most accredited Lunar Tables — those of Hansen — what is the result? One of them (No. 3) turns out to have been invisible; another one (No. 15) happened one hour and fifteen minutes later than Ptolemy states; another obscuration of the moon (No. 6) amounted to *one* inch and fourteen minutes instead of *three* inches, as the Almagest says. Let us come nearer to the subject. Prof. Hartwich has taken upon himself to recalculate Ptolemy's 19 eclipses by means of Hansen's Tables (Schumacher's Astronom. Nachrichten, 1860, No. 1241, p. 257), and the result was that Ptolemy's statements differ very much from the computations. For,

No. 2. Obscuration 0.206 (2½ inches) instead of 0.250 (3 inches).
No. 4. Obscuration 0,071 (⅞ inch) instead of 0.250 (3 inches).
No. 6. Obscuration 0.113 (1⅜ inches) instead of 0.250 (3 inches).
No. 7. Obscuration 0.022 (¼ inch) instead of 0.167 (2 inches).
No. 14. Obscuration 0.671 (8½ inches) instead of 0.583 (7 inches), etc.

As to the times of Ptolemy's eclipses, the following incongruities came to light:

No. 9. Duration 2h. 5m. instead of 3h.
No. 15. Beginning 8h. 41m. instead of 9h. 56m.
No. 17. Middle 10h. 41m. instead of 11h. 8m.
No. 19. Middle 15h. 29m. instead of 16h. 13m., and so forth.

What astronomer would have committed so many and such gross mistakes!

The worst of all, however, is that the eclipse No. 2, so well described in the Almagest, concerning its time and magnitude, and which Ptolemy referred to the year —719, Sept. 1st, 6h. 0m., terminated, according to Hansen's Tables, prior to the rising of the moon in Babylonia. "This difficulty," says Prof. Hartwich, "we would overcome by lessening the longitude of the moon in —719; but, alas! in this case the ecliptic full moon (No. 8) —282, Dec. 22d, 18h. 53m., in the Almagest would drop out of existence." What then? Besides, the ☊ lay 16° W. of the earth's shadow on occasion of the aforesaid eclipse.

These mathematical facts will, as it appears to me, suffice to destroy the whole of the reliability of the Almagest thus far, as well as the groundwork of the present theory of the moon. Otherwise, I take the liberty to request the reader to answer the following questions: With what instruments may the Babylonian astronomers have seen an *invisible* eclipse, so *minutely* described in the Almagest? How could they notice an obscuration of the moon amounting to *one-quarter* of an inch only, i.e. to 37″? How could they take an eclipse of $\frac{1}{8}$ of an inch for an eclipse of 3 inches?

Moreover, the unfavorable statements of Hansen's Tables are confirmed by other ones. Ideler (Abhandlungen der Berlin. Academie d. W. 1814–15, p. 221), having computed anew the seven older Babylonian eclipses, discovered that the magnitude of one of them was only 1 inch 30 minutes, and not 3 inches, as the Almagest narrates, and so on. The same Tables brought to light errors of 1h. 4m., of 49m., of 35m., of 30m., of 15m., and so forth. In short, Ptolemy's eclipses could never have been observed by astronomers.

Finally, it is well known that Buerg, disregarding the Almagest, based his Lunar Tables upon 3200 Greenwich observations, and by means of these Tables Ideler recalculated the same seven eclipses. The result, however, was that 3 of the said 7 eclipses could not have been seen in Babylonia at all. And yet, on occasion of the total eclipse in 1851, it turned out that Buerg's Tables were more correct than those of Burckhardt and Damoiseau, based on the Almagest.

In one word, the Babylonian eclipses are by no means confirmed by Hansen's and other Tables. It is Ptolemy who, A.D. 140, determined the times and magnitudes of the Babylonian eclipses, having referred them to wrong years.

4. The present theory of the moon's motions, especially Hansen's Lunar Tables, principally derived from the Babylonian eclipses, have been ratified by eminent astronomers. It is true, Prof. Airy determined the dates of three famous eclipses of the sun by means of Hansen's Tables. (See Phil. Transac. of the R. Astron. Soc. of London, vol. 8, p. 92; Monthly Not. of the London Astr. Soc. 1857, pp. 233–355.) First, he referred the total eclipse of the sun of Agathocles, Arch. Hieromnemon (p. 472, No. 24), to —309, Aug. 14th, 21h. 15m., ☊ 4° W.; but, unfortunately, this eclipse belonged to *χειμών*, and not, as history reports, to *θέρος* (p. 418), and the archons of this time ruled, as we have seen, two years later. Consequently, this eclipse ought to have been referred to —306, June 13, 22h., ☊ 0° 45′, correctly 4° W. of the sun. Accordingly, this eclipse does not confirm, but confutes, the present theory of the moon. — The second eclipse by which Prof. Airy intended to assure the present lunar theory, especially Hansen's Tables, is that observed on occasion of the expugnation of Nineveh and the destruction of the Medo-Babylonian supremacy in Asia, which eclipse he referred to —556, May 19th, 2h. 15m. P. T.; but this eclipse is refuted by a great many of the most reliably ascertained historical events, as we have seen (p. 483). For instance, the Babylonian Captivity, it is universally known, commenced in —601, the first year of Nebuchadnezzar (p. 498), and hence, had Cyrus destroyed Nineveh in —556, the captivity would have lasted 45 years only. Who is able to believe that the coëval Prophets and chroniclers did not know the duration of the Babylonian captivity, which lasted, as they narrate, seventy years and some months? And yet Airy persists that about that time, and during a period of forty years, only one total eclipse of the sun was, according to the present lunar theory and Hansen's Tables, possible in Nineveh, viz. that in —556. The

date of this eclipse, moreover, is refuted by the Apis periods, by the *turnus* of the priests, by Daniel, by the reports concerning the years in which Cyrus was born, in which he conquered Babylon and Nineveh, and died, as we have seen (p. 483). Finally, a short time since, Airy himself conceded the untenableness of Hansen's Tables, for our newspapers report the following item: "In his last report, Prof. Airy devotes a few words to the great work he has been engaged in, namely, the preparation for the formation of Lunar Tables, *according to a new treatment of the theory* by which he hopes to be able to give greater accuracy to the final results by means of operations which are entirely numerical throughout the work. Considerable progress has been made in these numerical developments, and he expects, at least, to put *his theory* in such a state that there will be no danger of its entire loss in the event of his death." In one word, Prof. Airy himself discovered Hansen's theory to be incorrect.—The third and last eclipse computed by Airy for vindicating the usual lunar motions and Hansen's Tables is that in —584, May 28th, 4h. 15m. P. T., ☊ 2° W., which he referred to the battle-field on the Halys (Her. i. 74). As this eclipse, however, was, according to Hansen's Tables, not total on the Halys, Airy was compelled to place the battle-field between Smyrna, Tarsus, Ancyra, Iconium, and Issus. Moreover, since Cyrus was born, as we have seen (p. 485), in —596; and since Mandane, the mother of Cyrus, was born one year after the battle on the Halys, viz. in —620, the strange event came to pass that Cyrus was wonderfully born thirteen years prior to his mother.

We proceed now to the 16 eclipses in "Nature," 1872, July 25, p. 251, carefully computed by Prof. Hind, by which the present theory of the moon and Hansen's Table appeared to be mathematically justified.

No. 1 relies on a cuneiform inscription, explained by Rawlinson. I do not know either how far the study of the cuneiform literature of the Assyrians has since my "Alphabeta genuina," Lipsiæ, 1840, p. 133, advanced, or what reasons led Hind to refer this presumed eclipse to —762, June 15. At that time no chronological eras existed except those of single kings; consequently the hypothetic eclipse must concern a certain year of a certain Assyrian king whose name is not determined by Rawlinson.

Moreover, the kings of that time, as we have seen (p. 507), reigning three years later than Ptolemy made the world believe, this eclipse would belong to —759. I fear, moreover, that the date of this eclipse was not fixed at all in the inscription, and that its epoch was made out only by the aid of Hansen's Tables. Pursuant to my approximate corrections of the latter, the longitude of the Lunar nodes was, about that time, shorter by 7° 14′. By the way, this solar eclipse is not all the "*terminus a quo* for researches on the historical eclipses"; for the eclipse observed during the building of Rome is of the same age, and, being fixed both by a planetary configuration and subsequent ascertained eclipses, it is much more reliably ascertained than that of Rawlinson. Moreover, the Chinese eclipse of —2192, likewise fixed by a planetary configuration, is 1300 years older than that in —762. (See page 494)

No. 2. I believe with Hind that the retrogression of the shadow on the dial of Ahaz signifies a solar eclipse. Hind refers it to —688, Jan. 10th, 22h. 15m., P. T.; but, since Hezekiah died in —696, and since 2 Kings xx. 6, reports the phenomenon to have taken place fifteen years prior to the king's death, it is apparent that Hind's eclipse cannot be the true one. (See the author's "Summary," Appendix, the year —696.) Consequently, the eclipse under consideration may have been that in —715, June 5, 21h. P. T., whilst the corrected place of the ☊ was nearly 5° W. of the sun. (See the eclipse p. 439, No. 3.) The proper eclipse in —710, March 13th, 23h., happened likely after sunset in Jerusalem.

No. 3. The total eclipse of the sun predicted to the Milesians by Thales, Hind took for the same which terminated the war between the Medians and Lydians, and hence he referred the eclipse to —584, May 28th, 4h. 15m. P. T., ☊ 2° W. This statement, however, stands in opposition to all ancient reports and established facts; for, in the first place, Herodotus (i. 74) expressly states that this eclipse predicted by Thales coincided with sunrise (εἶδον νύκτα ἀντὶ ἡμέρας γενομένην), whilst Hind's eclipse took place in the afternoon (4 hours p.m.) Even Eusebius, who notoriously begins the Olympian years with the preceding local newyears day, puts the Thalesian eclipse in Ol. 48, 3, i.e. in —581, and not in —584. Further, Pliny (H. N. ii. 12, 9) refers this eclipse to Ol. 48, 4, and at the same time to u.c. 170. Now, then, the Olympiads beginning two years earlier than was formerly believed (p. 79. 98), Ol. 48, 4, extended from June, —582, to the same in —581. And, moreover, the first year post urbem conditam commencing with the Julian month of January in —751, the year u.c. 170 began with January in —581. Thus the time of the famous Thalesian eclipse predicted to the Milesians is incontrovertibly fixed; it must have taken place between January and June in the year —581, namely, during sunrise, as Herodotus says. Indeed it was the ecliptic full moon in —581, March 27th, 17h. 45m. P. T., to which Pliny and Eusebius point us. According to the present theory of the moon's motions, however, this eclipse preceded sunrise in Miletus by nearly two hours; and it was there not total at all, because the ☊ lay 2° E. of the sun. But, according to the Table on p. 429, this eclipse commenced 4h. 9m. later, and the longitude of the ☊ was about 6° 23′ shorter; it lay nearly 4° W. of the sun. How came it to pass that chronologers, being acquainted both with the æra urbis conditæ and the Olympiads, referred the year u.c. 170 and Ol. 48, 4, to the year —584? But it is much stranger still that Hind's predecessors confounded the Milesian eclipse with that on the Halys, and terminating the Lydo-Median war; for Cambyses, the son of Cyrus, reigned, as we have repeatedly seen, since —526, because in his sixth year, i.e. in —520, a new Apis period commenced: accordingly, Cyrus must have died in —526. Further, in —532, the Jews, having returned to Jerusalem, rebuilt the Altar six years and some months prior to Cyrus's death (Cyrop. viii. 7, 1), which is confirmed by the *turnus* of the Hebrew priests. Two years earlier, in —535, Cyrus conquered Babylon (Cyrop. vii. 4, 16), and Daniel, his contemporary (Dan. vi. 1), testifies that in the same year Cyrus was 62 years old; consequently the latter must have been born in —596, and this date is confirmed by Cicero (De div. i. 33), who reports that Cyrus died 70 years old. Now, then, seeing that Cyrus died in —526, being 70 years of age, it follows that he must have been born in —596. Moreover, since Cyrus conquered Babylon 9 years prior to his death, as history reports, viz. in —535, "whilst he was 62 years old," we obtain once more the very same birth-year —596. Thus Cyrus's natal year is put beyond any question. Now, Mandane, Cyrus's mother, was born, as before asserted, one year after the battle on the Halys, during which a total obscuration of the sun happened there; and this Mandane was, at the time of marrying Cyrus's father, an adult virgin of about 25 years; consequently, the eclipse on the Halys and that predicted to the Milesians by Thales must have been two separate occurrences. If we refer the eclipse in —584 to the battle on the Halys, Cyrus, being born in —596, would have been born 13 years prior to his mother. The eclipse terminating the battle on the Halys near Lydia was that in —621, May 17, 20h. 25m. P. T., which happened, as Herodotus says, "some hours after the beginning of the battle," and not with sunrise, ☊ 2° E. of the sun; but according to our Table, p. 429, 4° W. of the sun. Consequently Mandane was born in —620; and one year prior to Cyrus's birth, in —595, she was 25 years of age. About that time no other eclipse of the sun could have been total on the southern Halys; consequently Hind's eclipse of —584 clearly refutes the present theory of the moon's motions.

No. 4. The notable total eclipse of the sun near Sardis, in sight of the whole army of Xerxes, coincided with sunrise (Her. viii. 51, vii. 37), for *ἀντὶ ἡμέρας νὺξ ἐγένετο*. It was, moreover, a total one (*ὁ ἥλιος ἀφανὴς ἦν*), and happened in the early spring, 1 year and 6 months prior to the Olympian games. The latter being celebrated, subsequent to the occupation of Attica by Xerxes, during June in —477, the epoch of this eclipse is evidently fixed. In the year —478, Feb. 17th, 15h. 30m. P. T., the ☋ lay 17°, but according to my Table, p. 429, only 12° east of the sun, and the conjunction happened almost four hours later. Petavius referred Xerxes' departure from Sardis to —480, but Hind, nevertheless, had recourse to the eclipse in —477, Feb. 16th, 23h. 10m. This eclipse, however, did not coincide with sunrise, as Herodotus, born in the same year, asserts, and it was not at all a total one near Sardis, but partial and "annular," and it disagrees apparently with the epochs of the Olympian games. Besides, the solar eclipse of Cleombrotus, mentioned by Hind, and observed near Corinth one year after the same Olympian games (Herod. ix. 10) subsequent to the battle at Salamis, was that in —476, Aug. 1st, 1h. 30m., P. T., ☊ 5° west of the sun (p. 488). Hind, on the contrary, computed the eclipse in —479, Oct. 2d, 1h. P. T., ☊ 9° W., which eclipse does not correspond with all other reports. It preceded both Xerxes' passage over the Hellespont and the battle near Thermopylæ.

No. 5. The eclipse in the seventh year of Agathocles, Archon Hieromnemon, noticed during *θέρος* between Syracuse and Carthage, was, according to Hind, that in —309, Aug. 17, 21h. 15m., ☊ 4° W., i.e. the same to which Airy recurred. Since, however, this eclipse belonged to *χειμών*, and not to *θέρος*, finishing with July 2d (p. 408), and since all the archons ruled from two to three years later (p. 412), it is apparent that Hind likewise referred the eclipse of Agathocles to a wrong year. The said eclipse belongs to —306, June 13th, 22h. P. T., ☊ 0° 43′; according to our Table, p. 429, 4° 15′ W.

No. 6. The total eclipse of the sun subsequent to J. Cæsar's passage over the "frozen" Rubicon, Hind refers to —50, March 7th, 0h. 50m.; yet this eclipse was annular, and it is contradicted by all ancient reports. First, during March no river in Italy is "*covered with ice.*" Moreover, Petronius testifies that during Cæsar's march against Rome a total eclipse of the moon also took place, which was, as Pingré shows, impossible in that year —50. Further, the same historians—Petronius, Lucanus, Dio Cassius (p. 448)—put these two eclipses, happening within fifteen days, in the year U.C. 705, that is to say, in —47, and not in —50: for Rome was founded in —752, and not in —755 (p. 439). Indeed, it was only in —47 that two total eclipses were possible, in the course of January, south of the Rubicon, viz. the solar eclipse on Jan. 3, 21h. 30m. P. T., ☋ 15° E., according to our Table (p. 429) 11° E. of the sun, and the total lunar eclipse on Jan. 18, 9h. 30m. P. T., ☊ 0°, correctly 3° W. of the sun. These two so important eclipses regulate, as we have seen, the whole of the Greek and Roman histories, and they are preëminently adapted to correct the present lunar theory. The dates of these eclipses, moreover, are confirmed by the eclipses observed about the time of Cæsar's assassination, March 15th. The Fasti Capitolini, Josephus, and other authors, record that Cæsar, subsequent to his crossing the Rubicon, ruled six years; he must, therefore, have died in —41. In the same year, on March 27th, 11h. 45m. P. T., a solar eclipse (☊ 7°, corr. 10° W.) occurred in Asia, because the conjunction happened 2h. 30m. later. The lunar eclipse took place in —41, March 13th, 1h. 45m. P. T., that is, 2h. 30m. later, ☊ 7°–3° 50′

E. (p. 429). Both eclipses were invisible in Italy, but the writers who mention these two eclipses only intended to record the singular phenomenon that two eclipses had occurred about the day of Cæsar's assassination, but visible in the eastern Roman provinces only. The same year is fixed by the recurrence of the Olympian games (p. 448).

No. 7. Hind correctly determines the date of Herod's lunar eclipse, witnessed by Josephus (Ant. xvii. 6, 4), which happened in the year 0, Jan. 9th, 12h., ☊ 0° W.; for Christ was born 16 days earlier, and Herod died a few months later. Thus the error of Ideler, who puts the same eclipse four years earlier, comes to light. Ideler's eclipse, moreover, was invisible (p. 454), and Christ was not born four years prior to the accepted date of his nativity.

No. 8. The total eclipse of the sun observed at Nicæa in Bithynia (40° 30′ N. Lat., 27° 30′ Long.), which Phlegon and other ancient authors refer to the 19th year of Tiberius, and to Ol. 202, 2, and to midday, was that of A.D. 33, Sept. 11th, 22h. 30m. P. T.; for, since Augustus died A.D. 16, on the 19th day of the month of August, the 1st year of Tiberius commenced on the very same day, and his 19th year began A.D. 34, according to Roman usage. But the Egyptians and oriental nations, as is known, reckoned the reigns of the emperors from the previous local newyears day, and hence the 19th year of Tiberius commenced in Asia Minor A.D. 33. Some authors put the same eclipse in Ol. 202, 4, instead of, as we have seen, Ol. 202, 2, because some ancient chronologers were in the habit of beginning the Olympiads two years earlier. The said ecliptic conjunction took place A.D. 33, on Sept. 11th, 22h. 30m. P. T., but, according to the Table on p. 429, 2h. 18m. later. The ☋ lay, according to Lalande, 8° E., and the curve described by the moon's shadow was, according to Pingré, 78°, 63°, 33°. Since, however, the longitude of the ☋ was shorter (p. 429) by 3° 32′, it will be found both that the obscuration was total, and that the eclipse commenced really with noon in Bithynia. Hind, on the contrary, referred Phlegon's eclipse to A.D. 29, Nov. 24th, 11h. 10m. a.m. Jerusalem time; but the obscuration of the sun was not total there, and was very partial in Bithynia; it did not, moreover, begin with "the sixth hour of the day"; it also remains irreconcilable with Roman history and with the 19th year of Tiberius.

On this occasion Hind mentions a lunar eclipse, observed A.D. 33, April 3d, 3h., in Jerusalem, ☊ 7° W., obscuration 7¼ inches; but no ancient author cites it. Calvisius's "Opus Chronologicum" only mentions, as something noteworthy, that about the time of Christ's crucifixion, which, according to ancient tradition, preceded the full of the moon by one day, an obscuration of the moon had taken place. The 14th day of Nisan, the day of the crucifixion, was always the 19th day of March, Julian style (p. 414).

No. 9. The really total eclipse of the sun in Chæronea, witnessed by Plutarch (p. 482), commenced with noon (ἐκ μεσημβρίας ἀρξαμένη), and happened A.D. 73, July 22d, 22h. P. T.; for about that time only one eclipse of the sun, viz. the one just mentioned, coincided with noon, ☋ 7° E.; curve 63°–64°, 61°, 24°. It must have been total in Chæronea (38° 30′ N. Lat., 20° 46′ Long.), because the longitude of the ☋ was shorter by nearly 3° 24′. Hind examined, by means of Hansen's Tables, all eclipses of the second half of the first century, as well as the ecliptic new moons of the first part of the second century, but failed, of course, to discover Plutarch's really total eclipse. Hence the inference is self-evident.

No. 10. Eye-witnesses have certified that A.D. 418, July 19th, 7 hrs. after sunrise in Constantinople, a total eclipse of the sun happened there, and a comet was discovered during the eclipse. Hansen's Tables, on the contrary, state this eclipse to have been partial in Constantinople; it was only total several miles south of Constantinople. Since, however, the western distance of the ☊ from the sun amounted to 5°, and not to 4°, the totality of this eclipse in Constantinople is saved. This important eclipse will contribute to establish the true secular acceleration of the moon's Nodes.

No. 11. Hind alleges a total eclipse of the sun, observed in Medina A.D. 671, Dec. 6th, 22h. 3m.—which was, however, annular according to Hansen's Tables—and the obscuration of the sun amounted to $\frac{85}{100}$ only, ☊ 6° W. The longitude of the ☊ being shorter by 1° 31′ (p. 429), and that of the perigee shorter by 4°, I conjecture that this eclipse was total in Medina.

No. 12. It is reported that A.D. 840, May 5th, 1h. 15m., a total eclipse of the sun was seen in Worms (Lat. 49° 38′), and yet its totality was visible, according to Hansen's Tables, but 100 miles south of Worms. As, however, the ☋ lay not 6° but 5° east of the sun, it is evident that the obscuration of the sun must have been smaller in Worms, probably total in Strasburg. The correction of the time (+46m.) and the parallax alter this result a little. According to Pingré, the central shadow of the moon described the following curve: 43°, 45°, 49°–37°.

No. 13. English and Holland chroniclers (Calvisius's Opus Chr.) narrate that A.D. 1133, Aug. 2d, about noon, a total eclipse of the sun happened both in London and Bruegge. Hansen's Tables state that the central line of the shadow traversed Northumberland, because the longitude of the ☋ (8° E. of the sun) was too great. Pingré's curve is 55°, 50°, 13°. Since the ☋, however, lay about 36′ nearer to the sun (p. 429), its total obscuration was nearer Bruegge by many miles. In recalculating this eclipse, the correction of the place of the apogee (–5′) is to be taken into account.

Nos. 14, 15, and 16, finally, refer to certain days of A.D. 1433, 1598, and 1652; and the magnitudes of these solar eclipses, computed by Hind, remain nearly the same. For my approximate corrections of the secular accelerations of the moon, her Nodes and Apsides, are not very important for epochs of such modern dates. Besides, the localities of the related solar eclipses, sometimes copied in the chronicles of that time, are not always accurate and reliable.

Summarily, the objection that the present theory of the moon, especially Hansen's Tables, has been confirmed by Hind's Ancient Eclipses, misses the mark; for, in the first place, the majority of the computed eclipses are irreconcilable with history, fixed by mathematical facts. Prof. Airy, moreover, being at present about to establish, as he himself says, another theory of the moon's motions, I do not doubt that Prof. Hind is now equally convinced that the usual theory concerning the accelerations of the moon, her Nodes and Apsides, deduced from the eclipses in the Almagest, is no longer tenable in face of the really observed eclipses of the Greeks and Romans; otherwise, Prof. Hind would certainly have disavowed, either publicly or privately, the contents of my missive of Feb. 11, 1873; for, *amicus Plato, magis amica veritas.*

5. The most important objection is, no doubt, the following:—

The present theory of the moon's motions is deduced from the unalterable law of gravitation; consequently all other lunar theories essentially differing from the present are to be rejected. To this argument of professed astronomers I have, of course, to respectfully submit. And yet, as a reminder, I venture to offer the following facts. Granted that Damoiseau's Tables, Airy's corrections inclusive, are based upon the immutable law of gravitation, including the influence of the planets,—how comes it to pass that these Tables disagreed so much with the observation of the total eclipse in 1851? How comes it that all the total eclipses of the sun, verified by Greek and Roman historians, are partial when computed by Damoiseau and Hansen's Tables? that, according to the same Tables, many ascertained ancient eclipses were invisible, that the latter preceded sunrise and the attested hours of the day? How is the phenomenon to be explained that the numberless Lunar Tables, constructed successively from Ptolemy to Hansen, differ so much from each other concerning the mean motions and the secular accelerations of the moon, her Apsides and Nodes; that, moreover, all these Tables, some years after their construction, proved useless? I refer to the statements of the Tabulæ Prudenicæ, of Rudolph, Marinus, Pagan, La Hire, Cassini, Clairaut, Halley, Mayer, Mason, Lalande, La Place, Wurm, Buerg, Voiron, Burckhardt, Bouvoir, Euler, Damoiseau, Hansteen, Airy, Hansen, which I compared with each other. How is it that Buerg's Tables, based on 3200 Greenwich observations, agree much better with the eclipse in 1851 than those of Burckhardt? that Prof. Airy, being down to 1875 fully convinced of the correctness of Hansen's theory, is just now occupied with a *new theory* of the moon's motions?

It is not yet known what will result from the researches of this distinguished astronomer. Should he, by a closer examination of the planetary attractions, arrive at the result that in −2300 the longitude of the moon was shorter by about 6°, that of the Nodes shorter by about 18°, that of the perigee nearly 8° shorter than the usual Tables state, then he will be under the necessity of abandoning the eclipses in the Almagest, as I have done, since 1846, in different places. On the other hand, should it be impracticable, in this way, to explain the greatly accelerating motions of the moon, as specified in our Table, p. 429, which is *approximately* deduced from the authenticity of classic eclipses, then it is to be borne in mind that the possibility of expounding the fact is not yet exhausted. The exploration of nature has not yet reached the end. In the first place, it is well known that the comets accelerate, and this phenomenon is put to account of the resistance of the ether. Humboldt (Kosm. p. 406, Philad. ed.) presumes this fluidity to move round the sun from west to east, and points us to the zodiacal light and to the evaporating tails of the comets. That ethereal substance, of whatsoever nature it may be, opposes the motion of all heavenly bodies of our system; and hence, the moon, more and more retarded, and consequently attracted, by the earth, must gradually perform shorter revolutions. In this way, perhaps, the aforesaid accelerating motions of the moon can be explained.

The following results of the preceding researches are true:

1. All eclipses in the Almagest, apart from the four later, occured in other years than was formerly accepted *bona fide*. Their times and other minutiæ are the result of Ptolemy's erroneous computations. Consequently, any theory of the lunar motions deduced from Ptolemy's Babylonian eclipses must necessarily be incorrect.

The chronology of the other eclipses discussed in the premises is true, because it is both based upon the reports of intelligent and honest eye-witnesses, and confirmed by infallible mathematical facts.

3. No Lunar Tables are to be considered correct as long as they disagree with the most reliably ascertained eclipses of the ancients, and as long as the former turn all total eclipses of the classics into partial or annular ones, many small eclipses into invisible ones, and according to which all eclipses coinciding with sunrise, and those of which the hours are fixed by ancient authorities, preceded sunrise and the attested times.

Catalogue of all Classic and other Eclipses discussed in the premises.

The following eclipses refer to astronomical and not to historical years before Christ.

The added hours and minutes of the eclipses are according to Paris time, wherever no local time is mentioned.

☉ signifies a solar eclipse, of which the magnitude and hours have not been fixed by ancient classic authors.

☽ signifies a lunar eclipse, of which the magnitude and hours are not specified by ancient classic authors.

☉ * and ☽ * signify a solar or lunar eclipse reported to have been total.

☉☽ a solar eclipse which has been nearly total (μηνοειδής) according to ancient classic authors.

☉ ⊎ and ☽ ⊎ eclipses of the sun and the moon reported to have been small ones.

☉ ☉ solar eclipses coinciding with sunrise according to ancient classic authorities.

☉ ‖ and ☽ ‖ solar and lunar eclipses, of which the hours have been specified by the same authors.

The ciphers below ☉ and ☽ signify the pages in the premises where the respective eclipses have been discussed.

The ciphers below the times of the eclipses mean the approximate corrections of the Lunar Table, p. 429–30.

The ciphers below ☊ and ☋ refer to the approximate corrections of the Lunar Table, p. 429–30.

E. (east), W. (west) of the centres of the sun's shadow respectively.

NO.	ECLIPSES.	LOCALITIES.	TIMES.	☊ ☋
1	☽ ⊎	Tanis, Northern	—2780, May 23, 15h. Par. T.	☊ 12° E.
	495........	Egypt.......	+ 14h. ±	—21° ±
2	☉ * ‖	Gan-y-hein and	—2192, Oct. 10, 7h.	☊ 10° E.
	492	Pekin, China	+ 11h.	—17° 15′
3	☉ * ‖	Rome, Latium.	— 771, Nov. 19, 0h. 45m.....	☋ 14° E.
	438........	....	+ 4h. 43m.....	—7° 17′
4	☉ ⊎ ‖	Rome and Teos,	— 752, May 25, 16h.	☊ 8° E.
	439........	Asia Minor ..	+ 4h. 40m....	—7° 11′
5	☽	Babylon	— 717, Jan. 16, 4h.........	☋ 5° E.
	507........	....	+ 4h. 33m.....	— 7° 3′
6	☽	Babylon	— 716, Nov. 26, 0h.	☋ 14° E.
	508	....	+ 4h. 33m.	— 7° 3′
7	☽	Babylon.	— 715, May 21, 5h.	☊ 19° E.
	508........	....	+ 4h. 33m.	— 7° 3′
8	☉ * ‖	Rome	— 715, June 5, 21h. 15m. ...	☊ 2° E.
	439........	....	+ 4h. 33m. ...	— 7° 3′
9	☉	Jerusalem.....	— 715, June 5, 21h. 15m....	☊ 2° E.
	516........	....	?—710, Mar. 13, 23h.........	☊ 4° W.
		....	+ 4h. 33m....	— 7° 3′
10	☉ * ?	Rome	— 642, Jan. 11, 18h.	☊ 1° E.
	440	....	+ 4h. 18m....	— 6° 38′

NO.	ECLIPSES.	LOCALITIES.	TIMES.	☊ ☋
11	☉ ✱ 485	Southern Halys, Asia Minor	— 621, May 17, 20h. 15m. + 4h. 14m.	☊ 2° 46′ E. — 6° 32′
12	☽ 507	Babylon	— 619, Oct. 6, 12h. + 4h. 14m.	☋ 0° E. — 6° 32′
13	☉ ✱ ☉ 441, 515, 516	Miletus, Asia Minor	— 581, March 27, 17h. 45m. + 4h. 9m.	☊ 2° E. — 6° 23°
14	☉ 486	Greece	— 538, Nov. 22, 19h. + 3h. 57m.	☊ 8° W. — 6° 6′
15	☉ ✱ ? 484, 514	Laryssa (Nine- veh), Mosul	— 532, Jan. 26, 15h. 45m. + 3h. 57m.	☋ 20° E. — 6° 5′
16	☽ 506	Babylon	— 519, Nov. 8, 2h. + 3h. 54m.	☋ 8° E. — 6°
17	☉ 486	Greece	— 519, Nov. 22, 17h. + 3h. 54m.	☊ 7° W. — 6°
18	☽ 507	Babylon	— 499, May 4, 10h. 15m. + 3h. 50m.	☊ 4° W. — 5° 55′
19	☽ 507	Babylon	— 489, Oct. 8, 4h. 30m. + 3h. 49m.	☋ 1° W. — 5° 52′
20	☉ ✱ ☉ 486, 518	Smyrna	— 478, Feb. 27, 15h. 30m. + 3h. 46m.	☋ 17° E. — 5° 49′
21	☉ ◡ 488	Corinth	— 476, Aug. 1, 1h. 30m. + 3h. 46m.	☊ 0° E. — 5° 49′
22	☉ ✱ 489	Thebes, Thes- salia	— 469, March 20, 1h. 30m. + 3h. 45m.	☊ 1° E. — 5° 47′
23	☉ ✱ 488	Athens	— 465, Dec. 25, 20h. + 3h. 44m.	☊ 6° W. — 5° 46′
24	☉ 488	Greece	— 460, March 9, 23h. 30m. + 3h. 43m.	☋ 16° E. — 5° 44′
25	☉ ☽ 473	Athens	— 429, Jan. 26, 22h. + 3h. 37m.	☊ 1° E. — 5° 35′
26	☽ 474	Athens	— 421, Aug. 8, 0h. 15m. + 3h. 35m.	☋ 10° E. — 5° 33′
27	☉ ◡ 475	Athens	— 420, Jan. 18, 2h. + 3h. 35m.	☋ 17° E. — 5° 32′
28	☽ ✱ 475	Athens	— 420, Feb. 2, 6h. + 3h. 35m.	☊ 2° E. — 5° 32′
29	☽ ✱ 476	Sicily	— 410, July 8, 7h. 45m. + 3h. 33m.	☊ 7° E. — 5° 29′
30	☽ 476	Athens	— 403, Feb. 23, 6h. 30m. + 3h. 31m.	☋ 9° E. — 5° 27′
31	☉ 476	Athens	— 401, Jan. 17, 21h. 30m. + 3h. 31m.	☋ 10° E. — 5° 26′
32	☉ ✱ 441	Rome	— 400, July 1, 17h. 45m. + 3h. 31m.	☊ 1° 45′ E. — 5° 26′
33	☉ ☽ 447	Bœotia, N.	— 391, Jan. 26, 22h. 30m. + 3h. 29m.	☊ 9° W. — 5° 24′
34	☽ 506	Babylon	— 381, Dec. 12, 9h. 30m. + 3h. 28m.	☋ 2° W. — 5° 20′
35	☽ 506	Babylon	— 380, June 6h. 7h. 45m. + 3h. 28m.	☊ 3° E. — 5° 20′
36	☽ 506	Babylon	— 380, Dec. 1, 0h. 30m. + 3h. 28m.	☋ 1° W. — 5° 20′
37	☉ (great) 478	Thebes, Bœotia	— 360, May 12, 3h. 15m. + 3h. 25m.	☊ 1° W. — 5° 14′
38	☉ 478	Syracuse, Sicily	— 356, Feb. 28, 23h. 30m. + 3h. 24m.	☊ 4° W. — 5° 14′

NO.	ECLIPSES.	LOCALITIES.	TIMES.	☊ ☋
39	☽ ✱ ...... 479........	Sicily	— 356, Aug. 9, 6h. 45m..... + 3h. 24m.....	☋ 10° E. — 5° 14′
40	⊙✱⊙.... 441........	Rome	— 340, Sept. 25, 18h........ + 3h. 20m..	☋ 10° E. — 5° 9′
41	☽ ✱ ‖ .... 480........	Arbela and Sicily	— 330, Sept. 20, 7h. 30m.... + 3h. 19m....	☊ 4° E. — 5° 6′
42	☽ ◡ ‖ ..... 480........	Arbela and Carthage........	— 328, Aug. 29, 12h......... + 3h. 18m...	☊ 9° W. — 5° 5′
43	⊙ ✱ 481, 514. 519	S.E. of Syracuse, Sicily ..	— 306, June 13, 22h. 45m... + 3h. 14m...	☊ 0°45′ E. — 5° 0′
44	⊙ .... 442........	Rome	— 293, March 23, 23h....... + 3h. 13m..	☋ 12° E. — 4° 55′
45	☽ .... 442........	Mysia, Asia Minor	— 217, March 9, 4h......... + 3h. 58m....	☋ 3° W. — 4° 34′
46	◡ ...... 442........	Sardinia.......	— 216, Feb. 11, 2h. 30m..... + 2h. 58m.....	☋ 5° E. — 4° 34′
47	⊙ ✱ ...... 443........	Zama, Africa	— 201, Oct. 18, 23h. 30m.... + 2h. 56m....	☊ 2° W. — 4° 31′
48	⊙ ◡ ...... 443........	Cumæ, near Rome	— 199, March 3, 22h........ + 2h. 56m...	☋ 13° E. — 4° 31′
49	☽ .... 505........	Greece	— 199, Sept. 12, 13h. 30m... + 2h. 55m...	☋ 3° W. — 4° 30′
50	☽ .. .. 505........	Greece	— 197, July 23, 11h. 45m.... + 2h. 55m....	☋ 11° E. — 4° 30′
51	⊙ ◡ ..... 444........	Rome	— 197, Aug. 6, 15h. 30m.... + 2h. 55m....	☊ 3° W. — 4° 30′
52	☽ .... 505........	Greece	— 196, Jan. 16, 5h. 30m..... + 2h. 55m.....	☋ 5° E. — 4° 30′
53	⊙ (great) ‖ 444........	Rome	— 187, July 16, 20h......... + 2h. 54m....	☋ 4° E. — 4° 27′
54	⊙ (great) ‖ 444........	Rome	— 186, Jan. 10, 23h. 30m.... — 2h. 54m....	☊ 3° W. — 4° 27′
55	☽ .... 508........	Egypt.........	— 171, Sept. 2, 11h. 30m.... — 2h. 50m....	☋ 5°59′ E. — 4° 24′
56	☽ ✱ ...... 445........	Apollonia, Greece	— 167, June 21, 7h. 45m.... — 2h. 50m....	☋ 3° E. — 4° 24′
57	☽ ‖ .... 445........	Pydna, Macedonia.	— 166, June 10, 13h. 30m... + 2h. 50m...	☋ 5° W. — 4° 24′
58	☽ .... 505........	Rhodus........	— 138, June 1, 10h. 15m..... + 2h. 45m.....	☋ 2° W. — 4° 15′
59	☽ .... 481........	Athens	— 126, Oct. 14. 13h. 30m.... + 2h. 44m....	☊ 9° W. — 4° 12′
60	⊙ (great) ‖ 445...... ..	Rome	— 102, Dec. 2, 19h.......... + 2h. 40m.....	☋ 15 E. — 4° 6′
61	☽ ✱ ...... 446........	Rome	— 62, Oct. 27, 7h. 30m...... + 2h. 34m......	☊ 5°37′ E. — 3° 57′
62	⊙ (great) ‖ 447........	Rome	— 60, March 27, 4h. 15m.... + 2h. 34m.....	☊ 0° W. — 3° 57′
63	⊙ ✱ ...... 447, 519 ...	N. of Rome....	— 47, Jan. 3, 21h. 30m...... + 2h. 30m......	☋ 15° E. — 3° 53′
64	☽ ✱ ...... 448........	Rome	— 47, Jan. 18, 9h. 30m...... + 2h. 30m......	☊ 0° W. — 3° 53′
65	☽ ✱ ...... 448........	Asia	— 41, March 13, 1h. 45m.... + 2h. 30m..... ..	☊ 7° E. — 3° 50′
66	⊙ ✱ ...... 448........	Asia	— 41, March 27, 11h. 45m... + 2h. 30m....	☊ 7° W — 3° 50′

NO.	ECLIPSES.	LOCALITIES.	TIMES.	☊ ☋
67	⊙ ⊍ …… 453 ……	Rome ………	— 40, Aug. 10, 16h. 15m…. + 2h. 29m….	☋ 14° E. — 3° 50′
68	⊙ (great).. 453 ……	Rome ………	— 39, July 30, 18h. 15m….. + 2h. 29m…..	☋ 6° E. — 3° 50′
69	⊙ …… 454 ……	Rome ………	— 37, Jan. 13, 21h. 30m….. + 2h. 28m…..	☊ 9° W. — 3° 50′
70	⊙ …… 454 ……	Rome ………	— 34, Oct. 31, 22h……… + 2h. 27m….	☊ 7° W. — 3° 49′
71	⊙ …… 454 ……	Rome ………	— 28, Jan. 4, 19h……… + 2h. 26m….	☋ 10° E. — 3° 48′
72	☽ …… 454 ……	Jerusalem …..	+ 0, Jan. 9, 11h. 30m…… + 2h. 24m……	☊ 0° W. — 3° 42′
73	⊙ ⊍ …… 455 ……	Rome ………	+ 7, Feb. 5, 23h……… + 2h. 23m……	☋ 15° E. — 3° 39′
74	⊙ …… 455 ……	Egypt? …….	+ 16, Aug. 20, 17h……… + 2h. 22m……	☋ 2° E. — 3° 37′
75	☽ * …… 455 ……	Laybach, Tyrol	+ 17, Jan. 30, 8h……… + 2h. 22m……	☊ 8° E. — 3° 37′
76	⊙ * ‖ …. 456, 520 …	Nicæa, Bithynia	+ 33, Sept. 11, 22h. 30m…. + 2h. 18m….	☋ 8° E. — 3° 32′
77	⊙ …… 456 ……	Rome ………	+ 45, July 31, 22h……… + 2h. 16m…..	☊ 0° W. — 3° 30′
78	☽ …… 457 ……	Rome ………	+ 47, June 25, 15h. 30m….. + 2h. 15m…..	☊ 1° E. — 3° 29′
79	☽ …… 457 ……	Rome. ………	+ 48, June 14, 6h……… + 2h. 15m…..	☊ 7° W. — 3° 29′
80	⊙ * ‖ …. 457 ……	Naples and Ar- menia …….	+ 60, Oct. 12, 19h……… + 2h. 13m…..	☊ 6° W. — 3° 27′
81	⊙ …… 458 ……	Rome ………	+ 67, May 31, 3h……… + 2h. 13m…..	☊ 3° W. — 3° 25′
82	☽ …… 460 ……	Rome ………	+ 68, May 5, 12h……… + 2h. 13m…..	☊ 2° E. — 3° 25′
83	☽ * …… 460 ……	Rome? ………	+ 68, Oct. 28, 18h. 30m….. + 2h. 13m…..	☊ 2° W. — 3° 25′
84	☽ …… 460 ……	Rome ………	+ 71, March 4, 8h……… + 2h. 12m…..	☋ 8° E. — 3° 25′
85	⊙ …… 640 ……	Rome ………	+ 71, March 19, 21h. 30m… + 2h. 12m…..	☊ 7° W. — 3° 25′
86	⊙ * ‖ …. 482, 461, 521	Chæronea, Bœ- tia ………	+ 73, July 22, 22h……… + 2h. 12m. …	☋ 7° E. — 3° 24′
87	⊙ ? …… 461 ……	Ephesus, Asia Minor …….	+ 95, May 21, 15h. 30m….. + 2h. 10m…..	☋ 5° E. — 3° 20′
88	⊙ …… 461 ……	Rome ………	+ 99, Sept. 2, 22h……… + 2h. 9m…..	☊ 2° E. — 3° 19′
89	⊙ …… 461 ……	Rome ………	+ 118, Sept. 2, 22h. 30m….. + 2h. 6m. ….	☊ 6° W. — 3° 15′
90	☽ …… 509 ……	Alexandria, Egypt …….	+ 125, April 5, 7h……… + 2h. 5m…..	☊ 10° E. — 3° 13′
91	☽ …… 509 ……	Alexandria, Egypt …….	+ 133, May 6, 9h. 15m…… + 2h. 4m……	☋ 5° E. — 3° 11′
92	☽ …… 509 ……	Alexandria, Egypt …….	+ 134, Oct. 20, 9h. 15m….. + 2h. 4m……	☊ 5° W. — 3° 11′
93	☽ …… 509 ……	Alexandria, Egypt …….	+ 136, March 5, 14h……… + 2h. 4m…….	☋ 9° E. — 3° 11′
94	⊙ * ? …. 461 ……	Utica, near Car- thage…….	+ 200, March 31, 21h. 30m.. + 1h. 55m..	☊ 4° E. — 2° 58′

NO.	ECLIPSES.	LOCALITIES.	TIMES.	☊ ☋
95	☉ ✱	Rome	+ 237, April 12, 3h. 30m....	☊ 2° W.
	462........	....	+ 1h. 49m....	— 2° 50′
96	☉.........	Rome	+ 239, Aug. 16, 22h.........	☋ 12° E.
	462........	....	+ 1h. 48m....	— 2° 49′
97	☉ (great) .	Rome	+ 291, May 15, 2h. 30m.....	☊ 0° W.
	463........	....	+ 1h. 43m.....	— 2° 39′
98	☽ ✱	Rome	+ 303, Sept. 11, 7h. 30m.....	☊ 5° E.
	463........	....	+ 1h. 42m.....	— 2° 37′
99	☉	Constantinople.	+ 316, Dec. 30, 19h. 30m....	☊ 2° W.
	464........	....	+ 1h. 39m....	— 2° 36′
100	☉ ‖	Constantinople.	+ 317, Dec. 20, 1h..........	☊ 11° W.
	464........	....	+ 1h. 39m.....	— 2° 36′
101	☉ ✱ ‖	Campania, Italy	+ 324, Aug. 6, 2h...........	☊ 4° W.
	464........	....	+ 1h. 37m......	— 2° 36′
102	☉ ‖	Rome or Sicily.	+ 334, July 16, 23h. 30m....	☊ 2° E.
	464........	....	+ 1h. 35m....	— 2° 35′
103	☉ ✱ ‖	Constantinople.	+ 345, June 16, 1h..........	☊ 1° E.
	465........	....	+ 1h. 34m.....	— 2° 34′
104	☉ ‖	Constantinople.	+ 346, June 5, 17h. 30m.....	☊ 7° W.
	465........	....	+ 1h. 34m.....	— 2° 33′
105	☉	Constantinople.	+ 347, Oct. 20, 3h...........	☋ 14° E.
	465........	....	+ 1h. 33m......	— 2° 33′
106	☉	Constantinople.	+ 348, Oct. 8, 20h	☋ 5° 17′ E.
	466........	....	+ 1h. 33m.....	— 2° 33′
107	☉ ✱ ☉....	Mesopotamia ..	+ 360, Aug. 27, 16h.........	☊ 3° W.
	466........	....	+ 1h. 32m....	— 2° 32′
108	☉	Alexandria,	+ 364, June 16, 1h..........	☊ 6° W.
	466........	Egypt.......	+ 1h. 32m....	— 2° 32′
109	☽ ✱	Alexandria,	+ 364, Nov. 25, 14h.........	☋ 6° E.
	467........	Egypt.......	+ 1h. 32m....	— 2° 32′
110	☉ ‖	Alexandria,	+ 374, Nov. 19, 22h. 30m....	☊ 2° W.
	467........	Egypt.......	+ 1h. 31m......	— 2° 32′
111	☉	Alexandria,	+ 378, Sept. 7, 23h. 30m.....	☊ 2° W.
	467........	Egypt.......	+ 1h. 31m.....	— 2° 31′
112	☉ ‖	Rome or Con-	+ 393, Nov. 19, 23h.........	☊ 10° W.
	458........	stantinople ..	+ 1h. 29m....	— 2° 29′
113	☉ ✱ ‖	Constantinople.	+ 418, July 18, 19h........ ..	☊ 4° W.
	521........	....	+ 1h. 26m....	— 2° 23′
114	☉ ✱	Medina, Arabia	+ 671, Dec. 6, 22h. 3m......	☊ 6° W.
	521........	....	+ 1h. 0m......	— 1° 31′
115	☉ ✱......	Worms? Stras-	+ 840, May 5, 1h. 15m......	☋ 6° E.
	521........	burg	+ 46m......	— 1° 7′
116	☉ ✱......	London and	+ 1133, Aug. 2, noon........	☋ 8° E.
	522........	Bruegge	+ 22m......	— 36′

To these 116 eclipses the following total obscurations of the sun, mentioned in later chronicles, and collected in "Calvisius's Opus Chronologicum," may be added, because they will probably confirm or amend our approximate corrections of the present theory of the secular accelerations of the moon's motions. Pingré's curves are enclosed in brackets.

NO.	ECLIPSES.	LOCALITIES.	TIMES.	☊ ☋
117	☉ *	Constantinople	+ 693, Oct. 3, 23h. [47°, 24°, 9° — 10°] .. + 58m	☊ 6° W. — 1° 29′
118	☉ *	Toledo, Spain	+ 718, June 3, 1h. [38° — 22°]. .... + 53m.	☊ 2° W. — 1° 25′
119	☉ * [A.]	Girwic, England..	+ 733, Aug. 21, 22h. [55°, 47°, 15°] ..... + 53 m.	☊ 5° W. — 1° 2?′
120	☉ *	Edessa, Syria.....	+ 812, May 14, 0h. [28°, 30°, 34°—23°] . + 48m.	☊ 2° W. — 1° 14′
121	☉ * (?)..	Paris.......	+ 878, Oct. 29, 1h. [57°, 55°, 48°—51°].. + 42m.	☊ 10° W. — 1° 3′
122	☉ *	Stiklastad, Norway..	+ 1030, Aug. 31, 2h. [73° — 27°]......... + 29m. .... (Schumacher's Astron. N. 1849, p. 46.)	☊ 10° W. — 46′
123	☉ *	Paris	+ 1230, May 12, 17h. [52°, 87°, 90°] .... + 17m.	☋ 10° E. — 27′
124	☉ *	Lesina and Mirabeau.	+ 1239, June 1, 23h. [36°, 42°, 43°—25°]. + 17m.	☊ 3° W. — 26′
125	☉ *	Rheims and Erfurt....	+ 1241, Oct. 6, 0h. [56°, 47°, 30°] .... + 27m.	☋ 10° E. — 26′
126	☉ * [A.]	Constantinople	+ 1255, Dec. 30, 2h. [31°, 32°, 51°]...... + 16m.	☋ 9° E. — 25′
127	☉ *	Constantinople	+ 1406, June 14, 18h. [41°, 69°, 58°] .... + 10m.	☊ 7° W. — 14′
128	☉ *	Constantinople	+ 1433, June 17, 5h [63°—32°] .... + 8m.	☋ 7° E. — 12′
129	☉ * 59	Danzig, Germany.	+ 1851, July 28, 2h. 30m. [70°—39°] .... + 24″ ±	Long. ☊ — 37″ ±

Specification of the most important Ancient Eclipses.

Solar eclipses reported to have been total in certain places: Nos. 2, 3, 8, 10(?), 11, 13, 15, 20, 22, 23, 32, 40, 43, 47, 63, 66, 76, 80, 86, 94, 95, 101, 103, 107, 113, 114, 115, 116–128.

Solar eclipses reported to have been nearly total (μηνοειδεῖς): Nos. 25, 33.

Solar eclipses reported to have been great ones: Nos. 37, 53, 54, 60, 62, 68.

Solar eclipses reported to have been small ones: Nos. 4, 21, 27, 46, 48, 51, 56, 67, 73.

Lunar eclipses reported to have been total: Nos. 28, 29, 39, 41, 61, 64, 65, 75, 83, 98, 109.

Lunar eclipses reported to have been small ones: Nos. 1, 42.

Solar eclipses reported to have coincided with sunrise: Nos. 13, 20, 40, 107.

Solar eclipses referred by the ancients to certain hours: Nos. 2, 3, 4, 8, 53, 54, 60, 62, 76, 80, 86, 100, 101, 102, 104, 110, 112, 113.

Lunar eclipses referred by the ancients to certain hours: Nos. 42, 57.

Total eclipses of the sun which were annular according to the present lunar theory: Nos. 13, 32, 43, 47, 76, 80, 119, 126.

THE END.

5

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Art. XLII.—*Introduction and Succession of Vertebrate Life in America;* by O. C. Marsh.

[Address before the American Association for the Advancement of Science, at Nashville, Tenn., August 30, 1877.]

The origin of life, and the order of succession in which its various forms have appeared upon the earth, offer to science its most inviting and most difficult field of research. Although the primal origin of life is unknown, and may perhaps never be known, yet no one has a right to say how much of the mystery now surrounding it science cannot remove. It is certainly within the domain of science to determine when the earth was first fitted to receive life, and in what form the earliest life began. To trace that life in its manifold changes through past ages to the present is a more difficult task, but one from which modern science does not shrink. In this wide field, every earnest effort will meet some degree of success; every year will add new and important facts; and every generation will bring to light some law, in accordance with which ancient life has been changed into life as we see it around us to-day. That such a development has taken place, no one will doubt who has carefully traced any single group of animals through its past history, as recorded in the crust of the earth. The evidence will be especially conclusive, if the group selected belongs to the higher forms of life, which are sensitive to every change in their surroundings. But I am sure I need offer here no argument for evolution; since to doubt evolution to-day is to doubt science, and science is only another name for truth.

Taking, then, evolution as a key to the mysteries of past life on the earth, I invite your attention to the subject I have chosen: THE INTRODUCTION AND SUCCESSION OF VERTEBRATE LIFE IN AMERICA.

In the brief hour allotted to me, I could hardly hope to give more than a very incomplete sketch of what is now known on this subject. I shall, therefore, pass rapidly over the lower groups, and speak more particularly of the higher vertebrates, which have an especial interest to us all, in so far as they approach man in structure, and thus indicate his probable origin. These higher vertebrates, moreover, are most important witnesses of the past, since their superior organization made them ready victims to slight climatic changes, which would otherwise have remained unrecorded.

In considering the ancient life of America, it is important to bear in mind that I can only offer you a brief record of a few of the countless forms that once occupied this continent. The review I can bring before you will not be like that of a great army, when regiment after regiment with full ranks moves by in orderly succession, until the entire host has passed. My review must be more like the roll-call after a battle, when only a few scarred and crippled veterans remain to answer to their names. Or rather, it must resemble an array of relics, dug from the field of some old Trojan combat, long after the contest, when no survivor remains to tell the tale of the strife. From such an ancient battle-field, a Schliemann might unearth together the bronze shield, lance-head, and gilded helmet of a prehistoric leader, and learn from them with certainty his race and rank. Perhaps the skull might still retain the barbaric stone weapon by which his northern foe had slain him. Near by, the explorer might bring to light the commingled coat of mail and trappings of a horse and rider, so strangely different from the equipment of the chief, as to suggest a foreign ally. From these, and from the more common implements of war that fill the soil, the antiquary could determine, by patient study, what nations fought, and, perhaps, when, and why.

By this same method of research, the more ancient strata of the earth have been explored, and, in our Western wilds, veritable battle-fields, strown with the fossil skeletons of the slain, and guarded faithfully by savage superstition, have been despoiled, yielding to science treasures more rare than bronze or gold. Without such spoils, from many fields, I could not have chosen the present theme for my address to-night.

According to present knowledge, no vertebrate life is known to have existed on this continent in the Archæan, Cambrian, or Silurian periods; yet during this time, more than half of the thickness of American stratified rocks was deposited. It by no means follows that vertebrate animals of some kind did not exist here in those remote ages. Fishes are known from the Upper Silurian of Europe, and there is every probability that they will yet be discovered in our strata of the same age, if not at a still lower horizon.

In the shore deposits of the early Devonian sea, known as the Schoharie Grit, characteristic remains of Fishes were preserved, and in the deeper sea that followed, in which the Corniferous limestone was laid down, this class was well represented. During the remainder of the Devonian, Fishes continue abundant in the shallower seas, and, so far as now known, were the only type of vertebrate life. These fishes were mainly Ganoids, a group, represented in our present waters by the Gar-pike (*Lepidosteus*) and Sturgeon (*Acipenser*), but, in the Devonian sea, chiefly by the Placoderms, the exact affinities of which are somewhat in doubt. With these were Elasmobranchs, or the Shark tribe, and among them a few Chimæroids, a peculiar type, of which one or two members still survive. The Placoderms were the monarchs of the ocean. All were well protected by a massive coat of armor, and some of them attained huge dimensions. The American Devonian fishes now known are not as numerous as those of Europe, but they were larger in size, and mostly inhabitants of the open sea. Some twenty genera and forty species have been described.

The more important genera of Placoderms are, *Dinichthys*, *Aspidichthys*, and *Diplognathus*, our largest Palæozoic fishes. Others are, *Acanthaspis*, *Acantholepis*, *Coccosteus*, *Macropetalichthys*, and *Onychodus*. Among the Elasmobranchs were, *Cladodus*, *Ctenacanthus*, *Machæracanthus*, *Rhynchodus*, and *Ptyctodus*, the last two being regarded as Chimæroids. In the Chemung epoch, the great Dipterian family was introduced with *Dipterus*, *Heliodus*, and possibly *Ceratodus*. Species of the European genera, *Bothriolepis* and *Holoptychius*, have likewise been found in our Devonian deposits.

With the close of the Devonian, came the almost total extinction of the great group of Placoderms, while the Elasmobranchs, which had hitherto occupied a subordinate position, increase in numbers and size, and appear to be represented by Sharks, Rays, and Chimæras. Among the members of this group from the Carboniferous, were numerous Cestracionts, species of *Cochliodus* of large size, with others of the genera *Deltodus*, *Helodus*, *Psammodus* and *Sandalodus*. Of the Petalodonts, there were *Antliodus*, *Chomatodus*, *Ctenoptychius*, *Petalodus* and

Petalorhynchus; and of the Hybodonts, the genera *Cladodus*, *Carcharopsis* and *Diplodus*. These Elasmobranchs were the rulers of the Carboniferous open sea, and more than one hundred species have been found in the lower part of this formation alone. The Ganoids, although still abundant, were of smaller size, and denizens of the more shallow and confined waters. The latter group of fishes was represented by true Lepidostidæ, of the genera *Palæoniscus*, *Amblypterus*, *Platysomus* and *Eurylepis*. Other genera are, *Rhizodus*, *Megalichthys*, *Ctenodus*, *Edestus*, *Orodus*, *Ctenacanthus*, *Gyracanthus*, and *Cœlacanthus*. Most of these genera occur also in Europe.

From the Permian rocks of America, no vertebrate remains are known, although in the same formation of Europe Ganoids are abundant; and with them are remains of Sharks, and some other fishes, the affinities of which are doubtful. The Palæozoic fishes at present known from this country are quite as numerous as those found in Europe.

In the Mesozoic age, the Fishes of America begin to show a decided approach to those of our present waters. From the Triassic rocks, Ganoids only are known, and they are all more or less closely related to the modern Gar-pike, or *Lepidosteus*. They are of small size, and the number of individuals preserved is very large. The characteristic genera are, *Catopterus*, *Ischypterus*, *Ptycholepis*, *Rhabdolepis*, and *Turseodus*. From the Jurassic deposits, no remains of fishes are known, but in the Cretaceous, ichthyic life assumed many and various forms; and the first representatives of the Teleosts, or bony fishes, the characteristic fishes of to-day, make their appearance. In the deep open sea of this age, Elasmobranchs were the prevailing forms, Sharks and Chimæroids being most numerous. In the great inland Cretaceous sea of North America, true osseous fishes were most abundant, and among them were some of carnivorous habits, and immense size. The more sheltered bays and rivers were shared by the Ganoids and Teleosts, as their remains testify. The more common genera of Cretaceous Elasmobranchs were, *Otodus*, *Oxyrhina*, *Galeocerdo*, *Lamna* and *Ptychodus*. Among the osseous fishes, *Beryx*, *Enchodus*, *Portheus* and *Saurocephalus* were especially common, while the most important genus of Ganoids was *Lepidotus*.

The Tertiary fishes are nearly all of modern types, and from the beginning of this period there was comparatively little change. In the marine beds, Sharks, Rays and Chimæroids maintained their supremacy, although Teleosts were abundant, and many of them of large size. The Ganoids were comparatively few in number. In the earliest Eocene fresh-water deposits, it is interesting to find that the modern Gar-pike, and *Amia*, the Dog-fish of our western lakes, which by their structure are seen to be remnants of a very early type, are well represented by species so closely allied to them that only an anatomist could separate the ancient from the modern. In the succeeding beds, these fishes are still abundant, and with them are Siluroids nearly related to the modern Cat-fish (*Pimelodus*). Many small fishes, allied apparently to the modern herring (*Clupea*), left their remains in great numbers in the same deposits, and, with them has been recently found a land-locked Ray (*Heliobatis*).

The almost total absence of remains of fishes from the Miocene lake-basins of the West is a remarkable fact, and perhaps may best be explained by the theory that these inland waters, like many of the smaller lakes in the same region to-day, were so impregnated with mineral matters as to render the existence of vertebrate life in them impossible. No one who has tasted such waters, or has attempted to ford one of the modern alkaline lakes which are often met with on the present surface of the same deposits, will doubt the efficiency of this cause, or the easy entombment of the higher vertebrates that ventured within their borders. In the Pliocene lake-basins of the same region, remains of fishes were not uncommon, and in some of them are very numerous. These are all of modern types, and most of them are Cyprinoids, related to the modern Carp. The Post-pliocene fishes are essentially those of to-day.

In this brief synopsis of the past ichthyic life of this Continent, I have mentioned only a few of the more important facts, but sufficient, I trust, to give an outline of its history. Of this history, it is evident that we have as yet only a very imperfect record. We have seen that the earliest remains of fishes known in this country, are from the lower Devonian; but these old fishes show so great a diversity of form and structure, as to clearly indicate for the class a much earlier origin. In this connection, we must bear in mind that the two lowest groups of existing fishes are entirely without osseous skeletons, and hence, however abundant, would leave no permanent record in the deposits in which remains of fishes are usually preserved. It is safe to infer, from the knowledge which we now possess of the simpler forms of life, that even more of the early fishes were cartilaginous, or so destitute of hard parts as to leave no enduring traces of their existence. Without positive knowledge of such forms, and considering the great diversity of those we have, it would seem a hopeless task at present to attempt to trace successfully the genealogy of this class. One line, however, appears to be direct, from our modern Gar-pike, through the lower Eocene *Lepidosteus* to the *Lepidotus* of the Cretaceous, and perhaps on through the Triassic *Ischypterus* and Carboniferous *Palæoniscus;* but beyond this, in our rocks, it is lost. The living Chimæra of our Pacific coast has nearly allied forms in the Tertiary and Cretaceous, more distant relatives in the Carboniferous, and a possible ancestor in the Devonian *Rhynchodus*. Our Sharks likewise can be traced with some certainty back to the Palæozoic; and even the *Lepidosiren*, of South America, although its immediate predecessors are unknown, has some peculiar characters which strongly point to a Devonian ancestry. These suggestive lines indicate a rich field for investigation in the ancient life-history of American fishes.

The Amphibians, the next higher class of vertebrates, are so closely related to the fishes in structure, that some peculiar forms of the latter have been considered by anatomists as belonging to this group. The earliest evidence of Amphibian existence, on this continent, is in the Sub-Carboniferous, where foot-prints have been found which were probably made by Labyrinthodonts, the most ancient representatives of the class. Well preserved remains are abundant in the Coal Measures, and show that the Labyrinthodonts differed in important particulars from all modern Amphibians, the group which includes our frogs and salamanders. Some of these ancient animals resembled a salamander in shape, while others were serpent-like in form. None of those yet discovered were frog-like, or without a tail, although the restored Labyrinthodont of the text books is thus represented. All were protected by large pectoral bony plates, and an armor of small scutes on the ventral surface of the body. The walls of their teeth were more or less folded, whence the name Labyrinthodont. The American Amphibians known from osseous remains are all of moderate size, but the foot-prints attributed to this group indicate animals larger than any of the class yet found in the old world. The Carboniferous Amphibians were abundant in the swampy tropical forests of that period, and their remains have been found imbedded in the coal then deposited, as well as in hollow stumps of the trees left standing.

The principal genera of this group from American Carboniferous rocks, are, *Sauropus*, known only from footprints, *Baphetes*, *Dendrerpeton*, *Hylonomus*, *Hylerpeton*, *Raniceps*, *Pelion*, *Leptophractus*, *Molgophis*, *Ptyonius*, *Amphibamus*, *Cocytinus*, and *Ceraterpeton*. The last genus occurs also in Europe. Certain of these genera have been considered by some writers to be more nearly related to the Lizards, among true reptiles. Some other genera known from fragmentary remains or footprints in this formation have likewise been referred to the true reptiles, but this question can perhaps be settled only by future discoveries.

No Amphibia are known from American Permian strata, but in the Triassic, a few characteristic remains have been found. The three genera, *Dictyocephalus*, *Dispelor* and *Pariostegus*, have been described, but, although apparently all Labyrinthodonts, the remains preserved are not sufficient to add much to our knowledge of the group. The Triassic foot-prints which have been attributed to Amphibians are still more unsatisfactory, and at present no important conclusions in regard to this class can be based upon them. From the Jurassic and Cretaceous beds of this Continent, no remains of Amphibians are known. A few only have been found in the Tertiary, and these are all of modern types.

The Amphibia are so nearly allied to the Ganoid fishes, that we can hardly doubt their descent from some member of that group. With our present limited knowledge of the extinct forms, however, it would be unprofitable to attempt to trace in detail their probable genealogy.

The authors to whom especial credit is due for our knowledge of American fossil Fishes and Amphibians, are Newberry, Leidy, Cope, Dawson, Agassiz, St. John, Gibbes, Wyman, Redfield, and Emmons, and the principal literature of the subject will be found in their publications.

Reptiles and Birds form the next great division of vertebrates, the Sauropsida, and of these the Reptiles are the older type, and may be first considered. While it may be stated with certainty that there is at present no evidence of the existence of this group in American rocks older than the Carboniferous, there is some doubt in regard to their appearance even in this period. Various foot-prints which strongly resemble those made by Lizards; a few well preserved remains similar to the corresponding bones in that group; and a few characteristic specimens, nearly identical with those from another order of this class, are known from American Coal Measures. These facts, and some others which point in the same direction, render it probable that we may soon have conclusive evidence of the presence of true Reptiles in this formation, and in our overlying Permian, which is essentially a part of the same series. In the Permian rocks of Europe, true Reptiles have been found.

The Mesozoic Period has been called the Age of Reptiles, and during its continuance some of the strangest forms of reptilian life made their appearance, and became extinct. Near its commencement, while the Triassic shales and sandstones were being deposited, true reptiles were abundant. Among the most characteristic remains discovered are those of the genus *Belodon*, which is well known also in the Trias of Europe. It belongs to the Thecodont division of Reptiles, which have teeth in distinct sockets, and its nearest affinities are with the Crocodilia, of which order it may be considered the oldest known representative. In the same strata in which the Belodonts occur, remains of Dinosaurs are found, and it is a most interesting fact that these highest of reptiles should make their appearance, even in a generalized form, at this stage of the earth's history. The Dinosaurs, although true reptiles in all their more important characters, show certain well marked points of resemblance to existing birds of the order *Ratitæ*, a group which includes the Ostriches; and it is not improbable that they were the parent stock from which birds originated.

During Triassic time, the Dinosaurs attained in America an enormous development both in variety of forms and in size. Although comparatively few of their bones have as yet been discovered in the rocks of this country, they have left unmistakable evidence of their presence in the foot-prints and other impressions upon the shores of the waters which they frequented. The Triassic sandstone of the Connecticut Valley has long been famous for its fossil foot-prints, especially the so-called "bird-tracks," which are generally supposed to have been made by birds, the tracks of which many of them closely resemble. A careful investigation, however, of nearly all the specimens yet discovered, has convinced me that there is not a particle of evidence that any of these fossil impressions were made by birds. Most of these three-toed tracks were certainly not made by birds; but by quadrupeds, which usually walked upon their hind feet alone, and only occasionally put to the ground their smaller anterior extremities. I have myself detected the impressions of these anterior limbs in connection with the posterior foot-prints of nearly all of the supposed "bird-tracks" described, and have little doubt that they will eventually be found with all. These double impressions are precisely the kind which Dinosaurian reptiles would make, and as the only characteristic bones yet found in the same rocks belong to animals of this group, it is but fair to attribute all these foot-prints to Dinosaurs, even where no impressions of fore-feet have been detected, until some evidence appears that they were made by Birds. I have no doubt that Birds existed at this time, although at present the proof is wanting.

The principal genera of Triassic Reptiles known from osseous remains in this country are, *Amphisaurus* (*Megadactylus*), from the Connecticut Valley, *Bathygnathus*, from Prince Edward's Island, *Belodon* and *Clepsysaurus*. Other generic names which have been applied to foot-prints and to fragmentary remains, need not be here enumerated. A few remains of Reptiles have been found in undoubted Jurassic rocks of America, but they are not sufficiently well determined to be of service in this connection. Others have been reported from supposed Jurassic strata, which are now known to be Cretaceous. It will thus be seen that, although reptilian life was especially abundant during the Triassic and Jurassic periods, but few bones have been found. This is owing in part to the character of most of the rocks then formed, which were not well fitted for preserving such remains, although admirably adapted to retain foot-prints.

During the Cretaceous Period, Reptilian life in America attained its greatest development, and the sediments laid down in the open seas and estuaries were usually most favorable for the preservation of a faithful record of its various phases. Without such a perfect matrix as some of these deposits afford, many of the most interesting vertebrates recently brought to light from this formation would probably have remained unknown. The vast extent of these beds ensures, moreover, many future discoveries of interest.

In the lowest Cretaceous strata of the Rocky Mountain region, the Dakota group, part of which at least represents the Wealden of Europe, remains of *Chelonia*, or Turtles, Crocodiles, and Dinosaurs occur, the last being especially abundant. The *Chelonia*, although known from the Jurassic of Europe, here appear for the first time in American rocks. Some of the earliest forms are allied to the modern genus *Trionyx*. In the higher Cretaceous beds, some Chelonians of enormous size have been found. They belong to the genus *Atlantochelys*, which has the ribs separate, as in the existing *Sphargis*, and presents other embryonic characters. A few genera appear to be related to the modern genus *Chelone*. The remaining Cretaceous species were mostly of the Emydoid type; and others were related to *Chelydra*. The more important genera of Cretaceous Chelonians known from characteristic specimens are, *Atlantochelys* (*Protostega*), *Adocus*, *Bothremys*, *Compsemys*, *Plastomenus*, *Osteopygis*, *Propleura*, *Lytoloma*, and *Taphrosphys*. Most of these genera were represented by several species, and the individuals were numerous. No land Tortoises have as yet been found in this formation. In American Tertiary deposits, Chelonians are abundant, especially in the fresh-water beds. They all show near affinities with modern types, and most of them can be referred to existing genera. In the Tertiary lake-basins of the West, land Tortoises are very numerous, and with them are many fresh-water forms of *Trionyx* and allied genera.

A striking feature of the American Cretaceous fauna, as contrasted with that of Europe, is the almost entire absence in our strata of species of *Ichthyosaurus* and *Plesiosaurus*, which abound in many other regions, but here seem to be replaced by the Mosasaurs. A few fragmentary remains have indeed been referred to these genera, but the determination may fairly be questioned. This is more than true of the proposed new order *Streptosauria*, which was founded wholly on error. The order *Plesiosauria*, however, is well represented, but mainly by forms more nearly related to the genus *Pliosaurus* than to the type of the group. These were marine reptiles, all of large size, while some of them attained vast dimensions. So far as at present identified, they may be referred to the genera, *Cimoliosaurus*, *Discosaurus* (*Elasmosaurus*), and *Pliosaurus*. The number of species is comparatively few, and none are known above the Cretaceous. The important suggestion of Gegenbaur, that the *Halisauria*, which include the Plesiosaurs, branched off from the Fishes before the Amphibians, finds some support in American specimens recently discovered.

The Reptiles most characteristic of our American Cretaceous strata are the *Mosasauria*, a group with very few representatives in other parts of the world. In our Cretaceous seas, they ruled supreme, as their numbers, size, and carnivorous habits, enabled them to easily vanquish all rivals. Some were at least sixty feet in length, and the smallest ten or twelve. In the inland Cretaceous sea from which the Rocky Mountains were beginning to emerge, these ancient "Sea Serpents" abounded; and many were entombed in its muddy bottom. On one occasion, as I rode through a valley washed out of this old ocean bed, I saw no less than seven different skeletons of these monsters in sight at once. The Mosasaurs were essentially swimming Lizards, with four well developed paddles, and they had little affinity with modern serpents, to which they have been compared. The species are quite numerous, but they belong to comparatively few genera, of which *Mosasaurus*, *Tylosaurus*, *Lestosaurus* and *Edestosaurus*, have alone been identified with certainty. The genus *Mosasaurus* was first found in Europe. All the known species of the group are Cretaceous.

The *Crocodilia* are abundant in rocks of Cretaceous age in America, and two distinct types are represented. The older type, which is foreshadowed by *Belodon* of the Trias, has biconcave vertebræ, and shows marked affinities with the genus *Teleosaurus*, from the Jura of Europe. The best known genus is *Hyposaurus*, of which there are several species, all more or less resembling in form the modern Gavial of the Ganges. A peculiar intermediate form is seen in *Diplosaurus*, from the Wealden of the Rocky Mountains. The second type, which now makes its appearance for the first time, has procœlian vertebræ, and in other respects resembles existing Crocodiles. The genera described are *Bottosaurus*, *Holops* and *Thoracosaurus*, none of which, so far as known, pass above the

Cretaceous. Of *Crocodilia* with opisthocœlous vertebræ, America, so far as we know, has none. Specimens similar to those so termed in Europe, are not uncommon here, but they pertain to Dinosaurs.

In the Eocene fresh-water beds of the West, Crocodilians are especially abundant, and all, with the exception of *Limnosaurus*, belong apparently to the genus *Crocodilus*, although some species show certain points of resemblance to existing Alligators. The Miocene lake-basins of the same region contain no remains of Crocodiles, so far as known, and the Pliocene deposits have afforded only a single species. The Tertiary marine beds of the Atlantic Coast contain comparatively few Crocodilian remains, and all are of modern types; the genus *Gavialis* having one Eocene species, and the Alligator being represented only in the latest deposits.

It is worthy of special mention in this connection, that no true *Lacertilia*, or Lizards, and no *Ophidia*, or Serpents, have yet been detected in American Cretaceous beds; although their remains, if present, would hardly have escaped observation in the regions explored. The former will doubtless be found, as several species occur in the Mesozoic of Europe; and perhaps the latter, although the Ophidians are apparently a more modern type. In the Eocene lake-basins of Western America, remains of Lizards are very numerous, and indicate species much larger than any existing to-day. Some of these, the *Glyptosauridæ*, were protected by a highly ornamented bony coat of mail, and others were covered with scales, like recent Lizards. A few resembled, in their more important characters, the modern Iguana. The genera best represented in the Eocene, are, *Glyptosaurus*, *Iguanavus*, *Oreosaurus*, *Thinosaurus*, *Tinosaurus* and *Saniva*. Some of these genera appear to have continued into the Miocene, but here, as well as in the Pliocene, few remains of this group have been found. It is not improbable that some of our extinct Reptiles may prove to belong to *Rynchocephala*, but at present this is uncertain. The genus *Notosaurus*, from Brazil, has biconcave vertebræ, and some other characters which point to that group. No Dicynodonts or Theriodonts have as yet been found in this country.

The first American Serpents, so far as now known, appear in the Eocene, which contains also the oldest European species. On the Atlantic border, the genus *Titanophis* (*Dinophis*) is represented by several species of large size, one at least thirty feet in length, and all doubtless inhabitants of the sea. In the fresh-water Western Eocene, remains of snakes are abundant, but all are of moderate size. The largest of these were related to the modern Boa Constrictors. The genera described are *Boavus*, *Lithophis* and *Limnophis*. The Miocene and Pliocene

Snakes from the same region are known only from a few fragmentary remains.

The *Pterosauria*, or flying Lizards, are among the most interesting Reptiles of Mesozoic time, and many of them left their remains in the soft sediments of our inland Cretaceous sea. These were veritable Dragons, having a spread of wings of from ten to twenty-five feet. They differed essentially from the smaller Pterodactyls found in the old world, in the entire absence of teeth, showing in this respect a resemblance to modern birds; and they possess other distinctive characters. They have therefore been placed in a new order, *Pteranodontia*, from the typical genus *Pteranodon*, of which five species are known. The only other genus is *Nyctosaurus*, represented by a single species. All the specimens yet found are from essentially the same horizon, in the Chalk of Kansas. The reported discovery of remains of this order from older formations in this country is without foundation.

The strange Reptiles known as *Dinosauria*, which, as we have seen, were numerous during the deposition of our Triassic shales and sandstones, have not yet been found in American Jurassic, but were well represented here throughout the Cretaceous, and at its close became extinct. These animals possess a peculiar interest to the anatomist, since, although reptilian in all their main characters, they show clear affinities with the Birds, and have some features which may point to Mammals. The Cretaceous Dinosaurs were all of large size, and most of them walked on the hind feet alone, like modern Struthious birds. Two well marked types may be distinguished among the remains discovered in deposits of this age: the herbivorous forms, represented mainly by *Hadrosaurus*, a near ally of the *Iguanodon* of Europe; and their carnivorous enemies, of which *Dryptosaurus* (*Lælaps*) may be considered typical in this country, and *Megalosaurus* in Europe. Near the base of our Cretaceous formation, in beds which I regard as the equivalent of the European Wealden, the most gigantic forms of this order yet discovered have recently been brought to light. One of these monsters (*Titanosaurus montanus*), from Colorado, is by far the largest land animal yet discovered; its dimensions being greater than was supposed possible, in an animal that lived and moved upon the land. It was some fifty or sixty feet in length, and, when erect, at least thirty feet in height. It doubtless fed upon the foliage of the mountain forests, portions of which are preserved with its remains. With *Titanosaurus*, the bones of smaller Dinosaurs, one (*Nanosaurus*) not larger than a Cat, as well as those of Crocodiles and Turtles, are not uncommon. The recent discovery of these interesting remains, many and various, in strata that had long been pronounced by professional explorers barren of vertebrate fossils, should teach caution to those who decline to accept the imperfection of our knowledge to-day as a fair plea for the supposed absence of intermediate forms.

In the marine Cretaceous beds of the West, only a single Dinosaur (*Hadrosaurus agilis*), has been found, but in the higher fresh-water beds, which mark the close of this formation, their remains are numerous, and indicate several well marked species, if not genera. In the marine beds on the Atlantic Coast, the bones of Dinosaurs are frequently met with, and in the Upper Cretaceous Greensand of New Jersey, the type specimens of *Hadrosaurus* and *Dryptosaurus* were found. In Cretaceous fresh-water deposits on the coast of Brazil, remains of this order occur, but the specimens hitherto discovered are not sufficiently characteristic for accurate determination. This is unfortunately true of many Dinosaurian fossils from North America, but the great number of these Reptiles which lived here during the Cretaceous Period promises many future discoveries, and substantial additions to our present knowledge of the group.

The first appearance of Birds in America, according to our present knowledge, was during the Cretaceous Period, although many announcements have been made of their existence in preceding epochs. The evidence of their presence in the Trias, based on footprints and other impressions, is, at present, as we have seen, without value; although we may confidently await their discovery there, if not in older formations. *Archæopteryx*, from the European Jura, the oldest bird known, and now fortunately represented by more than a single specimen, clearly indicates a much higher antiquity for the class. The earliest American forms, at present known, are the *Odontornithes*, or Birds with teeth, which have been exhumed within the last few years, from the Chalk of Kansas. The two genera, *Hesperornis* and *Ichthyornis*, are types of distinct orders, and differ from each other and from *Archæopteryx* much more than do any existing birds among themselves; thus showing that Birds are now a closed type, and that the key to the history of the class must be sought for in the distant past.

In *Hesperornis*, we have a large aquatic bird, nearly six feet in length, with a strange combination of characters. The jaws are provided with teeth, set in grooves; the wings were rudimentary, and useless; while the legs were very similar to those of modern diving birds. This last feature was merely an adaptation, as the more important characters are Struthious, showing that *Hesperornis* was essentially a carnivorous swimming Ostrich. *Ichthyornis*, a small flying bird, was stranger still, as the teeth were in sockets; and the vertebræ biconcave, as in Fishes, and a few Reptiles. *Apatornis* and other allied forms occur in the same beds, and probably all were provided with teeth. It is strange that the companions of these ancient toothed Birds should have been Pterodactyls without teeth. In the later Cretaceous beds of the Atlantic Coast, various remains of aquatic Birds have been found, but all are apparently distinct from those of the West. The known genera of American Cretaceous birds are, *Apatornis*, *Baptornis*, *Graculavus*, *Hesperornis*, *Ichthyornis*, *Laornis*, *Lestornis*, *Palæotringa* and *Telmatornis*. These are represented by some twenty species. In Europe, but two species of Cretaceous birds are known, and both are based upon fragmentary specimens.

During the Tertiary period, Birds were numerous in this country, and all yet discovered appear to have belonged to modern types. The Eocene species described are mostly wading birds, but here, and in the later Tertiary deposits, some characteristic American forms make their appearance, strongly foreshadowing our present avian fauna. The extinct genera are the Eocene *Uintornis*, related to the Woodpeckers, and *Aletornis*, which includes several species of Waders. Among the existing genera found in our Tertiary beds are, *Aquila*, *Bubo*, *Meleagris*, *Grus*, *Graculus*, *Puffinus*, *and Catarractes*. The Great Auk (*Alca impennis*), which was once very abundant on our Northeast Coast, has become extinct within a few years.

In this brief summary of the past life of Reptiles and Birds in America, I have endeavored to exclude doubtful forms, and those very imperfectly known, preferring to present the conclusions reached by careful study, incomplete though they be, rather than weary you with a descriptive catalogue of all the fossils to which names have been applied. Even this condensed review can hardly fail to give you some conception of the wealth of our continent in the extinct forms of these groups, and thus to suggest what its actual life must have been.

Although the Trias offers at present the first unquestioned evidence of true Reptiles, we certainly should not be justified in supposing for a moment that older forms did not exist. So too in considering the different groups of Reptiles, which seem to make their first appearance at certain horizons, flourish for a time, and then decline, or disappear, every day brings evidence to show that they are but fragments of the unraveled strands which converge in the past to form the mystic cord uniting all life. If the attempt is made to follow back any single thread, and thus trace the lineage of a group, we are met by difficulties which the science of to-day can only partially remove. And yet the anatomist constantly sees in the fragments which he studies hints of relationship which are to him sure prophecies of future discoveries.

The genealogy of the *Chelonia* is at present unknown, and our American extinct forms, so far as we now have them, throw little light on their ancestry. This is essentially true, also, of our *Plesiosauria*, *Lacertilia* and *Ophidia*, although suggestive facts are not wanting to indicate possible lines of descent. With the *Crocodilia*, however, the case seems to be different, and Huxley has clearly pointed out the path for investigation. It is probable that material already exists in our museums for tracing the group through several important steps in its development. We have already seen that the modern procœlian type of this order goes back only to the Upper Cretaceous, while the *Belodonts*, of our Triassic rocks, with their biconcave vertebræ, are the oldest known Crocodilians. Our Jurassic, unfortunately, throws but little light on the intermediate forms, but we know that the line was continued, as it was in the old world through *Teleosaurus*. The beds of the Rocky Mountain Wealden have just furnished us with a genuine "missing link," a saurian (*Diplosaurus*) with essentially the skull and teeth of a modern Crocodile, and the vertebræ of its predecessor from the Trias. This peculiar reptile clearly represents an important stage in the progressive series, and evidently one soon after the separation of the Crocodile branch from the main stem. The modern Gavial type appears to have been developed about the same time, as the form was well established in the Upper Cretaceous genus, *Thoracosaurus*. The Teleosaurian group, with biconcave vertebræ, evidently the parent stock of Crocodilians, became extinct with *Hyposaurus* of the same horizon, leaving the Crocodile and Gavial, with their more perfect procœlian vertebræ, to contend for the supremacy. In the early Eocene, both of these types were abundant, but some of the Crocodiles possessed characters pointing towards the Alligators, which do not appear to have been completely differentiated until later.

Nothing is really known to-day of the earlier genealogy of the *Pterosauria*, but our American forms, without teeth, are clearly the last stage in their development before this peculiar group became extinct. The oldest European form, *Dimorphodon*, from the Lower Lias, had the entire jaws armed with teeth, and was provided with a long tail. The later genus *Pterodactylus* retained the teeth, but had essentially lost the tail; while *Ramphorhynchus* had retained the elongated tail, but had lost the teeth from the fore part of both jaws. In the genus *Pteranodon* from the American Cretaceous, the teeth are entirely absent, and the tail is a mere rudiment. In the gradual loss of the teeth and tail, these reptiles followed the same path as Birds, and might thus seem to approach them, as many have supposed. This resemblance, however, is only a superficial one, as a study of the more important characters of the Pterodactyls shows that they are an aberrant type of Reptiles, totally off the line through which the Birds were developed. The announcement made not long since in Europe, and accepted by some American authors, that the *Pterosauria*, in consequence of certain points in their structure, were essentially Birds, is directly disproved by American specimens, far more perfect than those on which the conclusion was based.

It is now generally admitted by biologists who have made a study of the vertebrates, that Birds have come down to us through the Dinosaurs, and the close affinity of the latter with recent Struthious Birds will hardly be questioned. The case amounts almost to a demonstration, if we compare, with Dinosaurs, their contemporaries, the Mesozoic Birds. The classes of Birds and Reptiles, as now living, are separated by a gulf so profound that a few years since it was cited by the opponents of evolution as the most important break in the animal series, and one which that doctrine could not bridge over. Since then, as Huxley has clearly shown, this gap has been virtually filled by the discovery of bird-like Reptiles and reptilian Birds. *Compsognathus* and *Archæopteryx* of the Old World, and *Ichthyornis* and *Hesperornis* of the New, are the stepping stones by which the evolutionist of to-day leads the doubting brother across the shallow remnant of the gulf, once thought impassable.

It remains now to consider the highest group of the Animal Kingdom, the class *Mammalia*, which includes Man. Of the existence of this class before the Trias we have no evidence, either in this country or in the Old World, and it is a significant fact that at essentially the same horizon in each hemisphere, similar low forms of Mammals make their appearance. Although only a few incomplete specimens have been discovered, they are characteristic and well preserved, and all are apparently Marsupials, the lowest Mammalian group which we know in this country, living or fossil. The American Triassic Mammals are known at present only from two small lower jaws, on which is based the genus *Dromotherium*, supposed to be related to the insect-eating *Myrmecobius*, now living in Australia.

Although the Jura of Europe has yielded other similar Mammals, we have as yet none of this class from that formation; while, from rocks of Cretaceous age, no Mammals are known in any part of the world. This is especially to be regretted, as it is evidently to the Cretaceous that we must look for the first representatives of many of our present groups of Mammals, as well as for indications of their more ancient lineage. That some discovery of this nature from the Cretaceous is near at hand, I cannot doubt, when I consider what the last few years have brought to light in the Eocene.

In the lowest Tertiary beds of this country, a rich Mammalian fauna suddenly makes its appearance, and from that time through the Age of Mammals to the present, America has been constantly occupied by this type of life in the greatest diversity of form. Fortunately, a nearly continuous record of this life, as preserved, is now accessible to us, and ensures great additions to our knowledge of the genealogy of Mammals, and perhaps the solution of more profound problems. Before proceeding to discuss in detail American fossil *Mammalia*, it is important to define the divisions of time indicated in our Tertiary and Post-Tertiary deposits, as these in many cases mark successive stages in the development of the mammals.

The boundary line between the Cretaceous and Tertiary in the region of the Rocky Mountains has been much in dispute during the last few years, mainly in consequence of the uncertain geological bearings of the fossil plants found near this horizon. The accompanying invertebrate fossils have thrown little light on the question, which is essentially, whether the great Lignite series of the West is uppermost Cretaceous, or lowest Eocene. The evidence of the numerous vertebrate remains is, in my judgment, decisive, and in favor of the former view.

This brings up an important point in Palæontology, one to which my attention was drawn several years since, namely: the comparative value of different groups of fossils in marking geological time. In examining the subject with some care, I found that, for this purpose, plants, as their nature indicates, are most unsatisfactory witnesses; that invertebrate animals are much better; and that vertebrates afford the most reliable evidence of climatic and other geological changes. The subdivisions of the latter group, moreover, and in fact all forms of animal life, are of value in this respect, mainly according to the perfection of their organization, or zoological rank. Fishes, for example, are but slightly affected by changes that would destroy Reptiles or Birds, and the higher Mammals succumb under influences that the lower forms pass through in safety. The more special applications of this general law, and its value in geology, will readily suggest themselves.

The evidence offered by fossil remains is, in the light of this law, conclusive, that the line, if line there be, separating our Cretaceous from the Tertiary, must at present be drawn where the Dinosaurs and other Mesozoic vertebrates disappear, and are replaced by the Mammals, henceforth the dominant type.

The Tertiary of Western America comprises the most extensive series of deposits of this age known to geologists, and important breaks in both the rocks and the fossils separate it into three well-marked divisions. These natural divisions are not the exact equivalents of the Eocene, Miocene, and Pliocene of Europe, although usually so considered, and known by the same names; but, in general, the fauna of each appears to be older than that of its corresponding representative in the other hemisphere; an important fact, not hitherto recognized. This partial resemblance of our extinct faunas to others in regions widely separated, where the formations are doubtless somewhat different in geological age, is precisely what we might expect, if, as was probable, the main migrations took place from this Continent. It is better at once to recognize this principle, rather than attempt to bring into exact parallelism, formations that were not strictly contemporaneous.

The freshwater Eocene deposits of our Western Territories, which are in the same region at least two miles in vertical thickness, may be separated into three distinct subdivisions. The lowest of these, resting unconformably on the Cretaceous, has been termed the Vermilion Creek, or Wahsatch, Group. It contains a well-marked mammalian fauna, the largest and most characteristic genus of which is the ungulate *Coryphodon*, and hence I have called these deposits the Coryphodon Beds. The middle Eocene strata, which have been termed the Green River and Bridger Series, may be designated as the Dinoceras Beds, as the gigantic animals of this order are only found here. The uppermost Eocene, or the Uintah Group, is especially well characterized by large mammals of the genus *Diplacodon*, and hence may be termed the Diplacodon Beds. The fauna of each of these three subdivisions was essentially distinct, and the fossil remains of each were entombed in different and successive ancient lakes. It is important to remember that these Eocene lake-basins all lie between the Rocky Mountains on the east and the Wahsatch Range on the west, or along the high central plateau of the Continent. As these mountain chains were elevated, the enclosed Cretaceous sea, cut off from the ocean, gradually freshened, and formed these extensive lakes, while the surrounding land was covered with a luxuriant tropical vegetation, and with many strange forms of animal life. As the upward movement of this region continued, these lake-basins, which for ages had been filling up, preserving in their sediments a faithful record of Eocene life-history, were slowly drained by the constant deepening of the outflowing rivers, and they have since remained essentially dry land.

The Miocene lake-basins are on the flanks of this region, where only land had been since the close of the Cretaceous. These basins contain three faunas, nearly or quite distinct. The lowest Miocene, which is only found east of the Rocky Mountains, alone contains the peculiar mammals known as the *Brontotheridæ*, and these deposits may be called the Brontotherium Beds. The strata next above, which represent the middle Miocene, have as their most characteristic fossil the genus *Oreodon*, and are known as the Oreodon Beds. The upper Miocene, which occurs in Oregon, is of great thickness, and from one of its most important fossils, *Miohippus*, may be designated as the Miohippus Series. The climate here during this period was warm temperate.

Above the Miocene, east of the Rocky Mountains and on the Pacific Coast, the Pliocene is well developed, and is rich in vertebrate remains. The strata rest unconformably on the Miocene, and there is a well-marked faunal change at this point, modern types now first making their appearance. For these reasons, we are justified in separating the Miocene from the Pliocene at this break; although in Europe where no marked break exists, the line seems to have been drawn at a somewhat higher horizon. Our Pliocene forms essentially a continuous series, although the upper beds may be distinguished from the lower by the presence of a true *Equus*, and some other existing genera. The Pliocene climate was similar to that of the Miocene. The Post-Pliocene beds contain many extinct mammals, and may thus be separated from recent deposits.

Returning now to our subject from this geological digression, —which will hardly be deemed unprofitable, since I have given you in few words the results of a great deal of hard mountain work,—let us consider the Tertiary mammals, as we know them from the remains already discovered, and attempt to trace the history of each order down to the present time. We have seen that a single small Marsupial, from the Trias, is the only mammal found in all the American rocks below the Eocene; and yet in beds of this age, immediately over the Chalk, fossil mammals of many different kinds abound.

The Marsupials, strange to say, are here few in number, and diminutive in size; and have as yet been identified only by fragmentary specimens, and most of them too imperfect for accurate description. In the higher Eocene deposits, this group is more abundant, but still represented by small animals, most of them insectivorous, or carnivorous in habit, like the existing Opossum. From the Miocene and Pliocene, no remains of Marsupials have been described. From the Post-Tertiary, only specimens nearly allied to those now living are known, and most of these were found in the caves of South America.

The Edentate Mammals are evidently an American type, and on this Continent attained a great development in numbers and size. No Eocene Edentates have been found here, and although their discovery in this formation has been announced, the identification proves to have been erroneous. In the Miocene of the Pacific Coast, a few fossils have been discovered which belong to animals of this group, and to the genus *Moropus*.

There are two species, one about as large as a Tapir, and the other nearly twice that size. This genus is the type of a distinct family, the *Moropodidæ*. In the lower Pliocene above, well preserved remains of Edentates of very large size have been found at several widely separated localities in Idaho and California. These belong to the genus *Morotherium*, of which two species are known. East of the Rocky Mountains, in the lower Pliocene of Nebraska, a large species apparently of the genus *Moropus* has been discovered. The horizon of these later fossils corresponds nearly with beds in Europe that have been called Miocene. In the Post-Pliocene of North America, gigantic Edentates were very numerous and widely distributed, but all disappeared with the close of that period. These forms were essentially huge Sloths, and the more important genera were *Megatherium*, *Mylodon* and *Megalonyx*. The genera *Megalocnus* and *Myomorphus* have been found only in Cuba.

In South America during the Pliocene or Post-Pliocene, enormous Edentates were still more abundant, and their remains are usually in such perfect preservation as to suggest a very recent period for their extinction. The Sloth tribe is represented by the huge *Mylodon*, *Megatherium*, *Megalonyx*, *Cœlodon*, *Ochotherium*, *Gnathopis*, *Lestodon*, *Scelidotherium*, and *Sphœnodon;* and among the Armadilloes were *Chlamydotherium*, *Eurydon*, *Glyptodon*, *Heterodon*, *Pachytherium* and *Schistopleurum*. *Glossotherium*, another extinct genus, is supposed to be allied to the Ant-eaters.

It is frequently asserted, and very generally believed, that the large number of huge *Edentata* which lived in North America during the Post-Pliocene, were the results of an extensive migration from South America soon after the elevation of the Isthmus of Panama, near the close of the Tertiary. No conclusive proof of such migration has been offered, and the evidence, it seems to me, so far as we now have it, is directly opposed to this view. No undoubted Tertiary Edentates have yet been discovered in South America, while we have at least two species in our Miocene, and during the deposition of our lower Pliocene, large individuals of this group were not uncommon as far north as the forty-third parallel of latitude, on both sides of the Rocky Mountains. In view of these facts, and others which I shall lay before you, it seems more natural to conclude from our present knowledge, that the migration, which no doubt took place, was from north to south. The Edentates finding thus in South America a congenial home flourished greatly for a time, and although the larger forms are now all extinct, diminutive representatives of the group still inhabit the same region.

The *Cetacea* first appear in the Eocene, as in Europe, and are comparatively abundant in deposits of this age on the Atlantic Coast. The most interesting remains of this order, yet found, belong to the *Zeuglodontidæ*, which are carnivorous whales, and the only animals of the order with teeth implanted by two roots. The principal genera of this family are *Zeuglodon* and *Squalodon*, the former genus being represented by gigantic forms, some of which were seventy feet in length. The genus *Saurocetes*, which includes some small animals of this group, has been found in South America. The Dolphin family (*Delphinidæ*) are well represented in the Miocene, both on the Atlantic and Pacific Coast. The best known genus is *Priscodelphinus*, of which several species have been described. Several other generic names which have been applied to fragments need not here be enumerated. In none of the Tertiary species of this family were the cervical vertebræ ankylosed. The Sperm Whales (*Catodontidæ*) were also abundant throughout the Tertiary, and with them in the earlier beds, various Ziphioid forms have been found. The toothless *Balænidæ* are only known with certainty as fossils from the later Tertiary and more recent deposits.

The Sirenians, which appear first in the Eocene of the Old World, occur in the Miocene of our Eastern Coast, and throughout the later Tertiary. The specimens described have all been referred to the genus *Manatus*, and seem closely related to our living species. In the Tertiary of Jamaica, a skull has been found which indicates a new genus, *Prorastomus*, also allied to the existing Manatee. The genus *Rhytina*, once abundant on our Northwest Coast, has recently become extinct.

The Ungulates are the most abundant Mammals in the Tertiary, and the most important; since they include a great variety of types, some of which we can trace through their various changes down to the modified forms that represent them to-day. Of the various divisions in this comprehensive group, the Perissodactyle, or odd-toed Ungulates, are evidently the oldest, and throughout the Eocene are the prevailing forms. Although all of the Perissodactyles of the earlier Tertiary are more or less generalized, they are still quite distinct from the Artiodactyles, even at the base of the Eocene. One family, however, the *Coryphodontidæ*, which is well represented at this horizon, both in America and Europe, although essentially *Perissodactyle*, possesses some characters which point to a primitive Ungulate type from which the present orders have been evolved. Among these characters are the diminutive brain, which in size and form approaches that of the Reptiles, and also the five-toed feet from which all the various forms of the mammalian foot have been derived. Of this family, only a single genus, *Coryphodon* (*Bathmodon*), is known, but there were several distinct species. They were the largest mammals of the lower Eocene, some exceeding in size the existing Tapirs.

In the middle Eocene, West of the Rocky Mountains, a remarkable group of ungulates makes its appearance. These animals nearly equaled the Elephant in size, but had shorter limbs. The skull was armed with two or three pairs of horn-cores, and with enormous canine tusks. The brain was proportionally smaller than in any other land mammal. The feet had five toes, and resembled in their general structure those of *Coryphodon*, thus indicating some affinity with that genus. These mammals resemble in some respects the Perissodactyles, and in others the Proboscidians, yet differ so widely from any known Ungulates, recent or fossil, that they must be regarded as forming a distinct order, the *Dinocerata*. Only three genera are known, *Dinoceras*, *Tinoceras* and *Uintatherium*, but quite a number of species have been described. During the later part of the middle Eocene, these animals were very abundant for a short time, and then became extinct, leaving apparently no successors, unless possibly we have in the Proboscidians their much modified descendants. Their genetic connection with the Coryphodonts is much more probable, in view of what we now know of the two groups.

Besides these peculiar Mammals, which are extinct, and mainly of interest to the Biologist, there were others in the early Tertiary which remind us of those at present living around us. When a student in Germany some twelve years ago, I heard a world-renowned Professor of Zoology gravely inform his pupils that the Horse was a gift of the Old World to the New, and was entirely unknown in America until introduced by the Spaniards. After the lecture, I asked him whether no earlier remains of horses had been found on this Continent, and was told in reply that the reports to that effect were too unsatisfactory to be presented as facts in science. This remark led me, on my return, to examine the subject myself, and I have since unearthed, with my own hands, not less than thirty distinct species of the horse tribe, in the Tertiary deposits of the West alone; and it is now, I think generally admitted that America is, after all, the true home of the Horse.

I can offer you no better illustration than this of the advance vertebrate palæontology has made during the last decade, or of the important contributions to this progress which our Rocky Mountain region has supplied.

The oldest representative of the horse. at present known, is the diminutive *Eohippus* from the lower Eocene. Several species have been found, all about the size of a fox. Like most of the early mammals, these Ungulates had forty-four teeth, the molars with short crowns, and quite distinct in form from the premolars. The ulna and the fibula were entire and distinct, and there were four well developed toes and a rudiment of another on the fore feet, and three toes behind. In the structure of the feet, and in the teeth, the *Eohippus* indicates unmistakably that the direct ancestral line to the modern horse has already separated from the other Perissodactyles. In the next higher division of the Eocene, another genus (*Orohippus*) makes its appearance, replacing *Eohippus*, and showing a greater, although still distant, resemblance to the Equine type. The rudimentary first digit of the fore foot has disappeared, and the last premolar has gone over to the molar series. *Orohippus* was but little larger than *Eohippus*, and in most other respects very similar. Several species have been found in the same horizon with *Dinoceras*, and others lived during the upper Eocene with *Diplacodon*, but none later.

Near the base of the Miocene, in the Brontotherium beds, we find a third closely allied genus, *Mesohippus*, which is about as large as a sheep, and one stage nearer the horse. There are only three toes and a rudimentary splint bone on the fore feet, and three toes behind. Two of the premolar teeth are quite like the molars. The ulna is no longer distinct, or the fibula entire, and other characters show clearly that the transition is advancing. In the upper Miocene, *Mesohippus* is not found, but in its place a fourth form, *Miohippus*, continues the line. This genus is near the *Anchitherium* of Europe, but presents several important differences. The three toes in each foot are more nearly of a size, and a rudiment of the fifth metacarpal bone is retained. All the known species of this genus are larger than those of *Mesohippus*, and none pass above the Miocene.

The genus *Protohippus* of the lower Pliocene, is yet more equine, and some of its species equaled the ass in size. There are still three toes on each foot, but only the middle one, corresponding to the single toe of the horse, comes to the ground. This genus resembles most nearly the *Hipparion* of Europe. In the Pliocene, we have the last stage of the series before reaching the horse, in the genus *Pliohippus*, which has lost the small hooflets, and in other respects is very equine. Only in the upper Pliocene, does the true *Equus* appear, and complete the genealogy of the Horse, which in the Post-Tertiary roamed over the whole of North and South America, and soon after became extinct. This occurred long before the discovery of the Continent by Europeans, and no satisfactory reason for the extinction has yet been given. Besides the characters I have mentioned, there are many others, in the skeleton, skull, teeth, and brain of the forty or more intermediate species, which show that the transition from the Eocene *Eohippus* to the modern *Equus*, has taken place in the order indicated, and I believe the specimens now at New Haven will demonstrate the fact to any anatomist. They certainly carried prompt conviction to the first of anatomists, who was the honored guest of the Association a year ago, whose genius had already indicated the later genealogy of the horse in Europe, and whose own researches so well qualified him to appreciate the evidence here laid before him. Did time permit, I might give you at least a probable explanation of this marvellous change, but justice to the comrades of the horse in his long struggle for existence demands that some notice of their efforts should be placed on record.

Beside the Horse and his congeners, the only existing Perissodactyles are the Rhinoceros and the Tapir. The last is the oldest type, but the Rhinoceros had near allies throughout the Tertiary; and, in view of the continuity of the equine line, it is well worth while to attempt to trace his pedigree. At the bottom of the Eocene, in our Western lake-basins, the tapiroid genus *Helaletes* is found, represented by numerous small mammals hardly larger than the diminutive horses of that day. In the following epoch of the Eocene, the closely allied *Hyrachyus* was one of the most abundant animals. This genus was nearly related to the *Lophiodon* of Europe, and in its teeth and skeleton strongly resembled the living Tapir; whose ancestry, to this point, seems to coincide with that of the Rhinoceros we are considering. Strangely enough, the Rhinoceros line, before it becomes distinct, separates into two branches. In the upper part of the Dinoceras Beds, we have the genus *Colonoceras*, which is really a *Hyrachyus* with a transverse pair of very rudimentary horn-cores on the nasal bones. In the lower Miocene west of the Rocky Mountains, this line seems to pass on through the genus *Diceratherium*, and in the higher Miocene this genus is well represented. Some of the species nearly equaled in size the existing Rhinoceros, which *Diceratherium* strongly resembled. The main difference between them is a most interesting one. The rudimentary horn-cores on the nasals, seen in *Colonoceras*, are in *Diceratherium* developed into strong bony supports for horns, which were placed transversely, as in the Ruminants, and not on the median line, as in all existing forms of Rhinoceros. In the Pliocene of the Pacific Coast, a large Rhinoceros has been discovered, which may be a descendant of *Diceratherium*, but as the nasal bones have not been found, we must wait for further evidence on this point. Returning now to the other branch of the Rhinoceros group, which left their remains mainly East of the Rocky Mountains, we find that all the known forms are hornless. The upper Eocene genus *Amynodon* is the oldest known Rhinoceros, and by far the most generalized of the family.

The premolars are all unlike the molars, the four canines are of large size, but the inner incisor in each jaw is lost in the fully adult animal. The nasals were without horns. There were four toes in front, and three behind. The genus *Hyracodon*, of the Miocene, which is essentially a Rhinoceros, has a full set of incisor and canine teeth; and the molars are so nearly like those of its predecessor *Hyrachyus*, that no one will question the transformation of the older into the newer type. *Hyracodon*, however, appears to be off the true line, for it has but three toes in front. In the higher Miocene beds, and possibly with *Hyracodon*, occurs a larger Rhinoceros, which has been referred to the genus *Aceratherium*. This form has lost the canine and one incisor above, and two incisors below. In the Pliocene are several species closely related, and of large size. Above the Pliocene in America, no vestiges of the Rhinoceros have been found, and our American forms doubtless became extinct at the close of this period.

The Tapir is clearly an old American type, and we have seen that, in the Eocene, the genera *Helaletes* and *Hyrachyus* were so strongly tapiroid in their principal characters, that the main line of descent probably passed through them. It is remarkable that the Miocene of the West, so greatly developed as it is on both sides of the Rocky Mountains, should have yielded but a few fragments of tapiroid mammals, and the same is true of the Pliocene of that region. In the Miocene of the Atlantic Coast, too, only a few imperfect specimens have been found. These forms all apparently belong to the genus *Tapiravus*, although most of them have been referred to *Lophiodon*, a lower Eocene type. In the Post-Tertiary, a true *Tapirus* was abundant, and its remains have been found in various parts of North America. The line of descent, although indistinct through the middle and upper Tertiary, was doubtless continuous in America, and several species exist at present, from Mexico southward. It is worthy of notice that the species North of the Isthmus of Panama appear all to be generically distinct from those of South America.

In addition to these three Perissodactyle types which, as the fittest, have alone survived, and whose lineage I have endeavored to trace, there were many others in early Tertiary times. Some of these disappeared with the close of the Eocene, while others continued, and assumed strange specialized shapes in the Miocene, before their decline and extinction. One series of the latter deserves especial mention, as it includes one of the most interesting families of our extinct animals. Among the large mammals in the lower Eocene is *Limnohyus*, a true Perissodactyle, but only known here from fragments of the skeleton. In the next higher beds, this genus is well represented, and with it is found a nearly allied form, *Palæosyops*. In the upper Eocene, both have left the field, and the genus *Diplacodon*, a very near relative, holds the supremacy. The line seems clear through these three genera, but on crossing the break into the Miocene, we have, apparently as next of kin, the huge *Brontotheridæ*. These strange beasts show in their dentition and some other characters the same transition steps beyond *Diplacodon*, which that genus had made beyond *Palæosyops*. The *Brontotheridæ* were nearly as large as the Elephant, but had much shorter limbs. The skull was elongated, and had a transverse pair of large horn-cores on the maxillaries, in front of the orbits, like the middle pair in *Dinoceras*. There were four toes in front, and three behind, and the feet were similar to those of the Rhinoceros. There are four genera in this group, *Brontotherium; Diconodon; Menodus* (*Titanotherium*); and *Megacerops*, which have been found only in the lowest Miocene, east of the Rocky Mountains.

In the higher Miocene beds of Oregon, an allied genus, *Chalicotherium*, makes its appearance. It is one stage further on in the transition, and perhaps a descendant of the *Brontotheridæ;* but here, so far as now known, the line disappears. It is a suggestive fact, that this genus has now been found in Western America, China, India, Greece, Germany and France, indicating thus, as I believe, the path by which many of our ancient mammals helped to people the so-called Old World.

The Artiodactyles, or even-toed Ungulates, are the most abundant of the larger mammals now living; and the group dates back at least to the lowest Eocene. Of the two well marked divisions of this order, the Bunodonts and the Selenodonts, as happily defined by Kowalevsky, the former is the older type, which must have separated from the Perissodactyle line after the latter had become differentiated from the primitive Ungulate. In the Coryphodon Beds of New Mexico, occurs the oldest Artiodactyle yet found, but it is at present known only from fragmentary specimens. These remains are clearly Suilline in character, and belong to the genus *Eohyus*. In the beds above, and possibly even in the same horizon, the genus *Helohyus* is not uncommon, and several species are known. The molar teeth of this genus are very similar to those of the Eocene *Hyracotherium*, of Europe, which is supposed to be a Perissodactyle, while *Helohyus* certainly is not, but apparently a true lineal ancestor of the existing pigs. In every vigorous primitive type which was destined to survive many geological changes, there seems to have been a tendency to throw off lateral branches, which became highly specialized and soon died out, because they are unable to adapt themselves to new conditions. The narrow path of the persistent Suilline type, throughout the whole Tertiary, is strown with the remains of such ambitious offshoots, while the typical pig, with an obstinacy never lost, has held on in spite of Catastrophes and Evolution, and still lives in America to-day. In the lower Eocene, we have in the genus *Parahyus* apparently one of these short-lived, specialized branches. It attained a much larger size than the true lineal forms, and the number of its teeth was reduced. In the Dinoceras Beds, or middle Eocene, we have still, on or near the true line, *Helohyus*, which is the last of the series known from the American Eocene. All these early Suillines, with the possible exception of *Parahyus*, appear to have had at least four toes, all of usable size.

In the lower Miocene, we find the genus *Perchœrus*, seemingly a true Suilline, and with it remains of a larger form, *Elotherium*, are abundant. The latter genus occurs in Europe in nearly the same horizon, and the specimens known from each Continent agree closely in general characters. The name *Pelonax* has been applied erroneously to some of the American forms; but the specimens on which it was based clearly belong to *Elotherium*. This genus affords another example of the aberrant Suilline offshoots, already mentioned. Some of the species were nearly as large as a Rhinoceros, and in all there were but two serviceable toes; the outer digits, seen in living animals of this group, being represented only by small rudiments concealed beneath the skin. In the upper Miocene of Oregon, Suillines are abundant, and almost all belong to the genus *Thinohyus*, a near ally of the modern Peccary (*Dicotyles*), but having a greater number of teeth, and a few other distinguishing features. In the Pliocene, Suillines are still numerous, and all the American forms yet discovered are closely related to *Dicotyles*. The genus *Platygonus* is represented by several species, one of which was very abundant in the Post-Tertiary of North America, and is apparently the last example of a side branch, before the American Suillines culminate in existing Peccaries. The feet in this species are more specialized than in the living forms, and approach some of the peculiar features of the ruminants; as for example a strong tendency to coalescence in the metapodial bones. The genus *Platygonus* became extinct in the Post-Tertiary, and the later and existing species are all true Peccaries. No authenticated remains of the genera *Sus*, *Porcus*, *Phacochœrus*, or the allied *Hippopotamus*, the Old World Suillines, have been found in America, although several announcements to that effect have been made.

In the series of generic forms between the lower Eocene *Eohyus* and the existing *Dicotyles*, which I have very briefly discussed, we have apparently the ancestral line ending in the typical American Suillines. Although the demonstration is not yet as complete as in the lineage of the Horse, this is not owing to want of material, but rather to the fact that the actual changes which transformed the early Tertiary pig into the modern Peccary were comparatively slight, so far as they are indicated in the skeletons preserved, while the lateral branches were so numerous as to confuse the line. It is clear, however, that from the close of the Cretaceous to the Post-Tertiary, the Bunodont Artiodactyles were especially abundant on this Continent, and only recently have approached extinction.

The Selenodont division of the Artiodactyles is a more interesting group and, so far as we now know, makes its first appearance in the upper Eocene of the West, although forms, apparently transitional, between it and the Bunodonts occur in the Dinoceras Beds, or middle Eocene. These belong to the genus *Homacodon*, which is very nearly allied to *Helohyus* and but a single step away from this genus toward the Selenodonts. By a fortunate discovery, a nearly complete skeleton of this rare intermediate form has been brought to light, and we are thus enabled to define its characters. Several species of *Homacodon* are known, all of small size. This primitive Selenodont had forty-four teeth, which formed a nearly continuous series.

The molar teeth are very similar to those of *Helohyus*, but the cones on the crowns have become partially triangular in outline, so that when worn, the Selenodont pattern is clearly recognizable. The first and second upper molars, moreover, have three distinct posterior cusps, and two in front; a peculiar feature, which is seen also in the European genera *Dichobune* and *Cainotherium*. There were four toes on each foot, and the metapodial bones were distinct. The type species of this genus was about as large as a cat. With *Helohyus*, this genus forms a well marked family, the *Helohyidæ*.

In the *Diplacodon* horizon of the upper Eocene, the Selenodont dentition is no longer doubtful, as it is seen in most of the *Artiodactyla* yet found in these beds. These animals are all small, and belong to at least three distinct genera. One of these, *Eomeryx*, closely resembles *Homacodon* in most of its skeleton, and has four toes, but its teeth show well marked crescents, and a partial transition to the teeth of *Hyopotamus*, from the Eocene of Europe. With this genus, is another (*Parameryx*), also closely allied to *Homacodon*, but apparently a straggler from the true line, as it has but three toes behind. The most pronounced Selenodont in the upper Eocene is the *Oromeryx*, which genus appears to be allied to the existing Deer family, or *Cervidæ*, and if so is the oldest known representative of the group. These facts are important, as it has been supposed, until very recently, that our Eocene contained no even-hoofed mammals.

In the lowest Miocene of the West, no true crescent-toothed *Artiodactyla* have as yet been identified, with the exception of a single species of *Hyopotamus;* but in the overlying beds of the middle Miocene, remains of the *Oreodontidæ* occur in such vast numbers as to indicate that these animals must have lived in large herds around the borders of the lake-basins in which their remains have been entombed. These basins are now the denuded deserts so well termed *Mauvaises Terres* by the early French trappers. The least specialized, and apparently the oldest, genus of this group is *Agriochœrus*, which so nearly resembles the older *Hyopotamus*, and the still more ancient *Eomeryx*, that we can hardly doubt that they all belonged to the same ancestral line. The typical Oreodonts are the genera *Oreodon* and *Eporeodon*, which have been aptly termed by Leidy, ruminating hogs. They had forty-four teeth, and four well developed toes on each foot. The true Oreodons, which were most numerous east of the Rocky Mountains, were about as large as the existing Peccary, while *Eporeodon*, which was nearly twice this size, was very abundant in the Miocene of the Pacific slope.

In the succeeding Pliocene formation, on each side of the Rocky Mountains, the genus *Merychyus* is one of the prevailing forms, and continues the line on from the Miocene, where the true Oreodons became extinct. Beyond this, we have the genus *Merycochœrus*, which is so nearly allied to the last, that they would be united by many naturalists. With the close of the Pliocene, this series of peculiar ruminants abruptly terminates, no member surviving until the Post-Tertiary, so far as known.

A most interesting line, that leading to the Camels and Llamas, separates from the primitive Selenodont branch in the Eocene, probably through the genus *Parameryx*. In the Miocene, we find in *Pœbrotherium* and some nearly allied forms unmistakable indications that the Cameloid type of ruminant had already become partially specialized, although there is a complete series of incisor teeth, and the metapodial bones are distinct. In the Pliocene, the Camel tribe was, next to the Horses, the most abundant of the larger mammals. The line is continued through the genus *Procamelus*, and perhaps others, and in this formation the incisors first begin to diminish, and the metapodials to unite. In the Post-Tertiary we have a true *Auchenia*, represented by several species, and others in South America, where the Alpacas and Llamas still survive. From the Eocene almost to the present time, North America has been the home of vast numbers of the *Camelidæ*, and there can be little doubt that they originated here, and migrated to the Old World.

Returning once more to the upper Eocene, we find another line of descent starting from *Oromeryx*, which, as we have seen, had apparently then just become differentiated from the older Bunodont type. Throughout the middle and upper Miocene, this line is carried forward by the genus *Leptomeryx* and its near allies, which resemble so strongly the Pliocene *Cervidæ* that they may fairly be regarded as their probable progenitors. Possibly some of these forms may be related to the *Tragulidæ*, but at present the evidence is against it.

The Deer family has representatives in the upper Miocene of Europe, which contains fossils strongly resembling the fauna of our lower Pliocene, a fact always to be borne in mind in comparing the horizon of any group in the two continents. Several species of *Cervidæ*, belonging to the genus *Cosoryx*, are known from the lower Pliocene of the West, and all have very small antlers, divided into a single pair of tynes. The statement recently published, that most of these antlers had been broken during the life of the animals, is unsupported by any evidence, and is erroneous. These primitive Deer do not have the orbit closed behind, and they have all the four metapodial bones entire, although the second and fifth are very slender. In the upper Pliocene, a true *Cervus* of large size has been discovered. In the Post-Tertiary, *Cervus*, *Alces*, and *Tarandus* have been met with, the latter far south of its present range. In the caves of South America, remains of *Cervus* have been found, and also two species of Antelopes, one referred to a new genus, *Leptotherium*.

The Hollow-horned Ruminants, in this country, appear to date back no further than to the lower Pliocene, and here only two species of *Bison* have as yet been discovered. In the Post-Tertiary this genus was represented by numerous individuals and several species, some of large size. The Musk Ox (*Ovibos*) was not uncommon during some parts of this epoch, and its remains are widely distributed.

No authentic fossil remains of true Sheep, Goats, or Giraffes have as yet been found on this continent.

The Proboscideans, which are now separated from the typical Ungulates as a distinct order, make their first appearance in North America in the lower Pliocene, where several species of *Mastodon* have been found. This genus occurs, also, in the upper Pliocene, and in the Post-Tertiary; although some of the remains attributed to the latter are undoubtedly older. The Pliocene species all have a band of enamel on the tusks, and some other peculiarities observed in the oldest Mastodons of Europe, which are from essentially the same horizon. Two species of this genus have been found in South America, in connection with the remains of extinct Llamas and Horses. The genus *Elephas* is a later form, and has not yet been identified in this country below the upper Pliocene, where one gigantic species was abundant. In the Post-Pliocene, remains of this genus are numerous. The hairy Mammoth of the Old World (*Elephas primigenius*) was once abundant in Alaska, and great numbers of its bones are now preserved in the frozen cliffs of that region. This species does not appear to have extended east of the Rocky Mountains, or south of the Columbia River, but was replaced there by the American Elephant, which preferred a milder climate. Remains of the latter have been met with in Canada, throughout the United States, and in Mexico. The last of the American Mastodons and Elephants became extinct in the Post-Tertiary.

The order *Toxodontia* includes two very peculiar genera, *Toxodon* and *Nesodon*, which have been found in the Post-Tertiary deposits of South America. These animals were of huge size, and possessed such mixed characters that their affinities are a matter of considerable doubt. They are thought to be related to the Ungulates, Rodents, and Edentates, but as the feet are unknown, this cannot at present be decided.

Macrauchenia and *Homalodontotherium* are two other peculiar genera from South America, now extinct, the exact affinities of which are uncertain. *Anoplotherium* and *Palæotherium*, so abundant in Europe, have not been found in our North American Tertiary deposits, although reported from South America.

Perhaps the most remarkable mammals yet found in America are the *Tillodontia*, which are comparatively abundant in the lower and middle Eocene. These animals seem to combine the characters of several different groups, viz: the Carnivores, Ungulates, and Rodents. In the genus *Tillotherium*, the type of the order, and of the family *Tillotheridæ*, the skull resembles that of the Bears; the molar teeth are of the ungulate type; while the large incisors are very similar to those of Rodents. The skeleton resembles that of the Carnivores, but the scaphoid and lunar bones are distinct, and there is a third trochanter on the femur. The feet are plantigrade, and each had five digits, all with long pointed claws. In the allied genus *Stylinodon*, which belongs to a distinct family, the *Stylinodontidæ*, all the teeth were rootless. Some of these animals were as large as a Tapir. The genus *Dryptodon* has been found only in the *Coryphodon* beds of New Mexico, while *Tillotherium* and *Stylinodon* occur in the middle Eocene of Wyoming. *Anchippodus* probably belongs to this group, which may perhaps include some other forms that have been named from fragmentary specimens.

The Rodents are an ancient type, and their remains are not unfrequently disinterred in the strata of our lowest fresh-water

Eocene. The earliest known forms are apparently all related to the Squirrels, and the most common genus is *Sciuravus*, which continued throughout the Eocene. A nearly allied form, which may prove to be the same, is *Paramys*, the species of which are larger than those of the older type. In the Dinoceras beds, the genus *Colonomys* is found, and the specimens preserved point to the *Muridæ*, as the nearest living allies. A peculiar genus, *Apatemys*, which also occurs in the middle Eocene, has gliriform incisors, but the molars resemble those of Insectivores. All the Eocene Rodents are of small size, the largest being about as large as a rabbit.

In the middle and upper Miocene lake-basins of the West, Rodents abound, but all are of moderate size. The Hares first appear in the Oreodon beds, and continue in considerable numbers through the rest of the Tertiary and Post-Tertiary to the present day. In these beds, the most common forms belong to the *Leporidæ*, and mainly to the genus *Palæolagus*. The Squirrel family is represented by *Ischyromys*, the *Muridæ* by the genus *Eumys*, and the Beavers by *Palæocastor*. In the upper Miocene of Oregon, most of the same genera are found, and with them some peculiar forms, very unlike anything now living. One of these is the genus *Allomys*, possibly related to the flying Squirrels, but having molar teeth somewhat like those of the Ungulates. In the Pliocene, east and west of the Rocky Mountains, Rodents continue abundant, but most of them belong to existing genera. Among these are *Castor*, *Hystrix*, *Cynomys*, *Geomys*, *Lepus* and *Hesperomys*. In the Post-Tertiary, the gigantic beaver, *Castoroides*, was abundant throughout most of North America. *Hydrochœrus* has been found in South Carolina. In the caves of the island of Anguilla, in the West Indies, remains of large extinct Rodents belonging to the *Chinchillidæ* have been discovered.

The early Tertiary Rodents known from South America are the genera *Megamys*, *Theridromys*, and a large species referred to *Arvicola*. In Brazil, the Pliocene Rodents found are referred to the existing genera *Cavia*, *Kerodon*, *Lagostomus*, *Ctenomys*, *Hesperomys*, *Oxymycterus*, *Arvicola* and *Lepus*. A new genus, *Cardiodus*, described from this horizon, is a true Rodent, but the peculiar *Typotherium*, which has been referred to this order by some authorities, has perhaps other affinities. In the Post-Tertiary, the Rodents were very abundant in South America, as they are at present. The species are in most instances distinct from those now living, but the genera are nearly the same. The *Caviidæ* were especially numerous. *Cercolabes*, *Myopotamus*, and *Lagostomus* are also found, and two extinct genera, *Phyllomys* and *Lonchophorus*.

The *Cheiroptera*, or Bats, have not been found in this country below the middle Eocene, where two extinct genera, *Nyctilestes* and *Nyctitherium*, are each represented by numerous remains. These fossils all belong to small animals, and, so far as they have been investigated, show no characters of more than generic importance to distinguish them from the Bats of to-day. No other members of this group are known from our Tertiary. In the Post-Tertiary, no extinct species of Bats have been found in North America, but from the caves of Brazil quite a number have been reported. These all belong to genera still living in South America, and most of them to the family *Phyllostomidæ*.

The Insectivores date back, in this country, at least to the middle Eocene. Here numerous remains occur, which have been described as belonging to this order, although it is possible that some of them were insect-eating Marsupials. The best known genera are, *Hemiacodon*, *Centetodon*, *Talpavus*, and *Entomacodon;* all represented by animals of small size. In the Miocene, the bones of Insectivores are comparatively abundant, and the genera best determined are *Ictops* and *Leptictis*. A few specimens only have been found in the Pliocene and Post-Pliocene, most of them related to the Moles. No extinct Insectivores are known from South America, and no member of the group exists there at present.

The *Carnivora*, or true flesh-eating animals, are an old type, well represented in the Eocene, and, as might be expected, these early forms are much less specialized than the living species. In the Coryphodon beds, the genus *Limnocyon*, allied to the *Pterodon* of the European Eocene, is abundant. Another genus, apparently distinct, is *Prototomus*, and several others have been named from fragmentary fossils. In the middle Eocene, Carnivores were still more numerous, and many genera have been discovered. One of these, *Limnofelis*, was nearly as large as a lion, and apparently allied to the cats, although the typical *Felidæ* seem not yet to have been differentiated. Another Carnivore of nearly equal size was *Orocyon*, which had short massive jaws and broad teeth. *Dromocyon* and *Mesonyx* were large animals, allied to *Hyænodon*. The teeth were narrow, and the jaws long and slender. Among the smaller Carnivores were, *Vulpavus*, *Viverravus*, *Sinopa*, *Thinocyon*, and *Ziphacodon*.

In our Western Miocene, Carnivores are abundant, and make an approach to modern types. The *Felidæ* are well represented, the most interesting genus being *Machairodus*, which is not uncommon in the Oreodon beds on both sides of the Rocky Mountains. An allied genus is *Dinictis*, and several smaller Cats are known from about the same horizon. The *Canidæ* are represented by *Amphicyon*, a European genus, and by several species of *Canis*, or a very nearly allied form. The peculiar genus *Hyænodon*, found also in Europe, and the type of a distinct family, is abundant in the Miocene east of the Rocky Mountains, but has not yet been found on the Pacific Coast. In the Pliocene of both regions, the *Canidæ* are numerous, and all apparently belong to the existing genus *Canis*. The genus *Machairodus* is still the dominant form of the Cats, which are abundant, and for the most part belong to the genus *Felis*. The extinct *Leptarctus* is supposed to belong to the *Ursidæ*, and if so, is the oldest American representative of this family. In the Post-Pliocene, the extinct *Felidæ* include species nearly as large as a lion, and smaller forms very similar to those still living. Bears, Raccoons and Weasels have also been found.

In the Pliocene of South America, *Machairodus* represents the *Felidæ*, while the genera *Arctotherium* and *Hyænarctus* belong to the Bear family. Species of *Mustela* and *Canis* have also been found. In the caves of Brazil, the fauna of which is regarded as Post-Pliocene, one species of *Machairodus* is known, and one of *Synælurus*. *Canis* and *Icticyon*, still living in Brazil, and the extinct genus *Speothos*, represent the *Canidæ*. *Mephitis* and *Galictis*, among the Weasels, were also present, and with them species of *Nasua* and *Arctotherium*.

We come now to the highest group of Mammals, the Primates, which includes the Lemurs, the Apes, and Man. This order has a great antiquity, and even at the base of the Eocene we find it represented by several genera belonging to the lower forms of the group. In considering these interesting fossils, it is important to have in mind that the Lemurs, which are usually regarded as Primates, although at the bottom of the scale, are only found at the present day in Madagascar and the adjacent regions of the globe. All the American Monkeys, moreover, belong to one group, much above the Lemurs, while the Old World Apes are higher still, and most nearly approach Man.

In the lower Eocene of New Mexico, we find a few representatives of the earliest known Primates, and among them are the genera *Lemuravus* and *Limnotherium*, each the type of a distinct family. These genera became very abundant in the middle Eocene of the West, and with them are found many others, all however, included in the two families, *Lemuravidæ* and *Limnotheridæ*. *Lemuravus* appears to have been most nearly allied to the Lemurs, and is the most generalized form of the Primates yet discovered. It had forty-four teeth, forming a continuous series above and below. The brain was nearly smooth, and of moderate size. The skeleton most resembles that of the Lemurs. A nearly allied genus, belonging to the same family, is *Hyopsodus*. *Limnotherium* (*Tomitherium*) also is nearly related to the Lemurs, but shows some affinities with the South American Marmosets. This genus had forty teeth. The brain was nearly smooth, and the cerebellum large, and placed mainly behind the cerebrum. The orbits are open behind, and the lachrymal foramen is outside the orbit. Other genera belonging to the *Limnotheridæ* are, *Notharctos*, *Hipposyus*, *Microsyops*, *Palæacodon*, *Thinolestes* and *Telmatolestes*. Besides these, *Antiacodon* (*Anaptomorphus*), *Bathrodon* and *Mesacodon* should probably be placed in the same group. In the Diplacodon Beds, or Upper Eocene, no remains of Primates have yet been detected, although they will doubtless be found there. All the Eocene Primates known from American strata are low generalized forms, with characters in the teeth, skeleton and feet that suggest relationships with the Carnivores, and even with the Ungulates. These resemblances have led palæontologists to refer some imperfect specimens to both these orders.

In the Miocene lake basins of the West, only a single species of the *Primates* has been identified with certainty. This was found in the Oreodon Beds of Nebraska, and belongs to the genus *Laopithecus*, apparently related both to the *Limnotheridæ* and to some existing South American Monkeys. In the Pliocene and Post-Pliocene of North America, no remains of Primates have yet been found.

In the Post-Pliocene deposits of the Brazilian caves, remains of Monkeys are numerous, and mainly belong to extinct species of *Callithrix*, *Cebus* and *Jacchus*, all living South American genera. Only one extinct genus, *Protopithecus*, which embraced animals of large size, has been found in this peculiar fauna.

It is a noteworthy fact, that no traces of any Anthropoid Apes, or indeed of any Old World Monkeys have yet been detected in America. Man, however, the highest of the Primates, has left his bones and his works from the Arctic Circle to Patagonia. Most of these specimens are clearly Post-Tertiary, although there is considerable evidence pointing to the existence of Man in our Pliocene. All the remains yet discovered belong to the well-marked genus *Homo*, and apparently to a single species, at present represented by the American Indian.

In this rapid review of Mammalian life in America, from its first known appearance in the Trias down to the present time, I have endeavored to state briefly the introduction and succession of the principal forms in each natural group. If time permitted, I might attempt the more difficult task of trying to indicate what relations these various groups may possibly bear to each other; what connection the ancient Mammals of this continent have with the corresponding forms of the Old World; and, most important of all, what real progress Mammalian life has here made since the beginning of the Eocene. As it is, I can only say in summing up, that the Marsupials are clearly the remnants of a very ancient fauna, which occupied this continent millions of years ago, and from which the other Mammals were doubtless all derived, although the direct evidence of the transformation is wanting.

Although the Marsupials are nearly related to the still lower Monotremes, now living in the Australian Region, we have as yet no hint of the path by which these two groups became separated from the inferior vertebrates. Neither have we to-day much light as to the genetic connection existing between Marsupials and the placental Mammalia, although it is possible that the different orders of the latter had their origin each from a separate group of the Marsupials.

The presence, however, of undoubted Marsupials in our lower and middle Eocene, some of them related to the genus *Didelphys*, although remotely, is important evidence as to the introduction of these animals into America. Against this, their supposed absence in our Miocene and Pliocene can have but limited weight, when taken in connection with the fact that they flourished in the Post-Tertiary, and are still abundant. The evidence we now have is quite as strongly in favor of a migration of Marsupials from America to the Old World, as the reverse, which has been supposed by some naturalists. Possibly, as Huxley has suggested, both countries were peopled with these low mammals from a continent now submerged.

The Edentate mammals have long been a puzzle to zoologists, and up to the present time no clew to their affinities with other groups seems to have been detected. A comparison of the peculiar Eocene Mammals which I have called the *Tillodontia*, with the least specialized Edentates, brings to light many curious resemblances in the skull, teeth, skeleton and feet. These suggest relationship, at least, and possibly we may yet find here the key to the Edentate genealogy. At present, the Tillodonts are all from the lower and middle Eocene, while *Moropus*, the oldest Edentate genus, is found in the middle Miocene, and one species in the lower Pliocene.

The Edentates have been usually regarded as an American type, but the few living forms in Africa, and the Tertiary species in Europe, the oldest known, have made the land of their nativity uncertain. I have already given you some reasons for believing that the Edentates had their first home in North America, and migrated thence to the southern portion of the continent. This movement could not have taken place in the Miocene period, as the Isthmus of Darien was then submerged; but near the close of the Tertiary, the elevation of this region left a much broader strip of land than now exists there, and over this, the Edentates and other mammals made their way, perhaps urged on by the increasing cold of the glacial winters. The evidence to-day is strongly in favor of such a southern migration. This, however, leaves the Old World Edentates, fossil and recent, unaccounted for; but I believe the solution of this problem is essentially the same, namely: a migration from North America. The Miocene representatives of this group, which I have recently obtained in Oregon, are older than any known in Europe, and, strangely enough, are more like the latter and the existing African types than like any of our living species. If, now, we bear in mind that an elevation of only 180 feet would, as has been said, close Behring's Straits, and give a road thirty miles wide from America to Asia, we can easily see how this migration might have taken place. That such a Tertiary bridge did exist, we have much independent testimony, and the known facts all point to extensive migrations of animals over it.

The *Cetacea* are connected with the marine Carnivores through the genus *Zeuglodon*, as Huxley has shown, and the points of resemblance are so marked that the affinity cannot be doubted. That the connection was a direct one, however, is hardly probable, since the diminutive brain, large number of simple teeth, and reduced limbs in the Whales, all indicate them to be an old type, which doubtless branched off from the more primitive stock leading to the Carnivores. Our American extinct Cetaceans, when carefully investigated, promise to throw much light upon the pedigree of these strange mammals. As most of the known forms were probably marine, their distribution is of little service in determining their origin.

That the Sirenians are allied to the Ungulates is now generally admitted by anatomists, and the separation of the existing species in distant localities suggests that they are the remnants of an extensive group, once widely distributed. The large number of teeth in some forms, the reduced limbs and other characters, point back to an ancestry near that of the earliest ungulates. The gradual loss of teeth in the specialized members of this group, and in the Cetaceans, is quite parallel with the same change in Edentates, as well as in Pterodactyls and Birds.

The Ungulates are so distinct from other groups that they must be one of the oldest natural divisions of mammals, and they probably originated from some herbivorous marsupial. Their large size, and great numbers during Tertiary and Post-tertiary time, render them most valuable in tracing migrations induced by climate, as well as in showing the changes of structure which such a contest for existence may produce.

In the review of the extinct Ungulates, I have endeavored to show that quite a number of genera usually supposed to belong originally to the Old World are in reality true American types. Among these were the Horse, Rhinoceros, and Tapir, all the existing odd-toed Ungulates, and besides these the Camel, Pig, and Deer. All these I believe, and many others, went to Asia from our North West Coast. It must, for the present, remain an open question whether we may not fairly claim the *Bovidæ*, and even the *Proboscidea*, since both occur in our strata at about the same horizon as on the other continent. On this point there is some confusion, at least in names. The Himalayan deposits called Upper Miocene, and so rich in Proboscideans, indicate in their entire fauna that they are more recent than our Niobrara River beds, which, for apparently good reasons, we regard as Lower Pliocene. The latter appear to be about the same horizon as the Pikermi deposits in Greece, also regarded as Miocene. Believing, however, that we have here a more complete Tertiary series, and a better standard for comparison of faunas, I have preferred to retain the names already applied to our divisions, until the strata of the two continents are more satisfactorily coördinated.

The extinct Rodents, Bats, and Insectivores of America, although offering many suggestive hints as to their relationship with other groups, and their various migrations, cannot now be fully discussed. There is little doubt, however, that the Rodents are a New World type, and, according to present evidence, they probably had their origin in North America. The resemblance in so many respects of this order to the Proboscideans is a striking fact, not yet explained by the imperfectly known genealogy of either group.

The Carnivores, too, I must pass by, except to call attention to a few special forms which accompanied the migrations of other groups. One of these is *Machairodus*, the saber-toothed Tiger, which flourished in our Miocene and Pliocene, and followed the huge Edentates to South America, and the Ungulates across Asia to Europe. With this genus went *Hyænodon*, and some typical Wolves and Cats, but the Bears came the other way with the Antelopes. That the Gazelle, Giraffe, Hippopotamus, Hyaena and other African types, once abundant in Asia, did not come, is doubtless because the Miocene bridge was submerged before they reached it.

The Edentates, in their southern migration, were probably accompanied by the Horse, Tapir and Rhinoceros, although no remains of the last have yet been found south of Mexico. The Mastodon, Elephant, Llama, Deer, Peccary, and other mammals, followed the same path. Why the Mastodon, Elephant, Rhinoceros, and especially the Horse, should have been selected with the huge Edentates for extinction, and the other Ungulates left, is at present a mystery, which their somewhat larger size hardly explains.

The relations of the American Primates, extinct and recent, to those of the other hemisphere, offer an inviting topic, but it is not in my present province to discuss them in their most suggestive phases. As we have here the oldest and most generalized members of the group, so far as now known, we may justly claim America for the birth-place of the order. That the development did not continue here until it culminated in Man, was due to causes which at present we can only surmise, although the genealogy of other surviving groups gives some data toward a solution. Why the old world Apes, when differentiated, did not come to the land of their earlier ancestry, is readily explained by the then intervening oceans, which likewise were a barrier to the return of the Horse and Rhinoceros.

Man, however, came; doubtless first across Behring's Straits; and at his advent became part of our fauna, as a mammal and primate. In these relations alone, it is my purpose here to treat him. The evidence, as it stands to-day, although not conclusive, seems to place the first appearance of Man in this country in the Pliocene, and the best proof of this has been found on the Pacific coast. During several visits to that region, many facts were brought to my knowledge which render this more than probable. Man at this time was a savage, and was doubtless forced by the great volcanic outbreaks to continue his migration. This was at first to the south, since mountain chains were barriers on the east. As the native Horses of America were now all extinct, and as the early man did not bring the old world animal with him, his migrations were slow. I believe, moreover, that his slow progress towards civilization was in no small degree due to this same cause, the absence of the Horse

It is far from my intention to add to the many theories extant in regard to the early civilizations in this country, and their connections with the primitive inhabitants, or the later Indians, but two or three facts have recently come to my knowledge which I think worth mentioning in this connection. On the Columbia River, I have found evidence of the former existence of inhabitants much superior to the Indians at present there, and of which no tradition remains. Among many stone carvings which I saw there, were a number of heads which so strongly resemble those of Apes, that the likeness at once suggests itself. Whence came these sculptures, and by whom were they made? Another fact that has interested me very much is the strong resemblance between the skulls of the typical Mound-builders of the Mississippi Valley and those of the Pueblo Indians. I had long been familiar with the former, and when I recently saw the latter, it required the positive assurance of a friend who had himself collected them in New Mexico, to convince me that they were not from the mounds. A third fact, and I leave Man to the Archæologists, on whose province I am even now trenching. In a large collection of Mound-builders' pottery, over a thousand specimens, which I have recently examined with some care, I found many pieces of elaborate workmanship so nearly like the ancient water-jars from Peru, that no one could fairly doubt that some intercourse had taken place between the widely separated people that made them.

The oldest known remains of Man on this continent differ in no important characters from the bones of the typical Indian, although in some minor details they indicate a much more primitive race. These early remains, some of which are true fossils, resemble much more closely the corresponding parts of the highest Old World Apes, than do the latter our Tertiary Primates, or even the recent American Monkeys. Various living and fossil forms of old world Primates fill up essentially the latter gap. The lesser gap between the primitive Man of America and the Anthropoid Apes is partially closed by still lower forms of men, and doubtless also by higher Apes, now extinct. Analogy, and many facts as well, indicate that this gap was smaller in the past. It certainly is becoming wider now with every generation, for the lowest races of men will soon become extinct, like the Tasmanians, and the highest Apes cannot long survive. Hence the intermediate forms of the past, if any there were, become of still greater importance. For such missing links, we must look to the caves and later Tertiary of Africa, which I regard as now the most promising field for exploration in the Old World. America, even in the Tropics, can promise no such inducements to ambitious explorers. We have, however, an equally important field, if less attractive, in the Cretaceous Mammals, which must have left their remains somewhere on this continent. In these two directions, as I believe, lie the most important future discoveries in Palæontology.

As a cause for many changes of structure in mammals during the Tertiary and Post-Tertiary, I regard, as the most potent, *Natural Selection*, in the broad sense in which that term is now used by American evolutionists. Under this head, I include not merely a Malthusian struggle for life among the animals themselves, but the equally important contest with the elements, and all surrounding nature. By changes in the environment, migrations are enforced, slowly in some cases, rapidly in others, and with change of locality must come adaptation to new conditions, or extinction. The life-history of Tertiary mammals illustrates this principle at every stage, and no other explanation meets the facts.

The real progress of mammalian life in America, from the beginning of the Tertiary to the present, is well illustrated by the Brain-growth, in which we have the key to many other changes. The earliest known Tertiary mammals all had very small brains, and in some forms this organ was proportionally less than in certain Reptiles. There was a gradual increase in the size of the brain during this period, and it is interesting to find that this growth was mainly confined to the cerebral hemispheres, or higher portion of the brain. In most groups of mammals, the brain has gradually become more convoluted, and thus increased in quality, as well as quantity. In some, also, the cerebellum, and olfactory lobes, the lower parts of the brain, have even diminished in size. In the long struggle for existence during Tertiary time, the big brains won, then as now; and the increasing power thus gained rendered useless many structures inherited from primitive ancestors, but no longer adapted to new conditions.

Another of the interesting changes in mammals during Tertiary time was in the teeth, which were gradually modified with other parts of the structure. The primitive form of tooth was clearly a cone, and all others are derived from this. All classes of vertebrates below mammals, namely, Fishes, Amphibians, Reptiles, and Birds, have conical teeth, if any, or some simple modification of this form. The Edentates and Cetaceans with teeth retain this type, except the Zeuglodonts, which approach the dentition of aquatic Carnivores. In the higher mammals, the incisors and canines retain the conical shape, and the premolars have only in part been transformed. The latter gradually change to the more complicated molar pattern, and hence are not reduced molars, but transition forms from the cone to more complex types. Most of the early Tertiary mammals had forty-four teeth, and in the oldest forms the premolars were all unlike the molars; while the crowns were short, covered with enamel, and without cement. Each stage of progress in the differentiation of the animal was, as a rule, marked by a change in the teeth; one of the most common being the transfer, in form at least, of a premolar to the molar series, and a gradual lengthening of the crown. Hence, it is often easy to decide from a fragment of a jaw, to what horizon of the Tertiary it belongs. The fossil Horses of this period, for example, gained a grinding tooth, for each toe they lost, one in each epoch. In the single-toed existing horses, all the premolars are like the molars, and the process is at an end. Other dental transformations are of equal interest, but this illustration must suffice.

The changes in the limbs and feet of mammals during the same period were quite as marked. The foot of the primitive mammal was doubtless plantigrade, and certainly five-toed.

Many of the early Tertiary forms show this feature, which is still seen in some existing forms. This generalized foot became modified by a gradual loss of the outer toes, and increase in size of the central ones; the reduction proceeding according to systematic methods, differing in each group. Corresponding changes took place in the limb bones. One result was a great increase in speed, as the power was applied so as to act only in the plane of motion. The best effect of this specialization is seen to-day in the Horse and Antelope, each representing a distinct group of Ungulates, with five-toed ancestors.

If the history of American Mammals as I have briefly sketched it, seems as a whole incomplete, and unsatisfactory, we must remember that the genealogical tree of this class has its trunk and larger limbs concealed beneath the *débris* of Mesozoic time, while its roots doubtless strike so deeply into the Paleozoic that for the present they are lost. A decade or two hence, we shall probably know something of the mammalian fauna of the Cretaceous, and the earlier lineage of our existing mammals can then be traced with more certainty.

The results I have presented to you are mainly derived from personal observation; and since a large part of the higher vertebrate remains found in this country have passed through my hands, I am willing to assume full responsibility for my presentation of the subject.

For our present knowledge of the extinct Mammals, Birds and Reptiles of North America, science is especially indebted to Leidy, whose careful, conscientious work has laid a secure foundation for our vertebrate palæontology. The energy of Cope has brought to notice many strange forms, and greatly enlarged our literature. Agassiz, Owen, Wyman, Baird, Hitchcock, Deane, Emmons, Lea, Allen, Gibbes, Jefferson, DeKay, and Harlan, deserve honorable mention in the history of this branch of science. The South American extinct vertebrates have been described by Lund, Owen, Burmeister, Gervais, Huxley, Flower, Desmarest, Aymard, Pictet, and Nodot. Darwin and Wallace have likewise contributed valuable information on this subject, as they have on nearly all forms of life.

In this long history of ancient life I have said nothing of what Life itself really is. And for the best of reasons, because I know nothing. Here at present our ignorance is dense, and yet we need not despair. Light, Heat, Electricity, and Magnetism, Chemical Affinity and Motion, are now considered different forms of the same force; and the opinion is rapidly gaining ground that Life, or vital force, is only another phase of the same power. Possibly the great mystery of Life may thus be solved, but whether it be or not, a true faith in Science knows no limit to its search for Truth.

ART. XLIII.—*Note on the Helderberg Formation of Bernardston, Massachusetts, and Vernon, Vermont;* by JAMES D. DANA.

IN examinations of the Bernardston Helderberg formation which were the basis of my former paper "On the Rocks of the Helderberg era in the Valley of the Connecticut"* my main purpose was lithological—that is, to ascertain and point out the kinds of crystalline rocks that were comprised within terranes of Helderberg (later Upper Silurian) age. The conformable position of the Bernardston limestone beneath strata of quartzyte and slate, first made known by Professor Edward Hitchcock,† I found to be, as I thought, a fact; and from there I traced the quartzyte at intervals, along with the slate—a peculiar mica slate easily distinguished by the minute garnets which gave its layers a pimpled surface, and the small crystals of mica set transversely to the lamination—over the country, to South Vernon in Vermont; and announced in my paper that the Helderberg formation included, besides the quartzyte and mica slate, beds of compact green hornblende rock, a rock of the composition of syenyte, staurolitic mica slate, coarse mica schist, whitish and grayish quartzose gneiss, and all stages of passage between quartzyte and gneiss.

Recently, Professor C. H. Hitchcock, in the Second Volume of his Report on the Geology of New Hampshire,‡ and more briefly in a note in this Journal,§ has suggested that the order of stratification at the limestone locality is not the true order; that the rocks may be "in an inverted position:" that the limestone stratum may have *overlaid* both the other formations, that is, the quartzyte and mica slate;‖ that "the limestone occupies a small valley in the quartzyte."¶ Having, through this supposition, made the limestone the newest of the formations, he concludes, further, that the mica slate, now lying over it, is not necessarily Helderberg; that the hornblende rocks and gneiss of Vernon are not necessarily of the Helderberg series,** and neither the staurolitic slate; that a long period

* This Journal, III, vi, 339.

† Report on the Geology, etc. of Massachusetts, by E. Hitchcock, 8vo, 1833, p. 295; Report of Amer. Assoc. for 1851, p. 299; Report Geol. of Vermont, 2 vols. 4to, 1861, p. 447. This last notice was prepared in conjunction with Mr. C. H. Hitchcock. It gives a section representing the limestone dipping beneath quartzyte and slate.

‡ Page 428 and beyond, 1877.

§ Vol. xiii, page 313, April, 1877.

‖ Ibid.

¶ Report New Hampshire, vol. ii, p. 455.

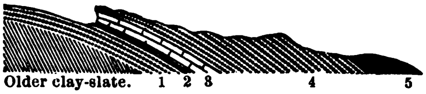
** It should be added here that the volume of the N. Hampshire Geological Report referred to conveys on an earlier page (p. 18), a different opinion as to the limits of the Helderberg, where it is stated that the Connecticut Valley Helderberg series consists of several thousand feet in thickness of quartzytes, limestones, slates, conglomerates, sandstones, flags, and probably hornblende schists.

intervened between the deposition of the hornblendic stratum and quartzyte.

While thus dissenting from my conclusions, Professor Hitchcock adopts my suggestion that the garnetiferous mica slate which overlies the quartzyte and limestone at the Bernardston limestone locality is identical with the Coös slate of the Connecticut Valley in all its characters and age, and hence that *if* the former should turn out after all to be Helderberg, the Coös formation (which extends up the valley to Canada, according to Professor Hitchcock) is also Helderberg or later Upper Silurian.

These differences of opinion, and the wide bearing of the facts on New England geology have led me to revisit the place and examine it anew. In my former paper I closed by stating my intention, another season, to study the stratigraphical details of the region, and trace out the limits or range of the formations southward along the Connecticut Valley. But other geological work in Western and Southern New England, and on the islands off its southern coast, have since occupied such leisure time as I could command. In my recent visit to Bernardston I was accompanied by Professor B. K. Emerson, of Amherst College; and it is a great satisfaction to know that he will give the whole subject a careful and thorough study, and connect it with a general geological survey of Central and Western Massachusetts—work for which he is eminently fitted.

To facilitate explanations I repeat the section of the strata at the Bernardston locality before published, with one correction. No. 3, the blocked area, represents the stratum of Crinoidal limestone; Nos. 1 and 4, dotted areas, an underlying and an overlying stratum of quartzyte; and Nos. 2 and 5, finely lined areas, an underlying and an overlying stratum of



Older clay-slate. 1 2 3 4 5

garnetiferous mica slate. The succession and position are the same as in the section by Professor Hitchcock in the Vermont Geological Report, excepting the omission here of a layer of slate immediately *over* the limestone.

The conclusions which the facts appear to me to sustain, in opposition to those set forth by Professor C. H. Hitchcock in the New Hampshire Report of 1877, but mostly in agreement with Mr. C. H. Hitchcock of the Vermont Report* (p. 598) of 1861—are the following:

* The Vermont Report makes no mention, in the chapter on the Bernardston limestone (p. 447), of the hornblende rocks, gneiss, etc., of the adjoining region on the east. But on page 598, in an account of "Section I," extending across the

1. That the quartzyte is Helderberg as much as the limestone.
2. That the garnetiferous mica slate is equally Helderberg.
3. That the limestone is a local deposit between the other members of the Helderberg formation.
4. That hornblende rocks, staurolitic slate, mica schist and gneiss of the adjoining region on the east and northeast are of one and the same geological formation.

1. *The Quartzyte is of the Helderberg formation.* The overlying quartzyte (No. 4) besides occurring in large outcrops over the hill-side, *constitutes the upper two to four feet* of the vertical section exposed in a portion of the limestone quarry. This alone proves its conformable position and close relation to the limestone. But further, while this overlying quartzyte is in part very compact and solid, some portions are very cellular from the removal of calcareous matter and pyrite, and also from the removal of fossils. The first of the fossils was found by Professor Emerson, while we were together, and was a cast of a *Pentamerus;* and both of us afterward obtained other specimens. The casts were too imperfect for a decision as to the species; but they appear to show that it was nearly equal in height and breadth, and without costæ, which are characteristics of the *P. pseudo-galeatus*, a Lower Helderberg species. Besides these brachiopods, there were in the same layers of the quartzyte numerous fragments of *crinoidal stems*, mostly of small species. Some of the laminæ of this quartzyte have between them mica in scales, so as to look in a surface view like mica schist.

The fact that there is conformability between the limestone and quartzyte is hence beyond question. And it is equally certain that, overturned or not, the quartzyte belongs to the same

south extremity of the State near the Massachusetts boundary, Mr. C. H. Hitchcock describes the hornblende rocks of Vernon and Bernardston as associated with gneiss and mica schists; states that the mica schist west of South Vernon contains chiastolite [staurolite?]; speaks of the gneiss as graduating insensibly into the quartz rock, as in some places difficult to be distinguished from granite, as also "invariably resting upon the quartz rock" and hence "newer" than it; as, therefore, probably of the Upper Helderberg age, like the Bernardston limestone (Hall's first determination), which rests on the same quartz rock; thus making the whole series Upper Helderberg. The unconformability of the quartz rock series on the clay slate is also recognized.

If these were still his views, excepting the change of Upper Helderberg to Lower Helderberg, we should be in close agreement. In my arguments above, I am in fact sustaining Mr. C. H. Hitchcock of 1861, against Professor C. H. Hitchcock of 1877) whose later conclusions have been influenced by his faith in the lithological test of geological age, and his unbelief in the existence of gneiss-like metamorphic rocks of later date than Cambrian.

The suspicion that C. H. Hitchcock may not have been the author of the notes on Section I is apparently set aside by the heading of the Chapter, "Notes on the Sections, by C. H. Hitchcock," and the absence of any statement that the notes on Section I were prepared by any other person.

era with the crinoidal limestone; for the former really graduates into the latter as its calcareous matter and fossils show.

2. *The garnetiferous mica slate is of the Helderberg formation.*—This is demonstrated by the existence of a stratum of garnetiferous mica slate (No. 2, in the section, p. 380,) *beneath* the limestone as well as one (No. 5) above. This inferior mica slate wants the little disseminated crystals of mica common in the other; but it is pimpled with garnets like that. The line of outcrops extends for several rods, and runs along within a few yards of the limestone, at the nearest point hardly a yard of earth intervening; and the strike and dip throughout correspond with that of the limestone adjoining. Having garnetiferous mica slate below the limestone as well as above, and the three strata conformable in dip, there can be no reasonable doubt that all are of one formation. The limestone stratum is so placed with reference to those above and below that it could not have been originally at the top, and the newest of the series. Whatever faulting or inversion be supposed, it must have had originally, as it has now, an overlying and an underlying mica slate.

3. *The limestone a local deposit in the Helderberg formation.*—The fact that the limestone has not been observed elsewhere in the region is no evidence of independent and later formation. It is plainly an isolated bed, of limited lateral extent, *like those that are so common in the widely spread "Calciferous mica schist" of the Connecticut Valley.* The fact that it is a wedge-shaped mass thus isolated is evident; for just north of the main quarry it soon thins out, (together with the thin underlying stratum of mica slate) through the approach and junction of the overlying and underlying quartzyte. Fossils, if found in any of the many isolated calcareous deposits in the "Calciferous mica schist," would be regarded as showing the age of the schist; and so it should be here.

Professor Hitchcock says that if the Coös slate is Helderberg, the Calciferous mica schist is unquestionably so too. Admitting this to be true, the parallelism between the Bernardston limestone and the isolated calcareous deposits in the schist becomes complete.*

4. *The hornblende rocks, staurolitic slate, mica schist and associated gneiss are of the Helderberg formation.*—As limestone has been found in the region only at the one locality in Bernardston, the evidence of equivalence has to be derived from the distribution of its associated rocks. This evidence, east and northeast of the Bernardston village-plain, is as follows:

(1.) The same garnetiferous mica slate with disseminated brown mica crystals set transversely that occurs associated

*** My knowledge of the rock formations of Western Vermont is not sufficient to warrant an independent opinion with regard to the "Calciferous mica schist."**

with quartzyte at the Bernardston locality on the west side of the Bernardston plain occurs associated with quartzyte at different points between Bernardston and Vernon. It sometimes dips beneath quartzyte and sometimes overlies it.

(2.) Outcrops of the quartzyte and the peculiar Bernardston mica slate together appear east and northeast of Bernardston within one to one and a half miles of the Crinoidal limestone locality (the intervening flat valley being under drift and alluvium), and at intervals beyond, to Vernon, with the same aspect and conformable superposition as at the limestone locality.

(3.) In the same region, hornblende rocks, staurolitic slate, gneiss and mica schist occur in alternating beds with the Bernardston mica slate and quartzyte.

A mile and a half east of Bernardston,* the Bernardston mica slate occurs in alternating beds with the hornblende rock, —a gray-green compact rock, not schistose—with so obvious junctions that the alternation cannot be questioned. The hornblende rock (1) dips beneath (2) mica slate; this beneath (3) hornblende rock; and the last beneath (4) mica slate again. Whether there is a fault between 2 and 3 is not certain; but it is unquestionable that 1 and 2, and 3 and 4 are strictly conformable. Part of the hornblende rock is speckled white with quartzyte and feldspar and is like a quartzytic syenite in constitution, though unlike true syenyte in aspect.

Again: a mile to the north of the last-mentioned locality and less than a mile and a half northeast of the Bernardston limestone locality, the same peculiar mica slate may be seen, dipping at a small angle beneath a stratum of quartzyte, the conformability, as in other cases in the region, unquestionable. Now this quartzyte stratum, while in part quartzyte, is partly a tough micaceous rock consisting chiefly of aggregated scales of brown mica, but with some hornblende and quartz.

Again: the Bernardston slate of the locality just mentioned extends eastward and is the same stratum, as well as the same kind of slate, with that of Purple's quarry, described in my former paper; which slate contains occasional crystals of *staurolite*. Hence the Bernardston slate which alternates with quartzyte is in some places a *staurolitic slate*. Another place, farther east, is mentioned in my former paper where the slate is abundantly staurolitic.

Again: in Vernon, four to five miles northwest of South Vernon, where the quartzyte is largely exposed to view, one of the quartzyte knolls consists partly of mica rock like that above described, made up mainly of aggregated scales of brown mica but containing distributed through it some quartz and

* For the position of this locality see my former paper.

hornblende. In other outcrops in the field adjoining, the rock is mostly true quartzyte, but partly a green compact hornblende rock, with insensible gradations between it and the quartzyte.

Again: at South Vernon, over the slopes nearly west of the hotel, there occur—first quartzyte, but with it, and graduating into it, the compact green hornblende rock; then, high up the slope, a coarse garnetiferous mica schist consisting mainly of brown mica, which is nothing but a coarse form of the Bernardston slate; and in this mica schist there are hornblendic layers; and some beds which consist of a quartzytic syenyte, though with the hornblende grains in slender crystals.

Again: between Vernon Center and South Vernon, there are outcrops showing the transition between the quartzyte and a quartzytic gneiss. The gneiss has the aspect of any ordinary light-colored thick-bedded gneiss. But it is all quartzytic, and in part very largely so.

Besides this light-colored quartzytic gneiss, there is also, north of South Vernon, quartzytic syenite, a whitish rock containing small grains of greenish hornblende, rather sparsely disseminated, without mica, and making a handsome rock which might at first be taken for a white granite.

5. *Conclusion.*—Thus, the region affords examples (1) of the *interstratification* of the quartzyte and Bernardston mica slate, with a green massive hornblende rock; with a syenytic rock; with gneiss; and with coarse mica schist; (2) of *transitions* of the Bernardston mica slate into staurolitic slate and mica schist; and (3) of *transitions* of the quartzyte into (*a*) micaceous quartzyte; (*b*) a tough quartzytic mica rock, more or less hornblendic; (*c*) quartzytic gneiss often granitoid: (4) green hornblende rock; and (5) syenyte, besides various intermediate forms. For some other examples of these transitions I refer to my former paper.

The demonstration is certainly complete that whatever the age of the quartzyte and the associated Bernardston mica slate, the same is the age of the rocks above mentioned; and that the fossils of the Bernardston locality decide the age approximately for the series; and finally, that all are of the Helderberg formation or later Upper Silurian, if that is true of the Crinoidal limestone.

6. *Lithological characteristics.*—In using the lithological test of geological age it must hence be noted that the following may be rocks of metamorphic Upper Silurian formations: mica slate and schist; staurolitic mica slate; hornblendic rocks, varying from a kind consisting mainly of green hornblende to a quartzytic syenyte, and hornblendic quartzyte; quartzytic gneiss; true gneiss; micaceous quartzyte; quartzyte.

The minerals included among the abundant metamorphic species are: brown and white mica, the former much the most common; staurolite; green and black hornblende; orthoclase; garnet; along with pyrite, magnetite, and granular limestone.

The mica slate and schist and the staurolitic mica slate are not distinguishable from kinds that are of earlier age.

The hornblende rocks are peculiar. Those which are made mainly of hornblende have a dark green color, and are massive, often indistinct in bedding instead of schistose and much jointed. The larger part consist of minute interlaced fibers; with sometimes whitish spots giving the rock a porphyritic aspect, which spots consist of quartzyte combined usually with grains of orthclase. Thin slices in polarized light under the microscope are beautiful objects. A cleavable granular variety occurs but is less common. Coarser kinds have the hornblende in oblong pointed crystals imbedded in fine grained quartzyte or quartzyte and orthoclase.

The whitish quartzytic syenite mentioned on the preceding page is peculiar in having a very fine grain; the hornblende dark green in color, and generally in oblong grains; and the whiter portions of the rock of an opaque white color and made up mostly of grains of orthoclase. In a thin slice, these whiter portions lie between others that are pellucid and consist of quartz grains with few of feldspar.

The gneiss also is peculiar. It generally consists very largely of grains of quartz, even where looking to the eye like a true gneiss. The mica is almost solely the brown kind and is like that of the mica slate, though often seeming to have as little elasticity as chlorite; and the regular disposition of the spots of mica, give to the most quartzytic varieties a strikingly gneiss-like look. Professor Hitchcock refers the rock to the Bethlehem gneiss. But "the most characteristic of the rocks comprising this formation," he says, speaking of the Bethlehem gneiss, "is a reddish granitic gneiss, the flesh-colored orthoclase predominating, with chloritic or some hydro-micaceous mineral in place of ordinary mica, and quartz in variable proportions" (Report, p. 105); and to such a rock it has little resemblance. Yet, it is to be admitted that there is nothing in the amount of orthoclase in characteristic "Bethlehem gneiss" which renders connection with a Helderberg formation improbable.

The quartzyte in some places—as two miles west of South Vernon,—contains much pearly mica (hydrous mica?); a weathered surface of such a specimen shows that the rock consists mainly of quartz. In other places the quartzyte is marked with dark-gray and blackish lines where the mica is not distinguishable without a glass. All of it indicates by its fineness of texture, and sometimes even flinty aspect, that the quartz sand of which it was made was very finely comminuted, and not coarse like that from which the Green Mountain quartzyte was made; and hence that the region, when the deposition took place, was not the border of the open sea.

7. *Origin of the Rocks.*—To understand the rocks of this Helderberg region, it must be borne in mind: that quartzyte beds in their original state, that is, beds of quartz sand, may have been formed at different times in the course of the era, owing to changes of level or of currents; that the sand beds—like those of any other era and of the present time—would, in many places, have had more or less earthy material (ground-up crystalline rock) with the quartz sand, so that metamorphism could not make pure quartzyte out of it, but might make a micaceous or gneissoid quartzyte, or a quartzytic gneiss, according to the nature of the earthy material present; that while sand-beds were formed where the currents were rapid enough for the purpose, mud-beds would have formed where the waters were more quiet, as now on all coasts (for sand-beds never exist along shores without cotemporaneous mud-beds within a distance that is not great); and that from these mud-beds, or beds of finely triturated rocks, the mica slate, mica schist, or mica rock, and the hornblende rocks would have been produced. The existence of some potash and alumina in the triturated rock or mud (both ingredients of orthoclase) would have favored the formation of brown mica (biotite) by metamorphism, while the presence of lime or rather a calcium compound, that of hornblende: magnesia and iron being in about the same proportion in the hornblende as in the mica. Analyses of average biotite and dark green hornblende afford:

	Biotite.	Hornblende.
Silica	40	45
Alumina	18	10–12
Iron protoxide, Iron sesquioxide	10	10
Magnesia	22	20
Lime	--	14
Potash	10	--

The magnesia would have come from the trituration of such older rocks as are made partly or wholly of minerals containing it; of which minerals, hornblende and biotite are the most common.

Admitting the Coös formation of Professor Hitchcock, and the Calciferous mica schist adjoining it, to be of the same formation with the mica slate, quartzyte, and hornblende rocks of the Bernardston and Vernon region, which Professor H. states to be a fact, this Helderberg formation stretches northward beyond the boundary of New England, with a breadth along the Connecticut Valley of fifteen to thirty miles or more:

breadth enough where narrowest—as at Bernardston—for a clear sea good for growing corals and crinoids. Whether Professor Hitchcock's Lisbon and Lyman groups, which he refers to the Huronian, are not to be included, remains to be ascertained, as indicated on page 317 of this volume. Adding them, it would follow that the Connecticut bay or channel of the Helderberg era covered a large part of Northern New Hampshire, and was connected with a great area in Maine marked off by the occurrence of Helderberg and Devonian fossils.

ART. XLIV.—*History of Cavern Exploration in Devonshire, England;* by W. PENGELLY, F.R.S., F.G.S., President of the Geological Section of the British Association at Plymouth.

[Continued from page 308.]

Brixham Cavern.—Early in 1858 an unsuspected cavern was broken into by quarrymen at the northwestern angle of Windmill Hill at Brixham, at a point seventy-five feet above the surface of the street, almost vertically below, and 100 feet above mean tide. On being found to contain bones, a lease in it was secured for the Geological Society of London, who appointed a committee of their members to undertake its exploration; funds were voted by the Royal Society, and supplemented by private subscriptions; the conduct of the investigation was intrusted to Mr. Prestwich and myself; and the work, under my superintendance, as the only resident member of the committee, was begun in July, 1858, and completed at midsummer, 1859.

The cavern, comprised within a space of 135 feet from north to south, and 100 from east to west, consisted of a series of tunnel galleries from six to eight feet in greatest width, and ten to fourteen feet in height, with two small chambers and five external entrances.

The deposits, in descending order, were:—

1st, or uppermost; a floor of stalagmite, from a few inches to a foot thick, and continuous over very considerable areas, but not throughout the entire cavern.

2d. A mass of small angular fragments of limestone, cemented into a firm concrete with carbonate of lime, commenced at the principal entrance, which it completely filled, and whence it extended thirty-four feet only. It was termed the *first bed.*

3d. A layer of blackish matter, about twelve feet long, and nowhere more than a foot thick, occurred immediately beneath the first bed, and was designated the *second bed.*

4th. A red, tenacious, clayey loam, containing a large number of angular and subangular fragments of limestone, varying from very small bits to blocks a ton in weight, made up the *third bed.* Pebbles of trap, quartz and limestone were somewhat prevalent, whilst nodules of brown hématite and blocks of stalagmite were occasionally met with in it. The usual depth of the bed was from two to four feet, but this was exceeded by four or five feet in two localities.

5th. The third bed lay immediately on an accumulation of pebbles of quartz, greenstone, grit and limestone, mixed with small fragments of shale. The depth of this, known as the *fourth* or *gravel bed,* was undetermined; for, excepting a few feet only, the limestone bottom was nowhere reached. There is abundant evidence that this bed, as well as a stalagmitic floor which had covered it, had been partially broken up and dislodged before the introduction of the third bed.

Organic remains were found in the stalagmitic floor and in each of the beds beneath it, with the exception of the second only; but as ninety-five per cent of the whole series occurred in the third, this was not unfrequently termed the *bone bed.*

The mammals represented in the stalagmite were bear, reindeer, *Rhinoceros tichorhinus,* mammoth, and cave lion.

The first bed yielded bear and fox only.

In the third bed were found relics of mammoth, *Rhinoceros tichorhinus,* horse, *Bos primigenius, B. longifrons,* red deer, reindeer, roebuck, cave lion, cave hyæna, cave bear, grizzly bear, brown bear, fox, hare, rabbit, *Lagomys spelæus,* water-vole, shrew, polecat and weasel.

The only remains met with in the fourth bed were those of bear, horse, ox and mammoth.

The human industrial remains exhumed in the cavern were flint implements and a hammer-stone, and occurred in the third and fourth beds only. The pieces of flint met with were thirty-six in number. Of these, fifteen are held to show evidence of having been artificially worked, in nine the workmanship is rude or doubtful, four have been mislaid, and the remainder are believed not to have been worked at all (see Phil. Trans., vol. clxiii, 1873, pp. 561, 562). Of the undoubted tools, eleven were found in the third and four in the fourth bed. Two of those yielded by the third bed, found forty feet apart, in two distinct but adjacent galleries, and one a month before the other, proved to be parts of one and the same *nodule*-tool; and I have little or no doubt that it had been washed out of the fourth bed and re-deposited in the third.

The hammer-stone was a quartzite pebble, found in the upper portion of the fourth bed, and bore distinct marks of the use to which it had been applied.

Speaking of the discovery of the tools just mentioned, Mr. Prestwich said in 1859:—"It was not until I had myself witnessed the conditions under which flint implements had been found at Brixham, that I became fully impressed with the validity of the doubts thrown upon the previously prevailing opinions with respect to such remains in caves" (Phil. Trans., 1860, p. 280); and according to Sir C. Lyell, writing in 1863: —" A sudden change of opinion was brought about in England respecting the probable coëxistence, at a former period, of man and many extinct mammalia, in consequence of the results obtained from the careful exploration of a cave at Brixham. . . . The new views very generally adopted by English geologists had no small influence on the subsequent progress of opinion in France" (" Antiquity of Man," pp. 96, 97).

Bench Cavern.—Early in 1861 information was brought me that an ossiferous cave had just been discovered at Brixham, and, on visiting the spot, I found that, of the limestone quarries worked from time to time in the northern slope of Furzeham Hill, one known as Bench Quarry, about half a mile due north of Windmill Hill Cavern, and almost overhanging Torbay, had been abandoned in 1839, and that work had been recently resumed in it. It appeared that in 1839 the workmen had laid bare the greater part of a vertical dike, composed of red clayey loam, and angular pieces of limestone, forming a coherent wall-like mass, twenty-seven feet high, twelve feet long, two feet in greatest thickness, and at its base 123 feet above sea-level. In the face of it lay several fine relics of the ordinary cave mammals, including an entire left lower jaw of *Hyæna spelæa* replete with teeth, but which had nevertheless failed to arrest the attention of the incurious workmen who exposed it, or of any one else.

Soon after the resumption of the work in 1861, the remnant of the outer wall of the fissure was removed, and caused the fall of an incoherent part of the dike, which it had previously supported. Amongst the *débris* the workmen collected some hundreds of specimens of skulls, jaws, teeth, vertebræ, portions of antlers, and bones, but no indications of man. Mr. Wolston, the proprietor, sent some of the choicest specimens to the British Museum, and submitted the remainder to Mr. Ayshford Sanford, F.G.S., from whom I learn that the principal portion of them are relics of the cave hyæna, from the unborn whelp to very aged animals. With them, however, were remains of bear, reindeer, ox, hare, *Arvicola ratticeps*, *A. agrestis*, wolf, fox, and part of a single maxillary with teeth not distinguishable from those of *Canis isatis*. To this list I may add rhinoceros, of which Mr. Wolston showed me at least one bone.

From the foregoing undesirably, but unavoidably, brief descriptions, it will be seen that the Devonshire caverns, to which attention has been now directed, belong to two classes,—those of Oreston, the Ash-Hole, and Bench being *Fissure Caves;* whilst those of Yealm Bridge, Windmill Hill at Brixham, Kent's Hole, and Ansty's Cove are *Tunnel Caves.*

Windmill Hill and Kent's Hole Caverns have alone been satisfactorily explored; and besides them none have yielded evidence of the contemporaneity of man with the extinct cave mammals.

Oreston is distinguished as the only known British cavern which has yielded remains of *Rhinoceros leptorhinus* (Quart. Journ. Geol. Soc., xxxvi, p. 456).

Yealm Bridge Cavern, if we may accept Mr. Bellamy's identification in 1835, was the first in this country in which relics of glutton were found (South Devon Monthly Museum, vi, pp. 218–223; see also "Nat. Hist. S. Devon," 1839, p. 89). The same species was found in the caves of Somerset and Glamorgan in 1865 (Pleist. Mam., Pal. Soc., pp. xxi, xxii), in Kent's Hole in 1869 (Rep. Brit. Assoc., 1869, p. 207), and near Plas Heaton, in North Wales, in 1870 (Quart. Jour. Geol. Soc.), xxvii, p. 407).

Kent's Hole is the only known British cave which has afforded remains of beaver (Rep. Brit. Assoc., 1869, p. 208), and up to the present year the only one in which the remains of *Machœrodus latidens* had been met with. Indeed Mr. MacEnery's statement, that he found in 1826 five canines and one incisor of this species in the famous Torquay Cavern was held by many paleontologists to be so very remarkable as, at least, to approach the incredible, until the Committee now engaged in the exploration exhumed, in 1872, an incisor of the same species, and thereby confirmed the announcement made by their distinguished predecessor nearly half a century before (Rep. Brit. Assoc., 1872, p. 46). In April last (1877) the Rev. J. M. Mello was able to inform the Geological Society of London that Derbyshire had shared with Devon the honor of having been a home of *Machœrodus latidens*, he having found its canine tooth in Robin Hood Cave in that county, and that there, as in Kent's Hole, it was commingled with remains of the cave hyæna and his contemporaries (Abs. Proc. Geol. Soc., No. 334, pp. 3, 4).

The Ash-Hole, as we have already seen, afforded the first good evidence of a British reindeer.

In looking at the published reports on the two famous Torbay caverns it will be found that they have certain points of resemblance as well as some of dissimilarity:

1st. The lowest known bed in each is composed of materials which, while they differ in the two cases, agree in being such as may have been furnished by the districts adjacent to the cavern-hills respectively, but not by the hills themselves, and must have been deposited prior to the existing local geographical conditions. In each, this bed contained flint implements and relics of bear, but in neither of them those of hyæna. In short, the *fourth bed* of Windmill Hill Cavern, Brixham, and the *breccia* of Kent's Hole, Torquay, are coëval, and belong to what I have called the *Ursine* period of the latter.

2d. The beds just mentioned were in each cavern sealed with a sheet of stalagmite, which was partially broken up, and considerable portions of the subjacent beds were dislodged before the introduction of the beds next deposited.

3d. The great bone bed, both at Brixham and Torquay, consisted of red clayey loam, with a large percentage of angular fragments of limestone; and contained *flake* implements of flint and chert, inosculating with remains of mammoth, the tichorhine rhinoceros, and hyæna. In fine, the *cave-earth* of Kent's Hole and the *third bed* of Brixham Cavern correspond in their materials, in their osseous contents, and in their flint tools. They both belong to what I have named the *Hyænine* period of the Torquay Cave.

But, as already stated, there are points in which the two caverns differ:

1st. While Kent's Hole was the home of man, as well as of the contemporary hyæna during the absences of the human occupant, there is no reason to suppose that either man or any of the lower animals ever did more than make occasional visits to Brixham cave. The latter contained no flint chips, no bone tools, no utilized *Pecten*-shells, no bits of charcoal, and no coprolites of hyæna, all of which occurred in the cave-earth of Kent's Hole.

2d. In the Torquay Cave, relics of hyæna were much more abundant in the cave-earth than those of any other species. Taking the teeth alone, of which vast numbers were found, those of the hyæna amounted to about 30 per cent of the entire series, notwithstanding the fact that, compared with most of the cave-mammals, his jaws, when furnished completely, possess but few teeth. At Brixham, on the other hand, his relics of all kinds amounted to no more than 8·5 per cent of all the osseous remains, while those of the bear rose to 53 per cent.

3d. The entrances of Brixham Cavern were completely filled up and its history suspended not later than the end of the Paleolithic era. Nothing occurred within it from the days when Devonshire was occupied by the cave and grizzly bears, reindeer, rhinoceros, cave lion, mammoth, and man, whose best tools were unpolished flints, until the quarrymen broke into it early in A. D. 1858. Kent's Cavern, on the contrary, seems to have never been closed, never unvisited by man, from the earliest Paleolithic times to our own, with the possible exception of the Neolithic era, of which it cannot be said to have yielded any certain evidence.

Though my "History of Cavern Exploration in Devonshire" is now completed, so far as the time at my disposal will allow, and so far as the materials are at present ripe for the historian, I venture to ask your further indulgence for a few brief moments while passing from the region of fact to that of inference.

That the Kent's Hôle men of the Hyænine period—to say nothing at present of their predecessors of the Breccia—belonged to the Pleistocene times of the biologist, is seen in the fact that they were contemporary with mammals peculiar to and characteristic of those times. This contemporaneity proves them to have belonged to the *Paleolithic* era of Britain and Western Europe generally, as defined by the archeologist; and this is fully confirmed by their unpolished tools of flint and chert. That they were prior to the deposition of even the oldest part of the peat bogs of Denmark, with their successive layers of beech, pedunculated oak, sessile oak, and Scotch fir, we learn from the facts that even the lowest zone of the bogs has yielded no bones of mammals but those of recent species, and no tools but those of *Neolithic* type; whilst even the granular stalagmite, the uppermost of the Hyænine beds in Kent's Hole, has afforded relics of mammoth, *Rhinoceros tichorhinus*, cave bear, and cave hyæna.

That the men of the Cave Breccia, or Ursine period, to whom we now turn, were of still higher antiquity, is obvious from the geological position of their industrial remains. That the two races of Troglodytes were separated by a wide interval of time we learn from the sheet of crystalline stalagmite, sometimes twelve feet thick, laid down after the deposition of the breccia had ceased, and before the introduction of the cave-earth had begun, as well as from the entire change in the materials composing the two deposits. But, perhaps, the fact which most emphatically indicates the chronological value of this interval is the difference in the faunas. In the cave-earth, as already stated, the remains of the hyæna greatly exceed in number those of any other mammal; and it may be added that he is also disclosed by almost every relic of his contemporaries—their jaws have, through his agency, lost their condyles and lower borders; their bones are fractured after a fashion known by experiment to be his; and the splinters into which they are broken are deeply scored with his teeth-marks. His presence is also attested by the abundance of his droppings in every branch of the cavern. In short, Kent's Hole was one of his *homes;* he dragged thither, piecemeal, such animals as he found dead near it; and the well-known habits of his representatives of our day have led us to expect all this from him. When, however, we turn to the breccia, a very different spectacle awaits us. We meet with no trace whatever of his presence, not a single relic of his skeleton, not a bone on which he has operated, not a coprolite to mark as much as a visit. Can it be doubted that had he then occupied our country he would have taken up his abode in our cavern? Need we hesitate to regard this entire absence of all traces of so decided a cave-dweller as a proof that he had not yet made his advent in Britain? Are we not compelled to believe that man formed part of the Devonshire fauna long before the hyæna did? Is there any method of escaping the conclusion that between the era of the Breccia and that of the Cave-earth it was possible for the hyæna to reach Britain?—in other words, that the last continental state of our country occurred during that interval? I confess that, in the present state of the evidence, I see no escape; and that the conclusion thus forced on me compels me to believe also that the earliest men of Kent's Hole were *interglacial*, if not *preglacial*.

The following table will serve to show at one view the coordinations and theoretical conclusions to which the facts of Kent's Cavern have led me, as stated briefly in the foregoing remarks. The table, it will be seen, consists of two divisions, separated with double vertical lines. The first, or left hand, division contains three columns, and relates exclusively to Kent's Cavern, as is indicated by the words heading it. The second, or right hand, division is of a more general character, and shows the recognized classification of well-known facts throughout western Europe. The horizontal lines are intended to convey the idea of more or less well-defined chronological horizons, and their occasional continuity through two or more columns denotes contemporaneity. Thus, to take an example from the two columns headed "Archæological" and "Danish-Bog," in the second division: the horizontal line passing continuously through both, under the words "Iron" and "Beech," is intended to suggest that the "Iron Age" of Western Europe and the "Beech" zone of the Danish Bogs take us back about equally far into antiquity; whilst the position of the line under the word "Bronze" indicates that the "Bronze age" (still of Western Europe) take us back from the ancient margin of the Beech era, through the whole of that of the Pedunculated Oak, and about half-way through the era of the Sessile Oak; and so on in all other cases.

KENT'S CAVERN.			PERIODS.				
Deposits.	Bones.	Implements.	Archeolog'al.	Danish-Bog.	Biological.	Geograph.	Climat
Black Mould.	Ovine.	Iron.	Iron.	Beech.	Recent.	Insular.	Post-Glacial
		Bronze.	Bronze.	Pedunculated Oak.			
		and (?)		Sessile Oak.			
		Neolithic.	Neolithic.	Scotch Fir.			
Granular Stalagmite.	Hyænine.	Paleolithic Flakes.	Paleolithic.		Pleistocene.	Continental.	Glacial and (?)
Black Band.							
Cave-earth.							
Crystalline Stalagmite.	Ursine.	Paleolithic Nodules.				Insular.	Inter-Glacial
Breccia.						Continental.	Pre-Glacial

ART. XLV.—*Is the Existence of Growth-rings in the Early Exogenous Plants proof of Alternating Seasons?* An extract from a paper read before the N. Y. Academy of Sciences, March 19, 1877; by CHARLES B. WARRING, PH.D.

WE are told that there must have been the same alternation of seasons before the Glacial Epoch as now, because the exogenous plants of those early times exhibit concentric growth-rings; and consequently the earth's axis must then have been inclined as at present.

But are seasons necessary to the formation of the rings? Until that is established their existence has no importance in this connection. Were it possible in some way to secure a temperature uniform through the year we might be able to determine the question experimentally. The nearest approach to such a condition in this latitude is to be found in green-houses.

The results thus far show that exogenous plants, e. g., the orange and lemon, so placed, form growth-rings as regularly as do the forest trees.

It would be interesting to know how generally exogenous plants in tropical regions exhibit these markings, and whether they are annual or whether they are made at longer or at shorter intervals. I have found it difficult to obtain any information on this point, either from books or from botanists. The latter tell me (I have applied to several botanists of distinction) that they know very little about it. Dr. Gray says, "I know of no exogenous tree that grows continuously. * * * Yet there are exogenous woody stems which do not make annual layers. There is a woody *Phytolacca* which makes more layers, at least twice as many, as it is years old—probably indicating two periods of growth and rest." To this I add that there now lies before me a section of *Chenopodium album* cut on the first of August, and consequently not more than four months old, in which are eight well defined rings. This section is as hard and compact and as well formed wood as if it were a section of ash or pine.

On the other hand there are exogens growing even in this climate, which, notwithstanding our cold winters and hot summers, show not the slightest trace of a ring. I have before me a section of *Akebia quinquefolia* cut by Dr. O. R. Willis on his own lawn from a plant five years old, which has no such markings. Then from a little further south I have a section of the Passion Vine in the same condition; also one of the Iron Wood (*Carpinus Americana*) which presents the faintest possible traces of them. For these also I am indebted to Dr. Willis.

Miss C. C. Haskell, of Vassar College, states the result of her examination of the tropical woods in their museum, as follows: In the *Moria atiara* of the Amazon, the circles are very apparent. In the *Aliso* or *Birch of the Indus*, the circles are evident. They are seen, too, in the *Brazilian Red-wood* (Upper Amazon), and in *Siphonia elastica* or Rubber tree, as well as in the *Moria peranya* of the Rio Negro. None are seen in the *Tortoise Shell Wood* or in the *Cow Tree*."

These suffice to show that, in the uniformly warm climate of the tropics, rings are formed as regularly as in the trees of our northern forests. But it may be said that although there is in these regions no alternation of hot and cold seasons, yet that they do undergo semi-annual changes from wet to dry, and from dry to wet, and that, these being dependent upon the earth's axial inclination, we are not at liberty to infer that the rings would have been formed had there been the absolutely seasonless condition which a perpendicular axis would produce. But there is evidence that exogenous trees would form these marks in a climate of absolutely no variation. I have before me a section of *Mangrove* also presented by Dr. Willis. This tree, as is well known, grows in the muddy margins of tropical rivers and all along the shores, forming dense forests even at the verge of the ocean and below high-water mark. In such a locality there can be no alternation of wet and dry seasons, and the changes of annual temperature must be less than the diurnal. It would seem impossible to conceive of greater uniformity of temperature and moisture, yet this tree presents the growth rings as broad and as well defined as those which are seen in any trees anywhere.

To dispel any vestige of belief that seasons and these markings are connected as cause and effect, I add that the *Cycads* require several years to form one ring.

The consideration of these facts leads to the conclusion that these circles have their origin in cycles of activity and repose, implanted in the constitution of the plant, which would continue to manifest themselves although there were no climatic variations—a conclusion strengthened by the experience of all who have attempted, by artificially equalizing the temperature, to make their plants bloom all the year. It is true that where seasonal variations exist, the successive stages of activity and rest are for obvious reasons synchronous with them, but they are not absolutely dependent upon them.

We may conclude, too, that the pre-glacial flora exhibited similar cycles of growth and rest, some of which may have been of short duration, measured perhaps by weeks, like those of the *Chenopodium*, while others like the *Cycads* may have required several years for their completion.

The following propositions appear to be established by the facts which have been presented.

1. Some exogens form rings at intervals much less than a year.
2. Others require intervals of several years.
3. Some form no rings.
4. The presence or absence of rings in exogens occurs in all climates.
5. Large and well defined rings are found under conditions in which there is absolutely no appreciable variation of temperature or moisture throughout the year.
6. An exogen naturally forming rings, will continue to form them although the climate become uniform through the year.

The existence, therefore, of these markings in the ancient flora gives no information as to the existence at that time of seasons, and so far as they are concerned we are left free to adopt any conclusion as to the inclination of the earth's axis which may appear to us most reasonable.

ART. XLVI.—*On Sipylite, a new Niobate, from Amherst County, Virginia;* by J. W. MALLET.

THE allanite found in Amherst County in this State, of which an analysis by Mr. J. A. Cabell, was published in the Chemical News, 1874, p. 141, occurs in large quantity, and furnishes an abundant source of supply of the cerium family of metals. In picking over a lot of three or four hundred pounds of it, I was struck with the appearance of a few fragments of an accompanying mineral, which on more careful examination turns out to be a new niobate.

The locality in question is on the northwest slope of Little Friar Mountain, about fifteen miles from the Virginia Midland Railway. The allanite is said to occur in a vein of more or less decomposed feldspar in a gneissoid rock, and is met with in large, but very imperfect crystals, and loose lumps of irregular shape, about four feet below the surface of the ground. Magnetite is found with it, the two minerals often forming parts of the same mass; and in going down the vein seems to become more compact, and tends to pass into solid magnetic iron ore. The vein is said to be about two feet wide, and runs about northeast and southwest, dipping at a large angle to the southeast.

Beside allanite, magnetite, and the new mineral now to be described, I have only noticed among the specimens which have reached me a few large crystals of hydrous zircon. One of these measured about 30×18×13 mm., was doubly terminated, of sp. gr.=4·217, and yielded on ignition 1·89 per cent of water.

The new mineral is decidedly rare; all the specimens I have collected were picked out from three lots of the allanite, two of them of several hundred pounds each, and would probably not weigh half a kilogram; the largest single piece weighs about forty grams; most of the fragments are much smaller. It is found imbedded in, or more commonly adherent to, the outside of the masses of allanite and magnetite, from which it is easily detached.

A few imperfect crystalline faces have been met with, but none of these brilliant, and only two dihedral angles that could be, even in a very rough way, measured with the application of a goniometer; each of these was about 125°, which is not far from $I \wedge I$ of the prism of yttro-tantalite, samarskite, and euxenite. There were observed also a few very imperfect cleavage planes. For the most part the mineral appears in little, irregularly shaped masses, very brittle, and exhibiting small, but distinct, conchoidal, as well as uneven fracture.

The color of the mineral in mass is brownish black, in thin splinters a red brown, like that of dark pine-rosin ; one or two small specimens display a gradual passage to a brownish orange, and even a yellow, but whether in these cases the chemical composition remains quite the same, there is not sufficient material to determine. The streak is light cinnamon-brown to pale gray. The luster resinous and pseudo-metallic. In general appearance to the eye the mineral is much like fergusonite from Greenland, euxenite from the neighborhood of Arendal, and samarskite from North Carolina, save that the last named is more distinctly pitchy black. Translucent in thin splinters. Hardness = nearly 6. Specific gravity may be considered = 4·89; one specimen gave 4·887 at 12°·5 C.; another 4·892 at 17°·5.

Heated alone in the ordinary blowpipe flame the mineral cracks, decrepitates, *glows brilliantly* (more brightly, I think, and at a lower temperature than any specimen of gadolinite I have ever seen), becomes pale greenish-yellow and opaque, like many specimens of good blast-furnace slag, and remains quite infusible. In the flame of one of Fletcher's hot-blast blowpipes, before which a stout blowpipe wire of platinum readily melts to a bead, thin splinters are fused merely on the edges. Heated in a closed glass tube, the same decrepitation, glowing and change of color are observed, and water is given off, which condensing on the surface of the tube is found to have an acid reaction, and slightly etches the glass. Fused with borax in the oxidizing flame, the mineral is dissolved, producing a yellow glass, which becomes pale on cooling, and assumes a greener tint in the reducing flame. With microcosmic salt, a yellowish green glass is obtained. Strong boiling hydrochloric acid attacks to some extent the mineral in fine powder, and the partial solution, if boiled with metallic tin and diluted with water, gives the fine sapphire-blue color due to niobium. This partial hydrochloric acid solution, if diluted, contains zirconium enough to brown turmeric paper to an extent quite sensible if a comparative experiment be made with similarly diluted hydrochloric acid alone. Boiling concentrated sulphuric acid decomposes the mineral completely, though somewhat slowly; and the diluted solution gives a blue color on addition of metallic zinc.

The chemical analysis was made, with much care and patience, under my direction by Mr. W. G. Brown, a student in this laboratory during the last winter. The details of the method used are given in a notice of his work in the Chemical News. Tantalum was found to be present, but in such small quantity, certainly less than one-twelfth of the niobium, that a satisfactory separation could not be obtained by Marignac's method. The sp. gr. of the mixed niobic and tantalic oxides was 4·60. By determination of the yttrium and erbium first as oxides and then as sulphates it was found that the mixture contained almost exclusively the latter metal, of which the absorption spectrum is obtainable with great distinctness from the crude solution. Iron and uranium were proved to exist as ferrous and uranous compounds. The following results were obtained:

Nb_2O_5, Ta_2O_5*	48·66
WO_3	·16
SnO_2	·08
ZrO_2	2·09
Eb_2O_3, Y_2O_3†	27·94
Ce_2O_3‡	1·37
La_2O_3§	3·92
Di_2O_3‖	4·06
UO	3·47
MnO	trace
FeO	2·04
BeO	·62
MgO	·05
CaO	2·61
Li_2O¶	trace
Na_2O	·16
K_2O	·06
F	trace
H_2O	3·19
	100·48

Throwing together, as Rammelsberg has done in his valuable paper** on the natural tantalates and niobates, the acid oxides of niobium, tantalum, tungsten, tin and zirconium, reducing all the basic oxides present to the equivalent amounts of dyad oxides, and leaving out the water, we have from the above figures the ratio,

$$R''O : M^{v}_2O_5 = 221 : 100$$

leading to the formula $R''_3M^{v}_2O_8 \cdot 4\, R''_2M^{v}_2O_7$, or, applying the common phosphate nomenclature, a single atomic group of ortho-niobates with four of pyro-niobates; while samarskite, according to the calculation of Professor O. D. Allen†† from his analysis, contains one to one, or is represented by the formula

* Ta_2O_5 may be assumed = about 2 per cent.

† Y_2O_3 may be assumed = about 1 per cent.

‡ Cerous oxide, but with Cléve's formulæ and atomic weights for this and the corresponding oxides of lanthanum and didymium.

§ Containing a trace of Di_2O_3. ‖ Containing a trace of Ce_2O_3.

¶ Spectroscopically detected.

** Jour. Chem. Soc., March, 1872, p. 189.

†† This Journal, August, 1877, p. 131.

$R''_3M^{v}_2O_8 \,.\, R''_2M^{v}_2O_7$, and Rammelsberg makes pyrochlore from Fredriksvärn solely the pyro-niobate, $R''_2M^{v}_2O_7$, and fergusonite, tyrite, etc., solely the ortho-salt, $R''_3M^{v}_2O_8$.

If, however, the water be included in the calculation, and considered basic, placing it on an equivalent footing with the dyad oxides we have the ratio,

$$R''O : M^{v}_2O_5 = 311 : 100, \text{ or nearly } 3 : 1,$$

which gives the simple formula of an ortho-salt, $R''_3M^{v}_2O_8$. This I confess I am inclined to think more probable, and, if so, it may be allowable to suppose that the very remarkable glow exhibited by the mineral when heated is connected with the loss of basic water and the change from ortho- to pyro-niobate, as in the well known incandescence of ammonio-magnesian ortho-phosphate at the moment of change by heat to the pyro-phosphate of the latter metal.*

Whichever formula be preferred, however, for the mineral now described, it differs essentially from that of any niobate hitherto on record, the one view making it the nearest approach to a simple pyro-*niobate* (since the Fredriksvärn pyrochlore contains largely of titanium) and the other making it an ortho-salt like fergusonite, etc., but one partially acid in character or containing basic hydrogen.

Not on chemical grounds alone, but in several respects as to physical properties, the mineral is new and distinct. Carrying out the fancy of Heinrich Rose, which led him to name niobium from the daughter of Tantalus, and remembering the number and complexity of the natural niobates which have been met with, I propose for this species the name Sipylite, from Sipylus, one of the numerous children of Niobe.

*** It may be worth remarking that from the analysis of Professor Allen (loc. cit.) of Professor J. Lawrence Smith's new mineral, hatchettolite, which accompanies samarskite in North Carolina, water seems to be present in it in definite proportion; and, although Rammelsberg has considered the water found in his analyses of tantalates and niobates as non-essential, and the formula, $R''_3M^{v}_2O_8$, which he has assigned in common to fergusonite, yttro-tantalite, tyrite and bragite, requires that water be excluded, if it be also taken into account his analyses of these minerals lead pretty closely to simple relations as to the extent of hydration (without considering the water basic), making**

Fergusonite, from Greenland (with very little water)—$R''_3M^{v}_2O_8$, or perhaps $2R''_3M^{v}_2O_8 \,.\, H_2O$.

Brown yttro-tantalite, from Ytterby	**$2\ R''_3M^{v}_2O_8 \,.\, 3H_2O$.**
Yellow " " "	
Tyrite	
Bragite	
Gray yttro-tantalite, from Gamle Kararfvet	**$2R''_3M^{v}_2O_8 \,.\, 5H_2O$.**

University of Virginia, Sept. 3, 1877.

ART. XLVII.—*On the Mean Motion of the Moon;* by SIMON NEWCOMB.

FOR some time after the appearance of Hansen's Lunar Tables, it was very generally considered that the theory of the moon, after occupying the attention of the mathematicians and astronomers of every century for two thousand years, was at length complete, and that the motion of that body could now be predicted with entire confidence. That Hansen's computation of the inequalities of short period produced by the sun not only far exceeded in accuracy any before made, but fulfilled all the requirements of modern astronomy, I conceive can hardly be doubted. But in the number of this Journal for September, 1870, I showed that this improvement did not extend to the inequalities of long period in the mean motion. While it was true that Hansen by an empirical term had secured a very good agreement with observations from 1750 to 1860, it was there shown that this agreement had been obtained by sacrificing the agreement before 1750, and that the moon had then begun to deviate from the tables at such a rate that they could not continue satisfactorily to represent the observations. During the seven years which have since elapsed, this suspicion has been entirely confirmed. So far as can be judged by the most recent observations, the error of the tables now exceeds ten seconds, and is increasing at a rate of not less than half a second a year.

Shortly after the publication of the short paper to which I have alluded, it was made a part of my official duty to investigate this question. In accordance with this arrangement, I have aimed at the complete discussion of all recorded observations of any astronomical value before the year 1750. These researches now being brought substantially to a close, so far as the observations are concerned, the object of the present article is to give some account of them, and of their results. The material consists in brief, of every observation of an eclipse or an occultation previous to 1750, which appears to be worthy of confidence, and calculated to throw any light upon the question of changes in the moon's mean motion. The available data may be classified as follows:—

I. Accounts of ancient historians from which it has been inferred that the shadow of the moon passed over certain points of the earth's surface during certain total eclipses of the sun. The celebrated eclipses of Thales, of Larissa, and of Agathocles have been very carefully discussed by Professor Airy in two papers which have appeared, the one in the Philosophical Transactions, and the other in the Memoirs of the Royal Astronomical

Society. After a careful examination of the six or eight eclipses in question, I was led to the conclusion that none of them could be safely relied upon as furnishing data for the error of the Lunar Tables at the times when they were observed. It is impossible, within the limited space of the present article, to enter into any details of the considerations which led me to this conclusion. It may be remarked, however, that among the eclipses in which I can feel but little confidence is the celebrated one of Thales. To prevent misapprehension I may say that I do not deny either that Thales predicted eclipses or that the shadow of the moon passed over Asia Minor, B.C., 585 as indicated by the Lunar Tables, or that a battle was stopped by some real or fancied advent of darkness, as described by Herodotus a century afterward; but I fail to see any good reason for maintaining that the extremely obscure account of Herodotus really refers to the total eclipse in question, or, in fact, to any eclipse whatever. Consequently, while these eclipses may be useful in throwing more or less of evidence on the question of the moon's secular acceleration, I do not think they can be considered reliable enough to be used for determining that quantity.

II. The second class comprises the nineteen eclipses of the moon quoted by Ptolemy in the Almagest, on which he founded his theory of the moon's motion. These eclipses appear to be worthy of some confidence, making due allowance for the very considerable errors of observation with which they are necessarily affected. The mode of treatment was this: from a very careful study of the account of each eclipse as given by Ptolemy, and without any knowledge of how it compared with the tables, I sought to make an estimate, first, of the most probable time of the phase described, and second, of the probable error of that time. These estimates I shall publish without any alteration suggested by the subsequent comparison with the tables. When this comparison was made, it was found that the general deviations of the tabular from the recorded times did not indicate a probable error essentially greater than that estimated, except in two cases.

There are five eclipses in which Ptolemy does not say to what phase the time which he gives refers. It has very generally been considered that in these cases the phase was that of the middle of the eclipse; but in all other cases the time which he gives is that of commencement; and there would be a certain probability in favor of the times where no phase was given being also those of commencement. The errors in question were systematically different from those of the other eclipses, and seemed to indicate that in these eclipses also, the beginning was referred to. Owing, however, to the uncertainty of this entire hypothesis, I judged it best to reject these eclipses entirely, and confine the discussion to the fourteen remaining ones.

Among these fourteen, which, in some cases, include the end of the eclipse as well as the beginning, there was a single one, that of B.C., 382, December 22, which was in contradiction with all the others. The other thirteen all agree in the most remarkable manner in assigning a correction of more than half an hour to the tabular times; while this one indicated a negative correction. This discordance, however, is not the most perplexing circumstance. It happened that this eclipse commenced just before sunrise, and therefore just before the moon set; and if the other eclipses were accurate, this one could not have been seen at all. If this one really was seen, it would almost necessitate a negative correction to the tabular times. We have then this dilemma: either the whole thirteen eclipses recorded by Ptolemy are, with a single exception, half an hour or more in error, or there is some mistake about this eclipse having been actually observed. Deeming the latter the more probable of the two hypotheses, I threw out this eclipse entirely. Of the twelve remaining eclipses, sixteen phases were observed, which were divided into four groups, and the mean result, by weight, of each group was taken. The mean corrections to the tabular times given by the several groups, are as follows:—

Epoch,	-687	$\delta t = +20^{m}$	$\delta\varepsilon = -11' \pm 4'$	3 phases.
	-381	$\delta t = +50$	$\delta\varepsilon = -27 \pm 5$	3 phases.
	-189	$\delta t = +36$	$\delta\varepsilon = -20 \pm 3$	8 phases.
	$+134$	$\delta t = +30$	$\delta\varepsilon = -16 \pm 4$	3 phases.

III. The next observations in order are the eclipses observed by the Arabian astronomers between the years 829 and 1004, which are published in the work entitled Le Livre de la Grande Table Hakémite, traduit par le C[en]. Caussin, Paris, 1804. This work is a translation of the Arabic manuscript belonging to the University of Leyden. A few of the observations were known to Tycho Brahe and were published by him in his Historia Cœlestis. As a slight indication of the value of these eclipses it may be remarked that the two or three given by Tycho Brahe furnished the first data from which the secular acceleration of the moon was deduced. It is therefore a singular fact that no comparison of them with modern tables has ever been seriously attempted.

There are, in all, in this book, observations of twenty-five eclipses including thirty-four phases of beginning or ending. They were all reduced and compared with the tables of Hansen. Three of them were so far discordant that they had to be rejected entirely. This ratio of three out of thirty-four will not appear great if we reflect that the manuscript from which the observations were translated, was frequently very difficult to decipher or to translate, owing not only to the fading of the writing, but to the uncertainty of some of the terms which the author used. Besides these three discordant observations, there were two which could not be used because the altitude assigned to the moon at the time of the observation actually exceeded its meridian altitude. Here it was evident that there was something wrong, in recording, transcribing or translating the observation. The general result was that each observation of a phase gave the mean longitude of the moon with a probable error ranging from three to five minutes of arc. The results were divided into three groups, each made by a separate observer or set of observers, and therefore worthy of being considered as entirely independent. The mean result of each of these groups was as follows:—

Epoch, 846	$\delta\varepsilon = -4'\cdot4$
926	$\delta\varepsilon = -1\cdot1$
986	$\delta\varepsilon = -4\cdot8$

IV. Observations made after the revival of science in Europe and before the invention of the telescope. These observations were made by various astronomers from Regiomontanus to Tycho Brahe. But after a careful and laborious examination of all their observations I could find, I was led to the conclusion that none of them would throw any light on the problem. Before Tycho Brahe the observations were no better than those of the Arabs, while the time elapsed was one half that which has elapsed since the Arabian observations. No doubt the observations of Tycho Brahe are more accurate; but the records are so confused that it is impossible to obtain any definite result from them. In fact they preceded the invention of the telescope by so short an interval that it can hardly be supposed that they would throw much light on the question under consideration, however carefully they had been made. I searched carefully to find whether Tycho Brahe had ever observed an occultation, especially of Aldebaran; but could find no trace of any such observation.

V. Observations of occultations and eclipses made with a telescope but without a clock, the time being determined by the altitude of the sun or of some star observed with a quadrant. This class comprises the observations of Bullialdus and Gassendus, as well as some of the earlier of Hevelius. Bullialdus seems to have been the first one who actually observed the occultation of a star by the moon, but he does not appear to have been a skillful observer. The observations of occultations have the great advantage that the only error to be feared is that of the determination of time, always supposing that the phenomenon was actually seen. The disappearance of the star behind the moon's limb is, in fact, a sudden phenomenon which does not require any measure of distance to be well observed.

VI. Observations of eclipses and occultations made by Hevelius with a very imperfect clock regulated by altitudes taken with a quadrant with pinnules. It is well known that Hevelius would never use a telescope with his quadrant; so that the results to be derived from the observations of this most indefatigable observer do not correspond to the labor which he spent in making them. His observations are much better than those of Gassendus, but far more inaccurate than those made with the telescopic sights.

VII. Observations of Flamsteed at Greenwich, and of the astronomers of the French school, from 1672 to 1750. Flamsteed's observations were published in the Historia Cœlestis. Those of the French astronomers are not only for the most part unpublished, but seem to have been totally forgotten from the time they were made until I was fortunate enough to find them in the archives of the Paris Observatory in 1871. Not only were they wholly unreduced, but in many cases not even the name of the occulted star was given. The reduction of these observations has been the most laborious part of my work. The observers have left no explanations whatever of their mass of observations, and it was necessary to learn this by induction from the observations themselves; and from the calculations scattered here and there through the books. The errors of the instruments and of the clocks had to be investigated from modern data; and the observations have proved to be well worthy of the pains which were taken with them. Thereby, the motion of the moon has been traced back to 1675, an epoch seventy-five years before observations upon it have heretofore been supposed to commence. In the same class with these Paris observations are to be included those of DeL'isle at St. Petersburgh, with which I was furnished by Struve.

The following are some independent mean corrections given by the observations of Bullialdus, Gassendus, Hevelius, Flamsteed, and the French astronomers. The list is incomplete, as the discussion of the solar eclipses has not been finished; but it will suffice for the purposes of the present discussion:—

1621	+ 77″	1661	+ 37″
1630	+ 30	1666	+ 24
1633	+ 53	1680	+ 30·4
1635	+ 55	1682	+ 2[illegible]·5
1639	+ 23	1715	+ 13·8
1645	+ 51	1725	+ 7·0
1652	+ 38		

The investigation is terminated at the epoch of 1750 so far as the reduction of observations is concerned, because there is reason to believe that Hansen's tables are not greatly in error from 1750 to 1865. We may, therefore, in this preliminary discussion consider the tabular errors zero between these epochs. For the epoch 1875 the correction given by some good observations of occultations is −8″0, a result 1″·7 less than that indicated by the observations at Greenwich and Washington. This discrepancy is quite surprising. It is, however, worthy of remark that Captain Tupman from a discussion of all the meridian observations made in Europe about the time in question obtained a mean result somewhat less than that given by Greenwich and Washington alone. It is well known that Hansen's term depending on eight times the mean motion of Venus minus thirteen times that of the earth is almost entirely empirical, being adjusted so as to satisfy the observations between 1750 and 1850. And since this term fails to satisfy the observations outside of these limits, in fact making the tables worse than they would be without it, it ought to be rejected from the comparison of theory with observation. Its effect upon the ancient results is, however, so small in comparison with the necessary error of the observations that its effect need not be taken into account.

From the individual corrections to the moon's mean longitude which have been given for the modern dates I have sought to obtain by a rough interpolation the actual corrections for every quarter of a century from 1625 to 1725. The general results are shown in the following table, of which I shall explain the several parts:

Table of residual corrections to the several theories of the mean motion of the moon.

	(1)	(2)	(3)	(4)	(5)
Epoch.	Hansen.	H′.	$s=8''\cdot8$.	$s=6''\cdot18$.	Δt.
−687	−11′	−11′	+16′	+39′	−70^m
−381	−27	−27	− 7	+10	−18
−189	−20	−20	− 4	+10	−17
+134	−16	−16	− 6	+ 4	− 6
846	− 4·4	− 4·4	− 2·4	− 0·2	0
926	− 1·1	− 1·1	+ 0·3	+ 2·1	− 4
986	− 4·8	− 4·8	− 3·8	− 1·3	+ 2
1625	+50″	+33″	+ 6″.1	− 6″·6	+12^s
1650	+39	+18	− 6 ·9	−19 ·0	+ 3
1675	+32	+15	− 7 ·4	−18 ·6	+34
1700	+21	+16	− 3 ·6	−13 ·5	+25
1725	+ 7	+16	− 0 ·3	− 8 ·6	+16
1750	0	+19	+ 6 ·4	0 ·0	0
1775	0	+21	+12 ·5	+ 8 ·4	−15
1800	0	+15	+11 ·1	+ 9 ·5	−17
1825	0	+ 2	+ 3 ·0	+ 4 ·4	− 8
1850	0	−11	− 4 ·6	0 ·0	0
1875	− 8	−28	−15 ·8	− 7 ·6	+14

In column (1) we have the mean correction indicated by observations to Hansen's tables of the moon without any modification whatever. In column (2) these corrections are modified by the effect of Hansen's empirical term, so as to show the corrections to the pure theory after this term is subtracted from the tables. If the theory is perfect, these numbers ought to be represented by corrections to the mean longitude and mean motion of the moon and the secular acceleration.

The following are the several corrections given by the method of least squares:

$$\left.\begin{array}{l}\delta e = +19''\cdot 57 \\ \delta n = -12\ \cdot 31 \\ \delta s = -\ \ 3\ \cdot 36\end{array}\right\}\text{Epoch, 1700.}$$

The value of the secular acceleration adopted by Hansen is 12″·17. Subtracting the correction it seems that the acceleration to which we are led by observation alone, is 8″·8.

Column (3) shows the outstanding corrections which remain after subtracting the result of the corrections we have just found. It is evident that the theory does not represent the observations, and that the most recent observations indicate a value of the secular acceleration much less than that indicated by the older ones. If we investigate the uniform variation of the acceleration which would best satisfy the whole of the observations, we shall find it to be −0″·9 in a century. The hypothesis of such a uniform variation is, however, too improbable to be admitted; and moreover, it still fails to represent the modern observations, although the ancient ones are thus greatly improved.

In recent times it has been generally considered that the difference between the theoretical acceleration and that given by observations arises from a change in the length of the day. It is worthy of remark that by supposing this change itself subject to variations, all the apparent changes in the mean motion of the moon can be accounted for. This is a hypothesis which I have suggested in former numbers of this Journal, as one by which the changes in question may be explained. Let us now see what the actual variations in the rotation of the earth must be to account for the difference between observation and theory. In the first place, the secular acceleration must be supposed to be uniform and equal to 6″·17. Two epochs at which we may suppose the time given by the rotation of the earth to be correct, being entirely arbitrary, we shall take 1750 and 1850 for these epochs. Having thus formed a theory of the moon's mean motion founded on gravitation alone, column (4) shows the apparent corrections indicated by observation. In column (5) these corrections are changed into time. The times here given are hypothetical errors of the earth's rotation which it is necessary to subtract from the times given by astronomical observations in order to reduce them to a perfectly uniform measure of time. The sign + indicates that the earth is ahead of its mean rotation, and the sign − that it is behind it. For some years past it has seemed to me that this was the most probable hypothesis on which to explain the deviations in question. It was evidently a most unwelcome one; for, granting its truth it would be no longer possible to predict the apparent motion of the moon, since the changes in the rotation of the earth could not be expected to follow any determinate law. It is therefore extremely gratifying to find that the comparisons we have just given lead to the hope that these deviations may, after all, be due to the action of some of the bodies of the solar system. A very cursory examination of the residuals given in column 3 shows that they have apparently a period not very far from 260 years. Now, it is remarkable that this differs very little from the period of Hansen's first inequality, which is 273 years. The question therefore arises whether the deviations in question may not be explained by a change in the constants of this inequality. The result is very surprising. By merely diminishing the argument of Hansen's first inequality by 60° 48′ without changing the co-efficients at all, the observations from 1625 to 1875 may all be represented within the limits of error. In fact, we see that the numbers in column (3) may be very nearly represented by the formula

$$-5''\cdot04-10''\cdot14\left(\frac{t-1800}{1800}\right)-15''\cdot50\cos A,$$

in which we have placed,

$$A=18V-16E-g,$$

V being the mean longitude of Venus counted from the equinox of 1800, E that of the earth counted in the same way, and g the mean anomaly of the moon. The comparison in question is shown in the following tables; the fourth column of which is taken from the corresponding column of the preceding table. The residuals still outstanding are shown in the last column.

Epoch.	A.	Computed terms.	Observ.	Diff.
1625	−47°·0	+2″·2	+6″·1	+3″·9
1650	−14 ·0	−4 ·7	−6 ·9	−2 ·2
1675	+19 ·0	−7 ·1	−7 ·4	−0 ·3
1700	52 ·0	−4 ·5	−3 ·6	+0 ·9
1725	85 ·0	+1 ·2	−0 ·3	−1 ·5
1750	118 ·0	+7 ·3	+6 ·4	−0 ·9
1775	151 ·0	+11 ·0	+12 ·5	+1 ·5
1800	184 ·0	+10 ·4	+11 ·1	+0 ·7
1825	217 ·0	+4 ·8	+3 ·0	−1 ·8
1850	250 ·0	−4 ·8	−4 ·6	+0 ·2
1875	283 ·0	−16 ·0	−15 ·8	+0 ·2

Correcting Hansen's term by this empirical addition, we find that instead of

$$15''\cdot34 \sin (A+30^\circ\cdot2),$$

the value given by Hansen, we shall have

$$15''\cdot3 \sin (A-30^\circ\cdot6),$$

as the result of observation.

As a test of this result, the sum of all the corrections here found to Hansen's tables has been taken and compared with the corrections given in column 1. It is to be remarked in the first place that the diminution of 10″ a century in the mean motion of the moon involves a further correction of $-0''\cdot4$ to the value of the secular acceleration in order that the ancient observations may still, on the average, be best represented. Thus the secular acceleration reduces to

$$8''\cdot4\,;$$

and the total correction to the acceleration of Hansen is

$$-3''\cdot76.$$

We put V_2 for the empirical term of Hansen,

$$21''\cdot47 \sin (8V-13E+274^\circ\ 14'),$$

the existence of which appears to have been entirely refuted by the researches of Delaunay; and T for the time counted in centuries after 1800. Then the total corrections to the tables of Hansen are as follows:—

$$-V_2-1''\cdot14-29''\cdot17\,T-3''\cdot76\,T^2-15''\cdot5 \cos A.$$

The following are the values of these corrections for the principal epochs from 1625 to 1900. The computation and comparison with observation is given so fully that any explanation of the table appears to be unnecessary.

Epoch.	$-V_2$	$-15''\cdot5$ cos A.	$-1''\cdot14$ $-29\cdot17$ T $-3\cdot77$ T².	Sum.	Observation.	Diff.
1625	+17″·1	−10″·6	+38″·4	+44″·9	+50″	+5″
1650	21·4	−15·0	34·1	40·5	39	−1·5
1675	16·9	−14·7	29·4	31·6	32	+0·4
1700	+ 5·2	− 9·5	24·3	20·0	21	+1·0
1725	− 8·6	− 1·4	18·6	8·6	7	−1·6
1750	−18·9	+ 7·3	12·5	+ 0·9	0	−0·9
1775	−21·2	13·6	+ 5·9	− 1·7	0	+1·7
1800	−14·7	15·4	− 1·1	− 0·4	0	+0·4
1825	− 2·1	12·4	− 8·7	+ 1·6	0	−1·6
1850	+11·4	5·3	−16·7	0·0	0	0·0
1860	15·7	+ 1·8	−20·0	− 2·5	+1·5	+4·0
1870	19·0	− 1·7	−23·4	− 6·1	−5·5	+0·6
1880	20·9	− 5·2	−26·9	−11·1	....	....
1890	21·4	− 8·4	−30·4	−17·4	....	....
1900	+20·6	−11·2	−34·1	−24·7	....	....

The only case in which the difference exceeds the possible error of the comparisons is at the epoch 1860. As an explanation of this I can only suggest that the term found by Mr. Neison as due to the action of Jupiter is at that time added to the result of a possible error in Hansen's value of the term which depends upon the ellipticity of the earth. The comparison may therefore be improved when the theory is suitably corrected.

The great question which now arises is this. Is it possible that this correction to the term produced by the action of Venus can really be a result of the attraction of that planet? We are struck by the fact that the proposed change can be expressed by a mere change of the algebraic sign of the constant term of the argument, leaving the value of the co-efficient unchanged. It may therefore be inquired whether it is possible that the sign of this quantity is erroneous in Hansen's formula. This question must be answered in the negative. I have found by an investigation still unpublished, substantially the same result as Hansen; while the researches of Delaunay published in the Connaissance des Temps for the year 1862 show that the approximate expression of the constant term in question, is

$$180°-2h'',$$

h'' being the longitude of the node of Venus, which does not differ much from 75°. It is, therefore, a mere chance that the change of Hansen's term can be expressed in this way.

Although Hansen, Delaunay, and myself have all arrived at the same result for the value of the term in question, I cannot confidently say that that result is complete. In all three computations the terms of the second order due to the mutual attraction of Venus and the earth are neglected. It is evident that in consequence of this mutual attraction, the direct action of Venus on the moon is different from what it would be if each planet moved in its elliptic orbit. It may be that this difference is sensible in terms of so high an order as those under consideration. I have actually computed the additional terms in $\frac{1}{\Delta^3}$ (Δ being the distance of Venus from the earth) which arise in this way and which depend upon the argument $18V-16E$. The result is that the values of the several parts which make up this term are quite comparable with those of the elliptic terms which depend on the same argument; but these co-efficients destroy each other in taking the sum. I have, however, always regarded my computations on this subject as incomplete, and have, in consequence, never published them.

As the case stands, the marked agreement between theory and observation which is produced by the introduction of this empirical term, seems to me such as to warrant its provisional use until a more careful investigation of the subject can be made.

Washington, Oct. 3, 1877.

SCIENTIFIC INTELLIGENCE.

I. Chemistry and Physics.

1. *On the Action of Saline solutions on Lead.*—Muir has examined the action exerted by various saline solutions upon lead, with and without access of air, with a view to explain the mechanism of the process. The lead used was sold as pure, and contained only traces of manganese, zinc and iron. Three parallel series of experiments were tried, one in corked flasks, another in beakers covered with paper, and a third in basins, similarly covered. Twenty-five square centimeters of lead were used in each experiment, being placed in a solution of 2·0 gram per liter, of one of the following salts: ammonium nitrate, potassium nitrate, calcium chloride, ammonium sulphate or potassium carbonate, for a time varying from 14 to 21 days. In the corked flask the maximum effect took place in the calcium chloride solution, 1·9 milligrams of lead being dissolved in 14, and 3 milligrams in 21 days; while in the open beaker, ammonium nitrate dissolved from 2 to 4 milligrams, and in the open basin, the nitrate and sulphate each dissolved from 8 to 16 milligrams. The order of solvent power is NH_4NO_3, $CaCl_2$, $(NH_4)_2SO_4$, KNO_3, K_2CO_3. In general, access of air increases the action. The lead deposit formed was a plumbic hydrocarbonate $(PbCO_3)_2$,

$$Pb(OH)_2 \text{ or } \begin{matrix} Pb\left\{\begin{matrix} OH \\ O \\ O \end{matrix}\right. \\ Pb\left\{\begin{matrix} O \\ O \end{matrix}\right. \\ Pb\left\{\begin{matrix} O \\ O \\ OH \end{matrix}\right. \end{matrix} \begin{matrix} \\ \}CO \\ \\ \}CO \\ \end{matrix}$$

, in fine silky scales. It was found to be more soluble in ammonium nitrate when air was excluded (one part in 4,600), and in calcium chloride with access of air (one part in 26,000), though its solubility was very great in carbonic acid water (one part in 4,300). The author believes that in the action of saline solutions upon lead, a soluble salt is first produced; that carbon dioxide is slowly absorbed from the air, converting the lead into hydrocarbonate, which is mostly precipitated; that in certain liquids the formation of the soluble salt proceeds at first more rapidly than its precipitation, but that later the latter action preponderates; and that carbonates precipitate the lead salt as fast as it is formed, in the form of hydrocarbonate.—*J. Ch. Soc.*, xxxi, 660, June, 1877. G. F. B.

2. *New Method for the Synthesis of Hydrocarbons.*—Friedel and Crafts, in examining the action of finely divided aluminum upon organic chlorides, which is at first very slow, but becomes more rapid, found that the aluminum chloride formed in the reaction was the active agent in evolving the hydrogen chloride, remaining itself unaltered. If, for example, amyl chloride be treated with the anhydrous chloride, hydrochloric acid gas is

evolved in the cold, as well as a mixture of gases not absorbable by bromine. In the residue, beside the unaltered Al_2Cl_6, are contained various hydrocarbons, some of high boiling point. If, however, the amyl chloride be mixed with a hydrocarbon, such as benzene in excess, the evolution of gas is regular, and the liquid separates into two layers, the upper one of which is a solution of amyl-benzene in excess of benzene, the lower, one of Al_2Cl_6. Iodides and bromides act similarly, though not as uniformly. Ethyl iodide treated as above gave ethylbenzene, methyl chloride gave toluene (methyl-benzene), xylene (dimethylbenzene) mesitylene (trimethylbenzene) and durene (tetramethylbenzene), benzyl chloride gave diphenylmethane, chloroform gave triphenylmethane and carbon tetrachloride gave tetraphenylmethane. Acid chlorides act in the same way. Benzoyl chloride dissolved in benzene, gives, by the action of aluminum chloride, the ketone benzophenone, acetyl chloride gives acetophenone, phthalyl chloride gives phthalophenone and another product, probably anthraquinone. Further examination showed that zinc and ferric chlorides acted similarly in the cold, ferrous chloride on warming. Copper, cobalt, and magnesium chlorides appeared to be without action. The authors explain the reaction by supposing an aluminum organic compound to be first formed and then decomposed, regenerating the chloride, thus:

$$C_6H_6+Al_2Cl_6=HCl+Al_2Cl_5(C_6H_5)$$

$$Al_2Cl_5.(C_6H_5)+(CH_2C_6H_5)Cl=Al_2Cl_6+(CH_2.C_6H_5)C_6H_5.$$

—*J. pr. Ch.*, II, xvi, 233, Aug., 1877. (*C. R.*, lxxxiv, 1392, 1450).

G. F. B.

3. *The Terpenes of Swedish Wood Tar.*—ATTERBERG has examined a so-called "wood oil," which is the first product of distillation of the wood tar made in Sweden by the destructive distillation of resinous woods, principally that of *Pinus sylvestris.* The oil was freed from creasote-like bodies and gummy acids by repeated treatment with potassium hydrate, and then submitted to repeated fractioning. In this way there was isolated a terpene boiling at 156·5°–157·5°, and having the properties of australene, and another boiling at 173°–175° having the odor of fresh pine wood, and not identifiable with any other terpene. This the author calls sylvestrene. These two terpenes constitute 80 per cent of the oil. Sylvestrene has a specific gravity of 0·8612 at 16°, is dextrorotatory, rotating +19·5° in sodium light, and forms mono- and di-hydrochlorates, the latter of which recrystallized from alcohol yields broad flat brilliant needles fusing at 72°–73°. Heated in sealed tubes with potassium hydrate, sylvestrene yields an oil having a strong pelargonium odor.—*Ber. Berl. Chem. Ges.*, x, 1202, July, 1877.

G. F. B.

4. *On the Amylene from Amyl Iodide.*—ELTEKOFF has investigated the action of alcoholic potash upon the amylene dibromide obtained from amyl iodide, and concludes that this amylene is really a mixture of two isomeric bodies, isopropylethylene

$\overset{CH[CH(CH_3)_2]}{\underset{CH_2}{|}}$ whose bromine derivative gives isopropylacetylene by the action of alcoholic potash; and methylethylethylene $\overset{CH[CH_2(C_2H_5)]}{\underset{CH_2}{|}}$ which does not yield valerylene under these circumstances, but is transformed into valeric ether. Isopropylacetylene boils at 35°, forms a crystalline addition product with silver nitrate $AgC{\equiv}C-CH\left\{\begin{matrix}CH_3\\CH_3\end{matrix}\right.$, which is decomposed by iodine, yielding $IC{\equiv}C-CH\left\{\begin{matrix}CH_3\\CH_3\end{matrix}\right.$, moniodisopropylacetylene. — *Bull. Soc. Ch.*, II, xxviii, Aug., 1877. G. F. B.

5. *On the Constitution of unsaturated Dibasic Acids.*—At the close of a series of researches upon certain unsaturated dibasic acids made in his laboratory, FITTIG sums up the results and discusses their bearing upon the constitution of the acids in question; i. e., fumaric and maleic acids in one group, and itaconic, citraconic and mesaconic acids in another. The facts are (1) the two former unite directly with hydrogen to yield succinic acid $\begin{matrix}CH_2.COOH\\|\\CH_2.COOH\end{matrix}$, as the three latter by the same treatment yield the same pyrotartaric acid, $\begin{matrix}CH_3\\|\\CH.COOH\\|\\CH_2.COOH\end{matrix}$; (2) the former acids by union with bromine give two different dibromosuccinic acids (both of which, however, are substitution products of ethylene-succinic acid), as the latter give in the same way three different dibromopyrotartaric acids (which must equally be regarded as substitution products of common pyrotartaric acid); (3) the former acids by union with hydrogen bromide yield the same bromosuccinic acid, the latter the same bromopyrotartaric acid, except itaconic acid, which yields an isomer; (4) while mono- or di-brom-citraconic and mesaconic acids lose with great ease on boiling with water or bases a molecule of carbon dioxide, becoming methacrylic or bromomethacrylic acids, probably identical, and easily reduced to isobutyric acid, the corresponding derivatives of itaconic acid are more permanent and yield no carbon dioxide when thus treated; (5) while fumarates and maleates yield on electrolysis the same acetylene, and citraconic and mesaconic acids the same allylene, itaconic acid gives a hydrocarbon not precipitating silver solutions. There is no constitutional formula which will satisfy all these conditions if the position be maintained that in unsaturated compounds the carbon atoms are *always* united by several bonds. Hence the facts compel the adoption of the view advanced by Kekulé, that beside these, there are other bodies, such as carbonous oxide for example, in which there are single carbon atoms whose attractions are not completely balanced. Rejecting also as unproved the existence in a compound of carbon atoms three of whose units are

balanced while the fourth is free, the author gives for maleic acid the formula $\begin{array}{l} CH_2COOH \\ \vert \\ =C.COOH \end{array}$ and for fumaric acid $\begin{array}{l} CH.COOH \\ \Vert \\ CH.COOH \end{array}$. For itaconic acid he gives $\begin{array}{l} =CH \\ \vert \\ CH.COOH \\ \vert \\ CH_2.COOH \end{array}$, or $\begin{array}{l} CH_2 \\ \Vert \\ C.COOH \\ \vert \\ CH_2.COOH \end{array}$, for citraconic acid $\begin{array}{l} CH_3 \\ \vert \\ CH.COOH \\ \vert \\ =C.COOH \end{array}$ and for mesaconic acid $\begin{array}{l} CH_3 \\ \vert \\ C.COOH \\ \Vert \\ CH.COOH \end{array}$. Hence the isobrommaleic acid of Kekulé is properly bromfumaric acid, and the dibrommaleic acid of Bourgoin, dibromfumaric acid. Of the two itaconic acid formulas, Fittig prefers the first.—*Liebig Ann.*, clxxxviii, 95, July, 1877. G. F. B.

6. *On a Phenol of Phenanthrene, Phenanthrol.*—Rehs has examined, under Graebe's direction, the product obtained by fusing phenanthrenemonosulphonic acid with potassium hydrate. After solution in water, the phenanthrol was separated in oily drops by the addition of sulphuric acid, which solidified on cooling. After boiling with ammonium carbonate, and recrystallization from a mixture of petroleum naphtha and benzene, it was obtained in beautiful blue-fluorescing plates, fusing at 112°, and giving on analysis the formula $C_{14}H_9(OH)$. It forms well crystallized compounds with alkalies, and ethers with acid oxides.—*Ber. Berl. Chem. Ges.*, x, 1252, July, 1877. G. F. B.

7. *Formation of Rosolic acid from Cresol and Phenol.*—The discovery of Caro and Wanklyn that by de-nitrogenizing rosaniline rosolic acid could be formed, and of Dale and Schlorlemmer, that aurin (rosolic acid) could be converted into rosaniline, led Zulkowsky to attempt the production of rosolic acid from cresol and phenol as rosaniline is produced from toluidine and aniline. A mixture of two molecules cresol, one phenol and three of sulphuric acid heated with arsenic acid to 120° C. became dark brown and thick and yielded to water a gummy body with a greenish metallic luster, having all the properties of rosolic acid. It is not produced with phenol alone. The author represents rosaniline as

$$NH_2.C_6H_3\begin{array}{l} \diagup C_6H_2\left\{\begin{array}{l}CH_3\\NH\end{array}\right. \\ \quad\vert \qquad\quad \vert \\ \diagdown C_6H_2\left\{\begin{array}{l}NH\\CH_3\end{array}\right. \end{array} \text{ and rosolic acid as } OH.C_6H_3\begin{array}{l} \diagup C_6H_2\left\{\begin{array}{l}CH_3\\O\end{array}\right. \\ \quad\vert \qquad\quad \vert \\ \diagdown C_6H_2\left\{\begin{array}{l}O\\CH_3\end{array}\right. \end{array}.$$

Corallin he separated into five different bodies.—*Ber. Berl. Chem. Ges.*, x, 1201, July, 1877. G. F. B.

8. *A new Coloring matter.*—Hofmann has examined a new brilliant red coloring matter, obtained from Martius. He found it to be the sodium salt of an organic acid, which was separated by concentrated hydrochloric acid. Fine red needles were obtained, easily soluble in alcohol, less so in water, having the formula $C_{16}H_{12}N_2SO_4$, and being monobasic. No doubt, therefore, that

this body was analogous to chrysöidin, and that it could be formed by diazobenzol and naphtolsulphonic acid, by azosulphanilic acid with a naphtol, by azonaphtylaminsulphonic acid and phenol, and by diazonapthalene and a phenolsulphonic acid. Using the first method, and mixing sodium α naphtolsulphonate with aniline nitrate and potassium nitrite, the new color was obtained.—*Ber. Berl. Chem. Ges.*, x, 1378, July, 1877. G. F. B.

9. *Examination of a Nickel magnet.*—H. WILD. (Abstract from the original memoir). The author has submitted to examination a nickel magnet presented to Kotschubey, President of the Russian Technological Society, by Jos. Wharton of Philadelphia. It had the form of a flat bar, 2 mm. thick, 9.5 mm. broad, and 155 mm. long, pointed at the ends, and had at its center an agate cap for supporting it on a pivot. Its weight was 25 grams. Its magnetic moment was determined by comparison with a bar of steel of about the same dimensions, and found to be per gram 112,000 units, the steel giving 245,000. After remagnetizing, the nickel gave 188,000, the steel 368,000. With wolfram steel, the moment went up to 594,000 in one instance. The nickel was analyzed by Butlerow and found to contain only one-third of one per cent of iron, with traces of cobalt. The effect of temperature and of time upon the magnetism of nickel was also noted. The following are the conclusions of the memoir: 1st. Pure nickel, unlike pure soft iron, may acquire a considerable amount of permanent magnetism; but the amount of this magnetism, as a maximum, is only from one-half to one-third of that which hardened steel can receive. 2d. The magnetism remaining in the nickel after the magnetizing force ceases, is less permanent than in well hardened steel; the slow loss of magnetism in the course of time, as well as that occasioned by heating and cooling, is proportionally greater than in hardened steel, even when like the steel, it is brought by repeated warming and cooling into a certain condition of permanence. 3d. The temperature-coefficient of a nickel magnet in this condition, is a little greater than that of well hardened steel. 4th. The temporary magnetism which pure nickel assumes is about double that of its permanent magnetic moment, about half of the temporary magnetism which hardened steel can acquire and one-fourth of that capable of being developed in soft iron.—*Bull. Ac. St. Pet.*, xxiv, 1, May, 1877.

10. *Spectroscope with a Fluorescent Eye-piece.*—M. J. L. SORET has published a detailed description of improvements which he has made in the application of the well known properties of fluorescent substances to the observation of the ultra violet portions of the spectrum. He places a screen of the fluorescent material at the focus of the object-glass of the spectroscope, and views the spectrum projected on this screen with an eye-piece placed obliquely, so that the diaphragms and blackened walls of the tube may extinguish the direct rays. As a screen he uses either a small plate of uranium glass or a cell fitted with an aqueous solution of esculine; and the spectroscope is best con-

structed with lenses of quartz and prisms of Iceland spar. With lenses of glass and prisms of flint the spectrum lines could not be distinguished beyond N, but with the spectroscope whose construction he describes in detail the principal lines could be distinguished as far as T. With a more portable spectroscope of similar construction M. Soret has made observations in the Alps at an altitude of 3180 meters, and draws from them the conclusions, that although the ultra violet spectrum is more brilliant at high elevations than on the plains it has no greater extent. In the observations referred to, he could not distinguish rays more refrangible than T. Whence he infers that it is the atmosphere of the sun, and not that of the earth which absorbs the most refrangible rays of the spectrum. The diminution in brilliancy of the more refrangible portion of the spectrum caused by the atmosphere he refers not to the selective absorption of its aeriform constituents, but to the effect of the floating liquid or solid particles, which when more abundant produce a distinct haze or collect in clouds. The general absorption of light due to this last cause affects all the rays of the spectrum, but to a greater extent in proportion as the rays are the more refrangible.—*Ann. Chim. et de Phys.*, V, xi, 72. J. P. C., JR.

11. *Sun's Heat.*—M. A. Crova has published a very extended paper on the calorific intensity of the solar radiation and its absorption by the atmosphere of the earth. In this paper the author discusses very exhaustively methods of observation and gives the results of a large number of measurements which are of great interest, but can not be described in a short abstract.—*Ann. Chim. et de Phys.*, V, xi, 433. J. P. C., JR.

12. *Changes in the Spectra of Gases caused by increasing tension.*—In the spectrum of a gas rendered luminous by an electric spark M. Wullner distinguishes two classes of effects as caused by an increasing tension. In the case of hydrogen only, the bands themselves broaden into a continuous spectrum. With other gases a continuous spectrum appears between the bands which remain meanwhile as definite as at first. In the case of compounds of carbon and markedly in the case of carbonic dioxide the brilliancy of the continuous spectrum soon becomes so great that the bands disappear, but with nitrogen and with air they can be distinguished until the pressure becomes much more considerable. This restatement of the results of previous observations is occasioned by a communication of M. Cazin who refers the continuous spectrum in such cases to the solid particles transported and rendered luminous during the electric discharge.—*Ann. Chim. et de Phys.*, V, xii, 143. J. P. C., JR.

13. "*The Influence of Light in Chemical Changes and chiefly in Oxidation,*" is the subject of a recent paper by M. P. Chastaing. The author distinguishes as a definite effect of the sun's rays the determining of the oxidation of inorganic metallic compounds, an influence which he locates in a different part of the solar spectrum from the well-known reducing action. He deduces this conclusion chiefly from the observation that the oxidation of

such substances as ferrous sulphate, alkaline solution of arsenious acid, and aqueous solutions of hydric sulphide or alkaline sulphides proceeds more rapidly in the light than in the dark, and he endeavors to estimate the action of the light by the difference in the rapidity of the process in the two cases under otherwise like conditions. He concludes that the chemical action of the solar spectrum on metallic compounds both binaries and salts while reducing at the more refrangible end is oxidizing at the less refrangible end. The general reducing action of white light he refers to the circumstance that in the rays as a whole the reducing action is the more powerful of the two. M. Chastaing finds that the green rays still exert a reducing action and he locates between the rays D and E a neutral point of the spectrum, at which chemical action takes place as in darkness. It appears however that the action exerted on organic compounds by the light is quite different from that just indicated. Its influence on such bodies is always oxidizing and this effect continually increases as we pass from the red to the violet end of the spectrum with some variation from this law in the green rays. For numerous details and subordinate conclusions we must refer to the original paper which is quite long but full of interest. We must add that results of our own do not accord, at least *apparently*, with those of M. Chastaing. We have recently discovered in the oxidation of a solution of antimonious iodide under the combined action of the air and light a direct effect of oxidation caused by the sun's rays, and this effect is produced chiefly, if not wholly, by the more refrangible rays.—*Ann. Chim. et de Phys.*, V, xi, 145. J. P. C., JR.

14. *Magnetic rotatory Polarization.*—M. HENRI BECQUEREL has very recently published the results of an important investigation on "Magnetic rotatory Polarization," which is especially interesting as supplementing the researches of his distinguished father on the same subject. His memoir is quite long and the results cannot be stated in a few words. The most important general conclusions are the following:—

(1.) That the positive rotation of the plane of polarization of a ray of light having a definite wave-length, in passing through the unit of thickness of a diamagnetic material under the influence of magnetism, is sensibly proportional to $n^2(n^2-1)$, a function of the index of refraction, and to a factor depending on the magnetism and on the diamagnetism of the body, this factor becoming the greater in proportion as the substances are more diamagnetic. (2.) That with substances chemically allied or containing the same radical the quotient of the magnetic rotation, and the corresponding value of $n^2(n^2-1)$, varies very slightly. (3.) That the chemical nature of the substance exerts an important influence on the phenomenon, and that the several constituents of a compound may produce an independent effect. (4.) That when in solution the specific effect of the molecules of diamagnetic bodies is not influenced by the concentration of the solution, while that of the molecules of magnetic bodies may be greatly affected by the closer proximity, which such a concentration would cause, (5.) That when the substances are very diamagnetic the dispersion of the rays caused by the magnetic rotation is sensibly proportional to $\frac{n^2 (n^2-1)}{\lambda^2}$ in which expression λ is the wave length and n the index of refraction. For various qualifications and details we must refer to the original paper, and also for a discussion of the theory advanced by M. Becquerel père, which refers the differences between magnetic and diamagnetic effects to the relative strength of the magnetic energy of the bodies experimented on and that of the medium by which they are surrounded. —*Ann. Chim. et de Phys.*, V, xiii, 5. J. P. C., JR.

15. *Rose-colored Sulphide of Manganese.*—The conditions of the transformation of the rose-colored sulphide of manganese obtained by precipitation into the green semi-crystalline modification of the same compound has been studied by MM. Ph. de Clermont et H. Guiot who come to the conclusion that the two substances are different states of hydration of the same body.—*Ann. Chim. et de Phys.*, V, xii, 111.

16. *Analysis of Alkaline Sulphides and Sulpho-carbonates.*—M.M. DELACHANAL et MERMET have described a new method for the complete chemical analysis of alkaline sulphides and sulphocarbonates based on the application of hypobromite of potassium as an oxidizing agent. The method offers certain marked advantages and the values obtained indicate that it yields accurate results.—*Ann. Chim. et de Phys.*, V, xii, 88.

17. *Separation of Potassium from Sodium.*—SCHLOESING has improved and simplified the process proposed by Serullas for separating potassium from sodium based on the circumstance that potassic prochlorate is insoluble in alcohol and that of the radicals most frequently occurring in analytical processes potassium is the only one whose prochlorate does not dissolve in this solvent. He uses for the purpose pure perchloric acid, and gives simple methods for preparing this reagent in the required quantity. Perchloric acid in excess readily replaces both nitric and hydrochloric acid, forming perchlorates of the bases present. Potassic perchlorate is then easily separated and washed with alcohol of forty degrees Baumè. In this process the potassium is weighed as perchlorate after the salt has been heated to 250°, while the sodium is converted into sulphate and weighed as such.—*Ann. Chim. et de Phys.*, V, xi, 561. J. P. C., JR.

18. *A volumetric method of determining the amount of Manganese in iron ores*, is described by M. GARCIA PARREÑO which will undoubtedly be found useful in many cases. The manganese in the ore is at the outset converted into Mn_3O_4 by roasting in a platinum crucible. About a gram is taken for each assay which is dissolved in thirty-five to forty cubic centimeters of hydrochloric acid. The process is conducted in a flask, and the chlorine gas conducted into a weak solution of potassic iodide, the last traces being driven over by boiling the acid. The amount of iodine

thus set free is then determined by a standard solution of hyposulphite of sodium which bleaches the solution colored by the iodine. The solution of the hyposulphite is standardized by a preliminary experiment with pure Mn_3O_4.—*Ann. Chim. et de Phys.*, V, xi, 571. J. P. C., JR.

19. *Light: A series of simple, entertaining and inexpensive experiments in the phenomena of light, for the use of students of every age;* by ALFRED M. MAYER and CHARLES BARNARD. 112 pp. 8vo. New York, 1877, (D. Appleton & Co.).—The purpose of this little volume is to present and illustrate the fundamental phenomena of light in a manner suited to the ready comprehension of the younger class of students, and to suggest means for verifying the laws of its action by actual experiment. The text, which was prepared by Mr. Barnard and revised by Professor Mayer, is written in vivacious and entertaining style, and with such clearness and accuracy of statement that the most inexperienced student cannot fail to understand it. The experiments, which were devised by Professor Mayer, are for the most part new, and have the merit of combining precision in the methods with extreme simplicity and elegance of design. Nearly all the apparatus figured can be made by the young experimenter himself, with the use of the most common and inexpensive materials, and at a very slight expense, the whole cost of the articles required being less than fifteen dollars. The aim of the authors has been to make their readers "experimenters, strict reasoners, and exact observers," and for the attainment of this end the book is admirably adapted. Its value is further enhanced by the numerous carefully drawn and well executed cuts, which add greatly to its beauty. It is to be hoped that many will avail themselves of this opportunity to become practically acquainted with the fundamental principles of Optics. A. W. W.

20. *A Manual of Inorganic Chemistry;* Vol. II. *The Metals;* by T. E. THORPE, Ph.D., F.R.S., Professor of Chemistry in the Yorkshire College of Science, Leeds. New edition, 406 pp. 8vo. New York, 1877, (G. P. Putnam & Sons).—The previous edition of this work, as well as the other volumes by the same author, are well known among chemists, and their value fully appreciated. A feature of this new edition is the collection of examination questions and exercises at the end of the volume.

21. *A System of Volumetric Analysis, by Dr. Emil Fleischer. Translated with notes and additions from the second German edition;* by M. M. PATTISON MUIR, F.R.S.E. 274 pp. 8vo. London, 1877, (Macmillan & Co).—The peculiar merit of this work lies in the fact that it gives not "a complete collection of receipts" for independent processes, but a system of general methods under which the individual cases may be brought. As remarked by the translator the work attempts to divide volumetric processes into a few great groups, to explain clearly the principles underlying each, and further to illustrate by examples the application of these principles. It will be readily seen how much greater benefit the

student will derive from studying this subject, when so systematically presented. We are indebted to the translator for the introduction of the chemical nomenclature and notation of modern chemistry, for some advantageous condensation, and for the addition of more or less new matter.

II. Geology and Mineralogy.

1. *Geological and Geographical Survey of the Territories;* by F. V. Hayden, U. S. Geologist in charge. Conducted under the authority of the Secretary of the Navy. Washington.—The following are notices of the recent publications of Dr. Hayden's survey, which has been so rich in results to science and the country.

(1.) *Ninth Annual Report, being a Report of Progress for the year* 1875. 810 pp. 8vo, with numerous plates.—This volume contains the following Reports: A letter to the Secretary of the Interior, by Dr. F. V. Hayden (30 pp.); Geology of the Grand River District, by Dr. A. C. Peale (70 pp.); Geology of the Southeastern District of Colorado, by F. M. Endlich (134 pp.); Geology of the San Juan Division, by W. H. Holmes (50 pp.); Notes on the Tertiary and Cretaceous of Kansas, by B. F. Mudge (18 pp.). Also Topographical and Geographical Reports by A. D. Wilson and F. B. Rhoda, H. Gannett, G. B. Chittenden and G. R. Bechler; and Zoological Reports on the History of the American Bison, by J. A. Allen; and on the Rocky Mountain Locust and other injurious insects of the West, by A. S. Packard, Jr. We cite a few facts from some of these reports.

Dr. Peale describes first the topography of his region and then the distribution and characters of the geological formations. These formations include the Archæan, (visible only where the other rocks have been removed), the Carboniferous or Permo-Carboniferous, the Triassic ?, Jurassic ?, and Cretaceous, and also volcanic rocks. Dr. Peale observes that the formations afford evidence of a gradual subsidence of the surface in this part of the Rocky Mountains (the Grand River District), from Pre-Silurian times to at least the end of the Cretaceous, and of no mountain-making era between. "The Archæan area along the northern edge of the San Juan Mountains and south of the Gunnison River probably formed a shore-line in Cretaceous times." "The area of the Archæan Continent was probably of some considerable extent, and the area of this district was probably an extension of that farther east, where the main chain of the Rocky Mountains is now." No Silurian or Devonian beds were observed; and over a great part of the district the red beds (Triassic) rest immediately on the Archæan. The thickness of the Carboniferous in Colorado is stated to be 4,000 to 5,000 feet; and of this only 500 to 1,000 feet consist of fragmental rocks. The accounts of the other formation contain interesting sections and many important details.

Dr. Endlich describes the Sangre de Cristo range and the San Luis and the Huerfano region. The highest mountains of this part of Colorado generally consist, he states, of metamorphic rocks, among which granites and gneisses are the prevailing kinds. Some of them, however, carry sedimentary beds to their summits, as is the case with the Trinchera group, 13,000 feet above the sea-level. The absence of Silurian beds among the unaltered strata in connection with their presence farther east, leads Dr. Endlich to suggest that the metamorphic rocks—granites, gneisses, etc.—may belong to the Silurian and perhaps also the lowest Carboniferous, and he adds, this point is at least "entitled to further investigation." The Carboniferous and Cretaceous rocks are described and various sections are given. The volcanic rocks are spoken of as either trachyte, dolerite or basalt. There are six volcanic areas situated—near the eastern entrance of the Sangre de Cristo Pass; on the Lower Huerfano, constituting small tables; the Spanish Peaks; and a trachytic area near the headwaters of Rio Culebra and Costella. Besides these areas there are numerous dikes. In the chapter on the Sawatch Range, several remarkable examples of erosion are finely represented on the plates, in which beds of trachytic conglomerate are reduced to clusters of slender columns or monuments from 50 to 400 feet in height. An account of the coal beds of the Trinidad region contains results of assays of the coal. Dr. Endlich also discusses the distribution of the Ancient Glaciers of Southern Colorado, and illustrates the subject with a map.

Dr. Holmes describes the geology of the La Plata mining region, giving several well-executed views and sections, and stating many facts of interest connected with the trachytic eruptions. Professor Mudge presents stratigraphical details respecting the Tertiary and Cretaceous beds of Kansas.

The Topographical and Geographical Reports treat of the Grand River District, (including the Uncompahgre valley and plateau,) the San Juan District, the Front Range of the Rocky Mountains, and the Middle and South Parks, giving details of the courses and heights of mountains, systems of drainage, soil and vegetation, and a large amount of information of practical as well as scientific value. Mr. Bechler's Report covers observations for the three years, 1873, 1874, 1875. It contains a general discussion of the mountain systems of the Middle and South Parks, and is illustrated by many outline sketches which are very effective.

The Zoological Report by Mr. Allen, which had previously appeared, has already been noticed in this Journal. Professor Packard's Report treats of a subject of the highest importance to the country—the injurious insects of the west—and occupies 220 pages of the volume. He remarks that in the United States the loss of agricultural products from this source is probably over two hundred millions of dollars each year, and that from one-quarter to one-half of this amount might be saved by preventive measures. Many figures are added to the text illustrating the species.

(2.) *Bulletin of the Survey*, vol. iii; No. 4, pp. 118, 8vo, (739-856 of vol. iii).—This number of the Bulletin contains the following articles: The first discovered traces of fossil insects in the American Tertiaries, by S. H. SCUDDER; description of two species of Carabidæ from Scarboro Heights, by S. H. SCUDDER; Report on insects collected in the explorations of 1875, by P. R. UHLER; on *Cambarus Couesi*, a new craw-fish from Dakota, by T. H. STREETS; on a Carnivorous Dinosaur (*Lœlaps trihedrodon*) from the Dakota beds of Colorado, by E. D. COPE; contribution to the Ichthyological fauna of the Green River Shales, by E. D. COPE; on the genus *Erisichthe*, by E. D. COPE.

Mr. Cope states that the fish of the Green River Shales represent families partly of fresh water and partly of brackish or salt water species. Material is needed to decide whether the Green River lake had communication with the sea.

(3.) *Monographs of North American Rodentia*, by ELLIOTT COUES, Assistant Surgeon U. S. A., and J. A. ALLEN, Assistant in the Museum of Comparative Zoology, Cambridge.—This large volume consists of a series of elaborate monographs of the several families of Rodents by Dr. Coues and Professor Allen. Those of the *Muridœ*, *Zapodidœ*, *Saccomyidœ*, *Haplodontidœ*, *Geomyidœ*. by Dr. Coues, and those of the *Leporidœ*, *Hystricidœ*, *Lagomyidœ*, *Castoroididœ*, *Castoridœ and Sciuridœ*, by Professor Allen. Specific and family distinctions, structural relations, geographical distribution past and present, divergences of varieties, and causes or conditions of variation, are among the subjects which come under discussion in this volume, and all the topics are treated with thoroughness and precision. The monograph on the Muridæ is accompanied by four plates. The volume closes with a synoptical list of the fossil Rodents of North America, by Professor Allen, and a Bibliography, by Mr. T. Gill and Dr. Coues.

(4.) *Miscellaneous Publications*, No. 7. *Ethnography and Philology of the Hidatsa Indians;* by WASHINGTON MATTHEWS, Assistant Surgeon, U. S. A. 240 pp. 8vo. Washington, 1877.—This volume, by one who has spent much time with the tribe treated of, contains a full account of the condition, habits, arts, history, etc., of the people, and also a discussion of the relations of its language, together with a grammar and vocabulary.

(5.) *Miscellaneous Publications*, No. 8. *Fur-bearing Animals. A Monograph of North American Mustelidœ;* by ELLIOTT COUES. 348 pp. 8vo, with 20 plates. Washington, 1877.—A work that is thoroughly scientific, while also in part popular in its character.

2. *Fifth Annual Report of the Geological and Natural History Survey of Minnesota, for the year* 1876, N. H. WINCHELL, State Geologist. 248 pp. 8vo. Saint Paul, 1877.—This volume contains reports on the Geology of Houston and Hennepin Counties, with colored maps of each, notes on the Trenton forests of Minnesota; a chemical report by Prof. S. F. PECKHAM; a list of the Fungi of the State, by Dr. A. E. JOHNSON; an entomological

report, treating of the locusts and other insects, by Allen Whitman.

The report on Hennepin County contains many valuable facts about the drift of the county, and also on the changes from erosion of the Falls of St. Anthony. According to the observations at the Falls from 1680 (by Hennepin) to the present time, the amount of recession is made out to be 906 feet, or, on an average, 5·15 feet per year. At this rate, it would have taken 8,202 years for its recession from Fort Snelling.

3. *The Geological Record for* 1875*; an account of works on Geology, Mineralogy and Palæontology*, published during the year. Edited by Wm. Whitaker, B.A., F.G.S., of the Geological Survey of England. 444 pp. 8vo. London, 1877.—This second volume of the Geological Record will be welcomed by all who are interested in the progress of geological or mineralogical science. The notices are brief, but yet they are so well prepared as to give a correct idea of the contents of publications. It thus enables the student to survey the year's progress at a glance, and to gather up references to the papers or works which he may need to consult in detail.

4. *Preliminary Notice of the Discovery of a new Mineral Species;* by Gideon E. Moore, Ph.D. (Communicated.)—The species here briefly described occurs associated with chalcophanite in ochreous limonite, at the Passaic Zinc Mine, Sterling Hill, New Jersey, and presents the following characters:

In botrioidal coatings of columnar radiate structure, usually coated with a thin layer of chalcophanite.

H.=5 (Mohs's scale); G.=4·933. Luster metallic to submetallic. Color black. Streak brownish black. Opaque. Brittle.

Before the blowpipe: in the forceps unchanged; in the closed tube yields a little water. With fluxes reactions for manganese and zinc.

The analyses lead to the formula $\dot{Zn}$, $\dot{Mn}$, $\overline{Mn}$ or $\dot{Zn}\,\overline{Mn}$. Whence the species is a zinc haussmanite.

From its invariable association with and close genetic relation to chalcophanite, I propose for the species the name *Hetærolite*, from ἑταῖρος, a companion.

Jersey City, Sept. 25, 1877.

5. *On some Tellurium and Vanadium Minerals;* by F. A. Genth.—Dr. Genth's paper contains descriptions of three new species, whose characters are here given.

Coloradoite. Not crystallized, without cleavage; massive, somewhat granular; sometimes having an imperfectly columnar structure. (Smuggler Mine). Hardness about 3; specific gravity = 8·627 (calculated for the pure mineral). Color iron-black, inclining to gray with a very faint purplish hue; luster metallic; surface frequently tarnished. Fracture uneven to subconchoidal. Composition: HgTe = Tellurium 39·02, mercury 60·98 = 100; all the specimens analyzed were more or less impure, as it was impossible to separate entirely the associated minerals. Found in Colorado at the Keystone and Mountain Lion Mines, with native tellurium and quartz; also at the Smuggler Mine, where it is often mixed with native gold, tellurium and tellurite.

Magnolite. Occurs in exceedingly fine needles, grouped in bundles or tufts, sometimes radiating. Color white; luster silky. Composition: Hg_2TeO_4. Found at the Keystone Mine, Magnolia District, Colorado; it occurs in the upper decomposed portion of the mine, with quartz, limonite and psilomelane, having been produced by the oxidation of coloradoite.

Ferrotellurite. A crystalline coating on quartz, associated with native tellurium; under the microscope it appears in very delicate tufts, sometimes radiating, or in cavities in minute prismatic crystals of a color between straw and lemon-yellow inclining to greenish-yellow. Composition probably $FeTeO_4$; the small quantity in hand did not allow of a reliable analysis. Found at the Keystone Mine, Colorado.

Dr. Genth also mentions the occurrence of *Tellurite* (tellurium dioxide, TeO_2) at the Keystone, Smuggler, and especially at the John Jay Mine, Colorado. It is found in minute white, yellowish-white and yellow crystals, mostly prismatic, isolated or aggregated into bundles. Cleavage eminent in one direction, luster on this face adamantine, elsewhere vitreous inclining to resinous. The same compound was observed by Petz with native tellurium at Transylvania. Native tellurium has been found, according to Dr. Genth, at the Keystone, Mountain Lion and Dun Raven Mines in Magnolia District, Boulder Co., Colorado, also at the Smuggler Mine in Ballerat District, Boulder Co.; hessite, (containing only ·1 per cent gold) has been found at the Kearsarge Mine, Dry Cañon, Utah; calaverite occurs at the Keystone and Mountain Lion Mines, Colorado.

Roscoelite is shown to contain not V_2O_5 (as assumed by Prof. Roscoe), but probably V_2O_3; a related "green mineral" has been found in an impure state in the gangue rock of the mines in Magnolia District, Colorado; it contains, however, more aluminum and less vanadium. An analysis of the Siberian volborthite is given, and its relation shown to the species psittacinite. E. S. D.

6. *Mineralogische Mittheilungen* von G. vom Rath.—The recent mineralogical memoirs of the eminent crystallographer, Prof. vom Rath of Bonn, contain descriptions of crystals of gold, of a remarkable twin of smaltite, of rutile in the form of hematite from Switzerland, and on compound rutile crystals from Arkansas; the explanation of the obscure crystallization of the plates, and thread-like forms of gold, is an especially valuable contribution, and is elucidated by a considerable number of figures. E. S. D.

7. *Elemente der Mineralogie* von Carl Friedrich Naumann. *Zehnte gänzlich neubearbeitete Auflage* von Dr. Ferdinand Zirkel. 714 pp. 8vo. Leipzig, 1877, (Wilhelm Engelmann).—The Mineralogy of Naumann has long been the standard work in Germany. The first edition was published in 1846; the ninth edition bears the date October 18, 1873, only a few weeks before the close of the author's long and active career. Fortunately this, his most

important work, did not end with his death, but its continuation has been undertaken by one well fitted to perform the task. The present edition, while including much new matter, retains the form and arrangement of those which have preceded except in one most important particular: the awkward and antiquated system of classification of mineral species employed by Naumann has been replaced by the now generally accepted method based upon their chemical composition. This will be seen to be a most important change for the better, increasing much the value of the work.

E. S. D.

III. Botany and Zoology.

1. *Occurrence of another gigantic Cephalopod on the coast of Newfoundland;* by A. E. Verrill.—A nearly perfect specimen of a large squid, was found cast ashore after a severe gale, at Catalina, Trinity Bay, Newfoundland, Sept. 24. It was living when found. It was exhibited for two or three days at St. Johns, and subsequently was carried in brine to New York, where it was purchased by Reiche & Bro. for the New York Aquarium, where I have had an opportunity to examine it.* Although somewhat mutilated, and not in a very good state of preservation when received, it is of great interest, being, without doubt, the largest and best specimen ever preserved. It proves to be *Architeuthis princeps*, formerly described by me, from the jaws alone, in this Journal.† The jaws agree well in form and color with the large pair there figured, and are fully equal to them in size, being apparently larger in proportion to the body than in *A. monachus*, so that my estimate of the probable size of the body of the former specimen was much too great. The Catalina specimen, when fresh,‡ was 9·5 feet from tip of tail to base of arms; circumference of body 7 feet, length of tentacular arms 30 feet; length of longest sessile arms (ventral ones) 11 feet; circumference at base 17 inches. Length of upper mandible 5·25 inches; diameter of large suckers 1 inch; diameter of eye-sockets 8 inches. The eyes were destroyed by the captors. It agrees in general appearance with *A. monachus*, but the caudal fin is broader and less acutely pointed; it was two feet and nine inches broad when fresh, and broadly sagittate in form. The rims of the large suckers are white, with very acutely serrate margins, and the small smooth-rimmed suckers, with their accompanying tubercles, are distantly scattered along most of the inner face of the tentacular arms, the last ones noticed being nineteen feet from the tips. The sessile arms present considerable disparity in length and size, the dorsal ones being somewhat shorter and smaller than the others; the serrations are smaller on the inner edge than on the outer of the

* When examined by me it was loose in a tank of alcohol. I learn that it has since been "prepared" for exhibition by a taxidermist, who has inserted two large, round, red eyes close together on the top of the head!

† Vol. ix, p. 181. Plate V, figs. 14, 15, March, 1875.

‡ Measurements of the freshly caught specimen were made by Rev. M. Harvey, at St. Johns, and communicated to me.

suckers. A more detailed description is deferred to a succeeding number, together with a description of another specimen.

2. *The Antelope and Deer of America;* by John Dean Eaton. 8vo, 426 pp., numerous cuts. New York, (Hurd & Houghton.) 1877.—This is an excellent treatise on the prong-horn antelope and the various species of deer, moose and elk, of which eight species are recognized. The descriptions of the species are detailed, and a large part of the book is devoted to their habits, domestication, hybridity, aliment, diseases, the chase, comparisons with congeners, and other kindred subjects. The illustrations are well executed and characteristic. V.

3. *Life Histories of the Birds of Eastern Pennsylvania;* by Thomas G. Gentry. Vol. ii, 8vo, 399 pp. Philadelphia. Published by the author, 1877.—The second volume of this work, which has just reached us, is, like the first, replete with details in respect to the habits of birds, and more especially as to their migrations, the time occupied in building nests and incubating, and the nature of food at different seasons of the year. In this consists its chief scientific value. The author has evidently spent a great amount of time and labor in making and recording observations of this kind. The volumes include all the families above the waders, and the author proposes to complete the work in another volume. V.

4. *Zoologische Wandtafeln;* by Dr. R. Leuckart and Dr. H. Nitsche. Cassel: Theodor Fischer.—We have received three examples (Protozoa, Cœlenterata, Arthropoda) of these lithographic zoological diagrams. They are admirably executed. The figures are large and clearly drawn, and printed in strong colors, so that they are excellently adapted for class rooms of large size. They will thus supply a want that has long been felt by teachers of zoology. The figures have been well selected from the best works, including very recent memoirs. Each diagram is printed in sections, on four sheets, and can easily be mounted on cloth backs. We heartily commend this series to all teachers who are in want of illustrative zoological diagrams. V.

5. *Bulletin of the U. S. National Museum.* Department of the Interior.—Numbers 7, 8 and 9 of this Bulletin have recently been issued. No. 7 contains contributions to the Natural History of the Hawaiian and Fanning Islands and Lower California, by T. H. Streets, M.D., 169 pp. 8vo, 1877; No. 8, Index to the Names which have been applied to the subdivisions of the Class Brachiopoda, by W. H. Dall, 88 pp. 8vo, 1877; No. 9, contributions to North American Ichthyology; No. 1, by David S. Jordan, 54 pp. 8vo, 1877.

6. *Notes on some Common Diseases [of plants] caused by Fungi;* by W. G. Farlow. Bulletin of the Bussey Institution, vol. ii, part 2, 1877.—Leaving to another department of the Journal to notice the interesting papers on the composition of pumpkins and squashes, the analyses of certain seeds occasionally used for human food, and two or three other papers by Prof. Storer which this new number

of the Bussey Bulletin contains, we call attention here only to Prof. Farlow's short article, which refers to some of the various questions and problems he has had to deal with during the past year. They mainly relate to the Black Knot of Plum trees, which was the subject of a former paper; to the American Grape-vine mildew (*Peronospora viticola*), and a disease caused by *Uncinula spiralis* the conidial form of which is practically undistinguishable from the notorious *Oidium Tuckeri;* to the *Fumago* of Orange and Lemon trees, in which it appears that the mischief produced is owing to a wooly plant-louse, upon the excretions of which, or the exudations of the leaf caused by the punctures, the fungus is thought to live; and, finally, there are some notes supplementary to Prof. Farlow's memoir on the Onion-smut. It seems that this *Urocystis Cepulæ* of Frost is not peculiar to the United States, but is to all appearance the same as *U. magica*, found on an Italian *Allium* in Italy. "A careful examination of the wild species of Onion growing in our own country should be made by fungologists, for it seems highly probable that the fungus which does so much injury to cultivated onions will also be found on the wild species." It is well that we have in this country an institution which furthers investigations of this sort, and intends to educate a generation of teachers capable of undertaking them. A. G.

7. *Flora Brasiliensis.*—The 70th fascicle, a large one, with 70 plates, is filled with Bentham's elaboration of the *Mimoseæ*, and concludes the sixteenth volume. The 71st contains the small orders, *Ochnaceæ*, *Anacardiaceæ*, *Sabiaceæ*, and *Rhizophoraceæ*, by Dr. Engler. As to *Ochnaceæ*, the author confirms Planchon's idea, announced in 1862 but not anywhere acted upon, that this order embraces *Sauvagesia;* and he insists that it is allied to *Dilleniaceæ* and not to *Rutaceæ*. For the large genus of the order, Engler, following Baillon, replaces Schreber's long received name of *Gomphia* by *Ouratea* of Aublet, and he appears to have seen and identified Aublet's original specimens. In *Anacardiaceæ* the genus *Lithrea* of Miers is reinstated. Brazil has but one species of the order *Sabiaceæ*, and only one Mangrove. Fascicle 72, which is rather large, continues the Grasses and includes the tribe *Paniceæ*. The editor, Dr. Dœll, describes 156 Brazilian species of *Panicum* (keeping *Helopus* distinct) and 105 of *Paspalum*. A. G.

8. *Botany of British Columbia and Northern Rocky Mountains.*—In the *Report of Progress* of the *Geological Survey of Canada for* 1875–76, issued in 1877, we find an interesting narrative, by Professor John Macoun, of a botanical exploration from Victoria to the Peace and Athabasca Rivers east of the Rocky Mountains, and thence to Canada, to which is appended a full Catalogue of the Plants collected, or known to Mr. Macoun to occur on this range, with a careful indication of their geographical distribution across the continent. A very useful and important paper. A. G.

9. *Sketch of the Vegetation of the Nicobar Islands; Enumeration of Burmese Palms; Contributions toward a knowledge of the*

Burmese Flora, etc.; by S. Kurz.—These are the titles of some of the principal papers recently contributed by Dr. Kurz to the Journal of the Asiatic Society of Bengal, which we have received, and which seem not to be as well known as they deserve to be. Besides critical systematic work, there are interesting observations upon the kinds and characteristics of tropical forests, etc. The subjoined note may be a caution to those who collect native names. "*Petal* occurs as a name for several different plants in Jelinck's journal. I fear it is meant for '*bětúl*' ('*just so*,' '*right so*'), a very usual reply of a Malay to a question regarding the pronunciation of a word." A. G.

10. *Arboretum Segrezianum* .... par Alph. Lavallée.—This handsome octavo volume, as its full title declares, is an enumeration of the trees and shrubs cultivated at Segrez (Seine and Oise), including their synonymy and their origin, with references to the works in which they are figured. It is published by Baillière, is edited with much care (yet not to the avoidance of sundry mistakes and misprints in other than French names, etc.), and is beautifully printed. It represents a great amount of conscientious work while the author was forming the Arboretum which now adorns his paternal estate, and which bids fair to be one of the very best in France, although as yet only twenty years old. The catalogue is prefaced by some notice of its formation, and of the origin and condition of several of the older French collections. Among the difficulties encountered, that of nomenclature was foremost and greatest, the same tree coming to him under various different names, and different trees under the same name, sometimes through traditional errors or oversights of the nurserymen, sometimes through less innocent practices of "*quelques horticulteurs*," which our author is constrained to denounce. In consequence he was obliged to study up the nomenclature and arrange the synonymy for himself throughout, with the best aid to be had from the *Jardin des Plantes* and elsewhere. Hence the *raison d'être* of this volume, and its value to all who have similar collections to make, or to care for. A. G.

11. *Systema Iridacearum*; by J. G. Baker.—This begins in the 90th number of the Journal of the Linnean Society, and is continued in the following. When concluded, we may give an abstract of the portions which relate to North American botany. Mr. Baker's work upon the monocotyledonous orders has been useful and timely, and the *Iridaceæ* particularly need revision, having been almost untouched by all later systematists, except by Dr. Platt, whose synopsis in the Linnea is far from complete, being founded mainly upon the materials in the Berlin herbarium, and whose views are not always to be adopted. A. G.

12. *Native "Artichokes."*—Upon reading the article upon *Helianthus tuberosus*, contributed to this Journal (in May last) by Messrs. Trumbull and Gray, the well-known Canadian botanist and explorer, Mr. John Macom, wrote as follows:

"It is a fact that a species of *Helianthus* which I took to be *H.*

tuberosus, grows in abundance in the valley of the Kaministiknia, on the right bank of the river, above Point Memon. This river discharges into Thunder Bay, Lake Superior, and is quite easy of access from the United States. I saw the plant growing in abundance on the alluvial flats on the 12th July, 1869, and found the tubers of a large size at that time. Whether my plant is the true tuberosus or not, it is certainly the parent of the Indian tuber. Where I got it was on the old (high) road to the northwest. It would be worth while to have one of the American tourists get a few plants late in August. I have no doubt but I have hit upon the exact locality from which the French took the tubers, and it only remains to identify my plant with *H. doronicoides*."

It is well to know of this station; but the Hurons of Sagard's narrative doubtless dwelt much farther east.

Dr. C. C. Parry has directed our attention to his list of plants of Wisconsin and Minnesota, published in Owen's Geographical Survey of Wisconsin, Iowa and Minnesota, in which, on page 614, is the entry: "*Helianthus tuberosus* L., Common Artichoke, river banks, St. Peter and St. Croix, certainly native, and a well-known article of diet among the Indians, called by the Chippewas Ush-ke-baug." An original specimen was kindly supplied by Dr. Parry, of which at this moment we can only say that it does not belong to *Helianthus tuberosus.* Thus there appears to be a second edible tuberiferous *Helianthus*, of which a further knowledge is a desideratum.

In regard to the other sort of root mentioned by Sagard as resembling parsnips, "which they call *Sondhratates*, and which are much better," Dr. Macom is confident that not *Sium lineare* but *Aralia racemosa* is meant: for the older inhabitants of that part of Canada affirm that the root of this *Aralia* was a favorite food of the Indians, and that they taught its use to the first settlers. In flavor this root (commonly called "spikenard" and "spignet") might well be said to resemble parsnips. A. G.

13. *Necrological.*—There have died during the past summer two European botanists of note, namely, Henry A. Weddell and Philip Parlatore; notices of whom will be given in the obituary record of the year. A. G.

IV. Astronomy.

1. *Discovery of a New Planet;* by C. H. F. Peters. (From a letter to the editors, dated Litchfield Observatory of Hamilton College, Clinton, N. Y., Oct. 15, 1877.)—I take pleasure in forwarding an observation of a new planet found last night, showing the brightness of a star of 10·5 magnitude:

Oct. 14. 12h 39m 51s H. C. m. t. $\alpha(175) = 1h\ 6m\ 4.73s.$ $\delta(175) = +8^\circ\ 6'\ 37{\cdot}4''$,

from 18 comparisons with Schj. 397. The daily motion of the planet is about 36^s in right ascension, and between 13′ and 14′ in declination toward the south. You are perhaps already aware, that the planet, of which the last number of the Journal contains a valuable series of observations with the number (174), is identical with the planet (141) *Lumen;* so that what is called there (175), receives the former, and the present planet the latter number.

2. *Comets in* 1877.—The number of comets for the present year already amounts to six. 1. Discovered by Borrelly, Feb. 8th. 2. Discovered by Winnecke, April 5th. 3. Discovered by Swift, April 11th. 4. Discovered by Coggia, Sept. 13th. 5. Discovered by Tempel, Oct. 2. To these is to be added d'Arrest's comet of short period.

3. *Observations and Orbit of Tempel's Comet.*—Comet *b*, 1877, discovered by Tempel October 2d, was first seen at the Observatory of the Sheffield Scientific School October 5th. From measurements made on the 5th, 7th and 9th, the following provisional orbit has been computed. The comet is receding from both the earth and sun, and will soon disappear. From places computed for August 1st and September 1st it appears that the comet must have been in reach of the telescope for a time after evening twilight as early as July, and has been ever since that time both favorably situated for observation and at least as bright as when discovered. It cannot have approached us at any time nearer than about ·8 of the earth's radius vector. These elements show no marked resemblance to those of any previously calculated orbit. The observations were made and orbit computed by Mr. W. Beebe and Mr. H. A. Hazen.

New Haven m. t.			Comet's α (m. eq. 1877·0).			Comet's δ.		
	h	m	h	m	s			
Oct. 5,	10	30·7	23	39	12·7	−13°	50′	9″
7,	9	48·3		32	31·0	15	40	47
9,	9	25·2		26	29·1	17	20	58
11,	7	45.4		21	7·7	18	48	53

T=1877, June 26·800 Wash. m. t.

π = 80° 43′·2 }
☊ = 184 17 ·9 } m. eq. 1877·0
i = 115 54 ·0
log. q = ·031956.

C.—O.

	$\Delta\alpha$	$\Delta\delta$
Oct. 7,	−·1s	+ 2″
11,	−·9	+38

V. Miscellaneous Scientific Intelligence.

1. *Proceedings of the Davenport Academy of Natural Sciences.* Vol. ii, Part 1. Jan., 1876 to June, 1877. 148 pp. 8vo.—The Davenport Academy publications are especially rich in papers on American Archæology. The number of the Proceedings before us contains several papers of this character by the following authors: W. H. Pratt, Rev. J. Gass, R. J. Farquharson, M.D., Rev. S. D. Peet, C. T. Lindley and Julia J. Wirt.

Mr. Gass announces the discovery of three engraved tablets in an Indian mound, which he calls "Mound No. 3 of the Cook's Farm Group;" and these are particularly described by Mr. Farquharson in a paper illustrated by excellent photographs of the tablets. One represents a calendar, another a sacrificial or cremation scene, and the third a hunting scene. In the last, thirty individuals are represented: of man, 8; bison, 4; deer, 4; birds, 3; hares, 3; big-horn or Rocky Mountain goat, 1; fishes, 1; prairie wolf, 1; non-descript animals, 3. One of the last three it is suggested may be the Mastodon, and hence, and in view of other published announcements, the cotemporaneity of Man and the Mastodon is deemed probable. The number contains also a few zoological papers by J. D. Putnam.

2. *U. S. Geographical and Geological Survey of the Rocky Mountain Region.* J. W. Powell, Geologist in charge. Department of the Interior.—This survey, under the Interior Department, has recently issued volume I of *Contributions to North American Ethnology*, a quarto volume of 362 pages, with many illustrations. Part I contains a Report on the tribes of the extreme Northwest; by W. H. Dall; Part II, on the tribes of Western Washington and Northwestern Oregon, with a map, by George Gibbs, including, in an appendix of 122 pages, comparative vocabularies, and a Niskwalli-English and English-Niskwalli Dictionary.

3. *Burial Customs of North American Indians;* Mr. H. C. Yarrow, of Washington, D. C., has issued a circular requesting information in aid of a memoir he is preparing, upon the "Burial Customs of the Indians of North America, both ancient and modern, and the disposal of their dead," and to this end invites attention to the following points in regard to which information is desired: Name of the tribe; locality; manner of burial, ancient and modern; funeral ceremonies; mourning observances, if any. The material obtained will be published under the auspices of Major J. W. Powell, in charge of the U. S. Geographical and Geological Survey of the Rocky Mountain Region. Communications may be addressed to Mr. Yarrow, 1747 F St., Washington, or at the Army Medical Museum, Washington, D. C. The circular contains more detailed statements.

4. *On the Science of Weighing and Measuring and Standards of Measure and Weight;* by H. W. Chisholm, Warden of the Standards. 192 pp. 8vo. London: 1877, (Macmillan & Co. Nature Series).—This little volume contains a very interesting account of the ancient standards of weight and measure, more particularly those of England; it also describes the methods employed in the restoration of the imperial pound and yard after their destruction by the burning of the Houses of Parliament in 1834. Another chapter is devoted to the Metric System, and one also to instruments for weighing and measuring. The numerous illustrations of the ancient standards add much to the interest of the description.

5. *How to draw a Straight Line; A Lecture on Linkages;* by A. B. Kempe, B.A. 51 pp. 8vo. London, 1877, (Macmillan & Co. Nature Series).—A description of the ingenious systems of linkages with which it is possible to draw a straight line, to trisect an angle, and to solve other geometrical problems.

6. *The Elements of Descriptive Geometry, Shadows and Perspective, with a brief statement of trihedrals, transversals, and*

spherical axonometric and oblique projections. For colleges and Scientific Schools; by S. EDWARD WARREN, C.E. 282 pp. 8vo. New York, 1877, (John Wiley & Sons).—A valuable work for students in descriptive geometry. It is intended as a concise text-book, and is rendered more useful in this work by the addition of numerous examples in connection with the successive problems.

7. *Smithsonian Report for* 1876.—Many are the ways in which the Smithsonian Institution is promoting the progress of science in the land. One among these, of very wide influence, is the publication, in its Annual Report, of an appendix containing memoirs on scientific subjects. Among the memoirs in the Report for 1876 there are the following:

Eulogy of Guy Lussac, by M. Arago; a biographical sketch of Dom Pedro II, by A. Fialho; a paper reviewing the kinetic theories of Gravitation, by W. B. Taylor; on the Revolutions of the Crust of the Earth, by Prof. G. Pilar, of Brussels; on the Asteroids between Mars and Jupiter, by D. Kirkwood; and various Ethnological papers, relating mostly to American relics. One of the papers, that on the Latimer Collection of Antiquities (from Porto Rico), now in the Smithsonian Museum, by O. T. Mason, is illustrated by sixty figures; and another, on Mounds in Wisconsin, by M. Strong, by five figures or diagrams.

RECEIVED TOO LATE FOR FURTHER NOTICE HERE.

Palæontology of the Geological Survey of the State of New York: Illustrations of Devonian Fossils, Gasteropoda, Pteropoda, Cephalopoda, Crustacea, and Corals of the Upper Helderberg, Hamilton and Chemung Groups; by JAMES HALL. Published in advance of the Palæontology of New York, by authority of the Legislature of the State of New York, nearly 150 plates, 4to. Albany, 1877.

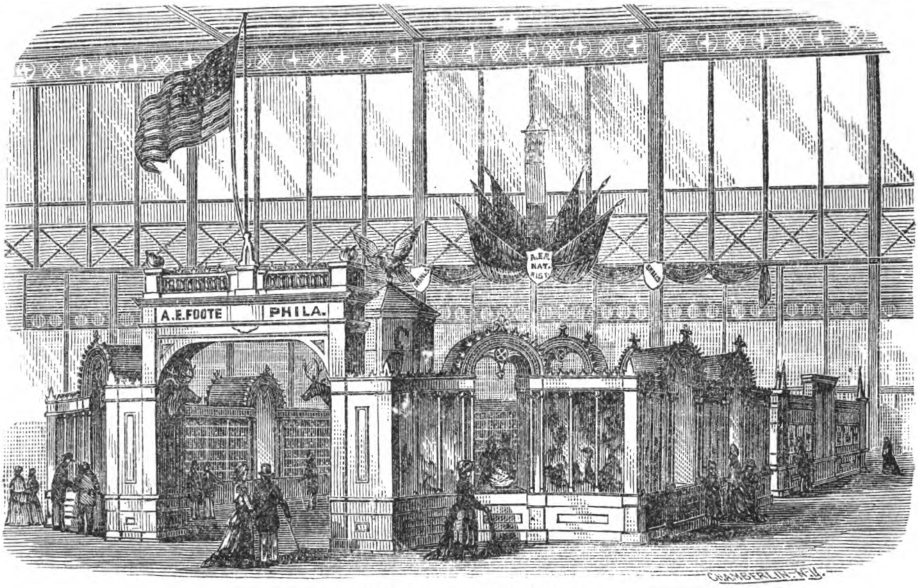
Twenty-eighth Annual Report of the New York State Museum of Natural History, by the Regents of the University of the State of New York (ex-officio Trustees of the Museum). Transmitted to the Legislature March 30, 1875; containing a paper by James Hall, consisting of 34 plates of fossils of the Niagara group of Central Indiana with explanations, and important papers on Trilobites, etc., by C. D. Walcott, with a Botanical Report, by C. H. Peck.

OBITUARY.

URBAN J. J. LEVERRIER, the French astronomer, died on Sunday, the 23d of September, at the age of sixty-six, having been born on the 11th of March, 1811.

JOHN G. ANTHONY, the conchologist, Professor in Harvard College, Cambridge, Mass., died on the 9th of October, aged seventy-three years. He was born in Providence, Rhode Island, May 17, 1804.

BENJAMIN HALLOWELL, author of a work on Geometrical Analysis, died at Nair Hill, Montgomery County, Maryland, in his seventy-eighth year.



INTERNATIONAL EXHIBITION.

THE above represents the principal one of the several exhibits I have in the Permanent Exhibition. A ground plan of this space is given on page 1 of the BULLETIN. It is undoubtedly the finest private exhibit ever made at any World's Exhibition. In Minerals, my specialty, no such exhibit can be made by any dealer in the world.

On the next page some details are given concerning the more interesting of the Minerals. Here are exhibited the only systematic collections of plants, shells and insects in the building. Every specimen here is offered for sale at the same price as at the store. A printed price label on each specimen puts all on an equal footing. The engraving on pages 13 and 14 of the April and May BULLETINS gave the location of my space in relation to the rest of the building.

In the Educational Section may be seen a collection of 300 specimens in an upright case. This is the academy or high school series, price $150. In the same case is seen a $10.00 collection which is not quite as good as I am putting up now. In flat cases near by will be found the students' ($5) collection. The collection of crystals and fragments, $1.00. Scale of hardness, scale of fusibility, Color series, Crystallographic and many other technical collections.

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REMOVAL.

1223 BELMONT AVENUE, the N. E. Corner of Belmont Avenue and Toledo Street, will be my location in the future. Here I hope all naturalists, especially those who visit the Permanent Exhibition, will find it a pleasure to come. It will certainly be a pleasure for me to see them, whether curiosity or a desire to purchase shall dictate their visits. 3725 Lancaster Avenue was the best that could be procured at a time of Centennial excitement. Now I have secured a grand work and storage room, 75 feet long and 24 feet wide. Besides plenty of open-air working and storage space, I have a large pleasant room fitted up with many cases of well-labelled specimens, in which to receive my friends. At the main building, three minutes' walk away, I have over 4,000 feet of floor-space well filled with upright glass cases and choice specimens. But the principal stock is kept at 1223 Belmont Avenue, where are 35 tons of crystallized minerals, 25,000 shells, and large numbers of fossils, plants, birds, insects, corals and other objects of Natural History. To the many scientific friends whose acquaintance I made last summer, and especially to those mine owners and smelters who have so kindly assisted me in my work, I extend a pressing invitation to call and see for themselves whether my work is worthy of their approval. They will find the permanent exhibition well worth their attention, and my place is less than three minutes' additional walk. By the door run the Race, Vine, Arch, Chestnut and Walnut Street cars, and the Pennsylvania Central and Reading R. R. Centennial Depots, to which trains still run, are near by. Nearly opposite me is the Girard Avenue P. R. R. station, where all trains stop. Persons coming by the Bound Brook route or any line leading into the north-east part of the city would do best to come by the Girard Avenue line, which, as well as the Market Street, stops on Belmont Avenue less than three minutes' walk away. While my present location is so desirable that I hope it will be permanent, it has been a very serious interruption to business from the length of time that it takes to pack, unpack and re-arrange such an immense number of specimens. By the time, however that this BULLETIN reaches its readers, regular work will again be resumed.

CONTENTS.

Holden no. 8

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THE

AMERICAN JOURNAL OF SCIENCE.

[THIRD SERIES.]

ART. I.—*On the Inequalities in the Moon's Motion produced by the Oblateness of the Earth;* by Professor J. N. STOCKWELL.

HAVING given in the November number of this Journal, an account of a secular inequality in the moon's motion, arising from the oblateness of the earth, I propose to give, in the present number, a somewhat detailed account of the principal periodic inequalities in the motions of the moon arising from the same cause. And I would here state that a careful study of the effect of the earth's ellipticity on the motions of the moon, through the medium of analysis, has presented some of the most curious and interesting cases of perturbation to be found in physical astronomy. The principal inequalities to which I would call attention, are chiefly remarkable as being the product of a number of important, though mutually antagonistic forces, the resultant of which, on any given coördinate of the moon, is of far less importance than would arise from the undisturbed operation of any one of the constituent forces; and I do not remember any cases of perturbation in the moon's motion, arising from the sun's attraction, in which the effect of the force on any given coördinate is so completely neutralized by the action of the same force on some of the other elements of motion. The analysis which I have employed has presented the results in such shape as to show: *First*, the separate effect of the earth's oblateness on the motions of the moon; and *Second*, the combined effect resulting from the earth's figure and the sun's attraction. It is from these two points of view that I shall now consider the subject.

In the paper referred to above, it was shown that the motion of a body moving in a circular orbit in the plane of the equator would be uniform; and that the motion in a circular orbit which was inclined to the equator would be subject to inequalities, the magnitude of which would depend on the inclination of the orbit to the equator. Now the ecliptic is inclined to the equator at an angle of 23° 27′, and the moon's orbit is inclined to the ecliptic at an angle of 5° 8′. If, then, the ascending node of the moon's orbit is at the vernal equinox, the inclination of the moon's orbit to the equator will be 28° 35′; and she will attain to that degree of declination twice during each sidereal revolution. In this position of the node, the inequalities of the earth's attraction on the moon attain their maximum values. Suppose, now, that the ascending node is at the autumnal equinox. It is evident that the inclination of the orbit to the equator will be only 18° 19′, and that she will only reach that declination twice during each revolution. The inequalities of the earth's attraction on the moon, in this last position of the node, will evidently be at their minimum values. As the node passes from one equinox to the other, it is plain that the inequalities of the earth's attraction on the moon pass through all the changes of value to which they are at any time subject. Now the moon's node makes a complete revolution on the ecliptic in a period of 18·6 years; consequently all the inequalities of the moon's motion arising from the oblateness of the earth will effect a complete restoration in that period. Since the motion of the moon's node is caused by the sun's attraction, we must, in the calculation of the separate effect of the spheroidal form of the earth on the moon's motion, neglect the sun's attraction and regard the elements of the moon's orbit as constant, except so far as they are affected by the form of the earth itself. From these general considerations it follows that the moon's declination, on which the inequalities of the earth's attractive force depends, is affected by two conditions: namely, the longitude of the moon and the longitude of the node. I shall therefore in the present paper consider only the inequalities which depend on these two elements, either separately or in combination.

In the calculations which I have made, the oblateness of the earth has been taken as $\frac{1}{300}$; and in the few equations which are given, the symbols have the following significations: a and nt denote the moon's mean distance and mean longitude; v and θ denote the moon's true longitude and latitude. γ and ☊ denote the inclination and longitude of node of moon's orbit; ϵ denotes the obliquity of the ecliptic, and D denotes the mean radius of the earth; ρ and φ denote the oblateness of the earth, and the ratio of the centrifugal force to the gravity at the earth's equator; R denotes the disturbing function; and δ placed before a quantity denotes the variation of that quantity arising from the oblateness of the earth. I also put for brevity

$$\beta = \{\rho - \tfrac{1}{2}\varphi\}\frac{D^2}{a^2}\sin\varepsilon\cos\varepsilon. \tag{1}$$

I now give the equations which express the variations of the elements and coördinates of the moon, and which are independent of the sun's action. In the first place I find that the position of the node and inclination of the orbit are affected by the following inequalities:

$$\delta\Omega = \tfrac{1}{2}\frac{\beta}{\gamma}\sin(2nt - \Omega) = +\,0''\cdot185\sin(2nt - \Omega), \tag{2}$$

$$\delta\gamma = \tfrac{1}{2}\beta\cos(2nt - \Omega) = +\,0''\cdot0165\cos(2nt - \Omega). \tag{3}$$

These two inequalities give rise to the following inequality in the moon's latitude:

$$\delta\theta = -\tfrac{1}{2}\beta\sin nt = -\,0''\cdot0165\sin nt. \tag{4}$$

But I find that the direct action of the earth on the moon produces the following inequality:

$$\delta\theta = +\,\tfrac{1}{2}\beta\sin nt, \tag{5}$$

The sum of these two inequalities gives $\delta\theta = 0$; whence it follows that there is no equation of the above form in the moon's latitude arising from the oblateness of the earth. In other words, the figure of the earth does not cause the moon to depart from the plane of the great circle in which its orbit is situated.

The above values of $\delta\Omega$ and $\delta\gamma$ also give the following inequality in the moon's longitude:

$$\delta v = +\,\tfrac{1}{4}\beta\gamma\sin\Omega = +\,0''\cdot00074\sin\Omega). \tag{6}$$

The direct action of the earth on the moon produces the following inequalities in the longitude:

$$\left.\begin{aligned}\delta v &= \tfrac{1}{12}\beta\tan\varepsilon\sin 2nt - \tfrac{1}{12}\beta\gamma\sin(2nt - \Omega)\\ &= +\,0''\cdot0012\sin 2nt - 0''\cdot00025\sin(2nt - \Omega).\end{aligned}\right\} \tag{7}$$

These inequalities in longitude are so excessively small as to be entirely insensible. The sum of the coefficients in these three terms of δv amounts to only $0''\cdot0022$, a quantity not exceeding *fourteen feet* if measured on the moon's orbit. From this calculation it follows that, if the moon were entirely free from solar disturbance, the effect of the oblateness of the earth on its motions would be so small that it would never be detected by observation.

Let us now examine into the effect of a combination of solar disturbance with that arising from the earth's oblateness. Since we have supposed the moon's orbit to be circular, it is evident that the earth's attraction on the moon is at a maximum whenever the moon is in the equator, or twice during each revolution of the moon. We will now suppose that the longitude of the moon's node is 90°, and examine into the consequences that must take place while it retrogrades through a semi-circumference, or from +90° to −90°.

When ☊ = +90°, the moon's orbit intersects the equator at a distance of 12° 45′ to the eastward of the equinox: and since the node retrogrades on the ecliptic about 1° 27′ during a sidereal revolution of the moon, it follows that the moon will arrive at the equator at a point a little to the westward of its previous crossing. In other words, the moon will make a complete revolution with respect to the center of force in a period somewhat shorter than the sidereal revolution. At the end of 9·8 years the longitude of the node will be −90°, and the orbit will intersect the equator at a distance of 12° 45′ to the westward of the equinox. Now the moon performs 124·3256 sidereal revolutions while the node is retrograding through an arc of 180°. But 124·3256 sidereal revolutions correspond to 124·3256 revolutions +23° 21′ with respect to the equator. Whence it appears that while the node is retrograding from +90° to −90°, the time of revolution with respect to the equator is shorter on an average by 20^{m} 32^{s} than the sidereal revolution. It is plain that while the node is retrograding from −90° through the autumnal equinox to +90°, the point of intersection of the orbit and equator will advance from −12° 45′ to +12° 45′, and the time of revolution of the moon with respect to the equator will exceed the time of the sidereal revolution by the same amount that it fell short of that quantity while retrograding through the other half of the orbit. It is also plain that the inclination of the orbit to the equator increases while the equatorial node is approaching the vernal equinox, at which point it is a maximum, and diminishing while it is receding from it. The constant retrograde motion of the ecliptic node of the moon's orbit, therefore, gives rise to a merely oscillatory motion of the equatorial node; and it is this pendulum-like motion of the equatorial node that gives rise to a number of inequalities in the moon's motion which I now proceed to consider; observing that the inequalities which are produced while the node is advancing are fully compensated by means of the retrograde motion which follows.

I now give the values of the perturbations of the elements and coördinates which I have obtained, as resulting from the motion of the moon's node, which is produced by the sun's attraction. I find

$$\left.\begin{aligned}\delta\Omega &= -\frac{\beta}{\alpha\gamma}\sin\Omega = +\,91''\cdot324\sin\Omega \\ \delta\gamma &= +\frac{\beta}{\alpha}\cos\Omega = -\,8''\cdot2233\cos\Omega\end{aligned}\right\}; \tag{8}$$

α being the ratio of the motion of the node to the moon's mean motion. These inequalities of the elements give rise to the following inequality in the latitude:

$$\delta\theta = \frac{\beta}{\alpha}\sin nt = -\,8''\cdot2233\sin nt. \tag{9}$$

This is exactly the same as LaPlace and subsequent investigators would have obtained had they used the same value of the earth's ellipticity. This inequality in the moon's latitude is equivalent to the supposition that the moon's orbit, instead of moving on the plane of the ecliptic with a constant inclination, moves, with the same condition, upon a plane passing always through the equinoxes, between the ecliptic and equator and inclined to the ecliptic by an angle which is equal to $\frac{\beta}{\alpha}$, as LaPlace has remarked.

I have not, however, been equally fortunate in reproducing the value of the inequality in the moon's longitude, which LaPlace and later investigators have obtained. I find as directly resulting from the motion of the moon's node, the following inequality in the longitude:

$$\delta v = -\frac{\beta}{\alpha^2}\gamma\sin\Omega = -\,184''\cdot3\sin\Omega. \tag{10}$$

But the preceding inequality in latitude gives rise to the two following inequalities in the longitude:

$$\left.\begin{aligned}\delta v &= +\frac{\beta}{\alpha_2}\gamma\sin\Omega - \tfrac{1}{2}\frac{\beta}{\alpha}\gamma\sin(2nt-\Omega) \\ &= +\,184''\cdot3\sin\Omega + 0''\cdot37\sin(2nt-\Omega)\end{aligned}\right\}. \tag{11}$$

The first of these two inequalities derived from the perturbations in latitude exactly cancels the preceding inequality in the longitude which is produced directly from the retrograde motion of the node; and we have as the resultant of the two forces,

$$\delta v = -\tfrac{1}{2}\frac{\beta}{\alpha}\gamma\sin(2nt-\Omega) = +\,0''\cdot37\sin(2nt-\Omega). \tag{12}$$

I find, however, that the radius vector of the moon's orbit is affected by the inequality

$$\delta r = 3a\beta\gamma\cos\Omega, \tag{13}$$

and this produces the following inequality in the longitude,

$$\delta v = -6\frac{\beta}{\alpha}\gamma\sin\Omega = +\,4''\cdot443\sin\Omega, \tag{14}$$

while LaPlace found

$$\delta v = -\tfrac{19}{2}\frac{\beta}{\alpha}\gamma \sin \Omega = +\,7''{\cdot}03 \sin \Omega. \qquad (15)$$

If, in the development of the inequalities depending on the oblateness of the earth we carry on the approximations so as to include terms of a higher order depending on the eccentricity and inclination of the orbit, we shall find two equations of sensible magnitude, having the same arguments as two empirical equations discovered by Airy about a third of a century ago. These equations depend on the arguments $nt - \omega - \Omega$, and $nt - \omega + \Omega$, in which ω denotes the longitude of the perigee. These arguments have periods of 27·4432 days, and 27·6661 days, respectively. The equation depending on the first of these arguments seems also to have been independently discovered quite recently, as an empirical equation, by Professor Newcomb, who attributes it to the attraction of some of the planets. From some calculations which I have made I am led to suspect that each of these equations has a value amounting to quite a large fraction of a second of arc; and I call attention to them here as being worthy of a more thorough investigation by astronomers.

It is but proper to add in this connection, that the mean motions of the perigee and node of the lunar orbit are affected by the oblateness of the earth; and are also affected by secular equations arising from the diminution of the obliquity of the ecliptic. The motions of the perigee and node which I have obtained agree in value with those obtained by LaPlace. I have therefore succeeded in reproducing exactly, by my method of computation, all the inequalities in the motions of the moon arising from the oblateness of the earth, which LaPlace discovered nearly a century ago, with the exception of the equation in longitude. The coefficient of LaPlace's equation exceeds the value which I have obtained, in the ratio of 19 to 12; and it has been a matter of surprise that two very dissimilar methods of computation should give so many results identically the same, and leave only a single one discordant. This has led me to make a critical examination of every step of LaPlace's calculation of this equation; and this examination has developed the fact that LaPlace has, in this instance, departed widely from the requirements of his own formulæ and methods; and that a correct calculation by his method gives a result identically the same as I have found by my own. I shall therefore now give the several steps of this examination, believing that it will not be without interest to the readers of this Journal. In this investigation it has been found convenient to use Bowditch's translation of the *Mécanique Céleste*, as the facilities for referring to any part of the work by means of the marginal numbers are much better than in the original. I shall also change the notation somewhat, putting ϵ for λ, and a for $g-1$ in some cases, and shall also put $f=1$. The numbers inclosed in brackets refer to the corresponding marginal numbers of the *Mécanique Céleste*.

The expression of the force R, [5362] becomes by using the value of β, which is given by equation (1) of this paper, and putting $f=1$,

$$R = 2\frac{a^2\beta}{r^3} s \sin v. \tag{16}$$

In this equation s denotes the tangent of the moon's latitude. This value of R gives the following values of the partial differential coefficients,

$$\left(\frac{dR}{dr}\right) = -6\frac{a^2\beta}{r^4} s \sin v, \tag{17}$$

$$\left(\frac{dR}{dv}\right) = \quad 2\frac{a^2\beta}{r^3} s \cos v, \tag{18}$$

$$\left(\frac{dR}{ds}\right) = \quad 2\frac{a^2\beta}{r^3} \sin v. \tag{19}$$

The equation which determines the value of δv, is [5367], namely,

$$d\delta v = 3\frac{dt^2}{r^2 dv}\int \text{d}R + 2\frac{dt^2}{r^2 dv} r\left(\frac{dR}{dr}\right); \tag{20}$$

and the value of dR is

$$\text{d}R = \left(\frac{dR}{dr}\right)dr + \left(\frac{dR}{dv}\right)dv + \left(\frac{dR}{ds}\right)ds. \tag{21}$$

The value of s is given by the equation [5376], namely,

$$s = \gamma \sin(gv - \Omega), \tag{22}$$

in which I have changed θ to Ω, in order to avoid confusion of symbols. Now (22) gives by differentiation

$$ds = g\gamma\, dv \cos(gv - \Omega). \tag{23}$$

If we neglect the eccentricity of the orbit we shall have $dr=0$, consequently the term $\left(\frac{dR}{dr}\right)dr$ will vanish from the value of dR. Now substituting the value of s, (22) in (18), and multiplying (19) by ds, which is given by (23), we shall get,

$$\left(\frac{dR}{dv}\right)dv = a^2\frac{\beta}{r^3}\gamma\, dv\left\{\sin(gv + v - \Omega) + \sin(gv - v - \Omega)\right\}. \tag{24}$$

$$\left(\frac{dR}{ds}\right)ds = a^2 g\frac{\beta}{r^3}\gamma\, dv\left\{\sin(gv + v - \Omega) - \sin(gv - v - \Omega)\right\}. \tag{25}$$

If we substitute these values in equation (21), and retain only the term depending on the angle $(gv - v - \Omega)$, it will become

$$dR = -a^2 \frac{\beta}{r^2} \gamma (g-1) dv \sin (gv - v - \Omega.) \tag{26}$$

This gives by integration

$$\int dR = a^2 \frac{\beta}{r^3} \gamma \cos (gv - v - \Omega). \tag{27}$$

If we substitute the value of s in equation (17) and multiply by r it will become

$$r\left(\frac{dR}{dr}\right) = -3a^2 \frac{\beta}{r^3} \gamma \cos (gv - v - \Omega). \tag{28}$$

Substituting (27) and (28) in (20), it becomes

$$d\delta v = -3a^2 \frac{\beta}{r^2 dv} \frac{\gamma}{r^3} \cos (gv - v - \Omega). \tag{29}$$

This is the same as LaPlace has given in [5368].

But LaPlace has given a second term depending on the same argument. This second term arises from the variation of the sun's disturbing force which is due to the variation of the moon's latitude produced by the earth's oblateness. The expression of this force is given in equation [5372] and is as follows:

$$\delta R = \tfrac{3}{2} m' u'^3 r^2 s\delta s. \tag{30}$$

I shall now show that this value of δR is the same as the value of R given by equation [5362], except that it has a contrary sign.

According to [5374] we have

$$\tfrac{3}{2} m' u'^3 r^2 = \tfrac{3}{2} \frac{m^2}{r} = 2\frac{g-1}{r}, \tag{31}$$

and if we substitute this in (30) or [5372] it becomes

$$\delta R = 2\frac{g-1}{r} s\delta s. \tag{32}$$

And if we substitute in this, the value of δs given by [5376], which reduced to the notation of this article is

$$\delta s = -a^2 \frac{\beta}{(g-1) r^3} \sin v, \tag{33}$$

it becomes

$$\delta R = -2a^2 \frac{\beta s}{r^3} \sin v. \tag{34}$$

This is the same as (16) or [5362] except that it has a contrary sign. This force is therefore the reaction of the force expended by the sun in giving motion to the moon's node, which in turn produces the inequality in the moon's latitude.

But in this second part of his work LaPlace seems to have committed a grave oversight, for he has treated his equation [5372] in the construction of [5373], as though δs were constant; whereas it is a function of both r and v, according to

[5376] which he afterwards uses in his reductions. However, as I have shown above that equation [5372] is the same as [5362], and has a contrary sign, it is unnecessary to pursue this part of the inquiry further, since it is evident that the whole value of δv must be derived from the value of R in [5362].

LaPlace has given the complete value of $d\delta v$ corresponding to the plane of the orbit, in [5379]; and he gives a correction in [5385] to reduce it to the plane of the ecliptic. It is apparent, however, that this correction does not exist, for LaPlace has shown in [923′] etc., where this subject is first investigated, that this correction is of the order of the square of the disturbing force; and as terms of that order have not been considered, it is evident that the value of that correction which he has given in [5385] is erroneous.

To complete this subject, it now remains to be shown that the value of R in equation [5362] gives the value of $d\delta v$ twice as great as LaPlace has found in [5368]. For this purpose I would remark that the value of $d\delta v$ given by means of [5367], is the correction to the *disturbed mean longitude*, and not to the undisturbed mean longitude. In order to correct for this condition it is necessary to add the term $3\frac{dt^2}{r^2dv}\int \mathrm{d}R$ to the first member, and this cancels the same term in the second member, thus leaving the correction to the *undisturbed mean longitude*, or δv equal to

$$\int 2\frac{dt^2}{r^2dv}\, r\left(\frac{dR}{dr}\right).$$

This will be apparent from the considerations given in § 54 of Book II of *Mécanique Céleste*, from which it appears that the function $\mathrm{d}R$ has a term of the form $\sin(\alpha t+\beta)$, in which α is very small, and gives by a double integration α^2 as a divisor; and LaPlace has shown in [1070′] that for this case we must increase the mean longitude by the quantity $3\frac{a}{\mu}\int ndt\int \mathrm{d}R$, which is equal to $3\int\frac{dt^2}{r^2dv}\int \mathrm{d}R$, or to the first term of the second member of equation [5367]. It therefore follows that the complete value of $d\delta v$ will be given by the equation

$$d\delta v = 2\frac{dt^2}{r^2dv}\, r\left(\frac{dR}{dr}\right). \qquad (35)$$

and if we substitute the value of $r\left(\frac{dR}{dr}\right)$ given by [5365] it becomes equal to twice equation [5368], or identically the same as I have obtained by an entirely different method.

Cleveland, Ohio, Oct. 29, 1879.

C/

no. 9

[From the American Journal of Science, Vol. XX, August, 1880.]

Recent Researches on the Lunar Theory; by John N. Stockwell.

In the November and January numbers of this Journal I have given some account of the inequalities in the moon's motion, arising from the oblateness of the earth. I now propose to give a somewhat detailed account of my investigations into the general theory of the moon's motion as affected by the sun's attraction. Although this problem has undoubtedly received more attention from mathematicians and astronomers, during the past century, than any other arising from the general gravitation of matter, it is nevertheless conceded by those who have given most attention to the subject, that the best lunar theories of the present day are essentially defective and erroneous; and that they signally fail to represent the motions of the moon with a precision at all commensurate with the refinements of calculation.

Early in the year 1876, I called the attention of my friend, the late Leonard Case, to the unsatisfactory state of the lunar theory, and he immediately suggested that I should undertake a thorough and systematic examination of the physical theory of the moon's motion, for the purpose of ascertaining whether the acknowledged defect arose from some oversight committed in the development of the theory, or was due to the omission of some of the smaller terms of the series produced by an otherwise correct development. At the same time Mr. Case, with characteristic generosity, offered to defray all the expenses arising from the prosecution of these researches.

The opportunity thus presented for a thorough investigation of the lunar theory was therefore very cheerfully accepted, although I had some misgivings as to my ability to do justice to a subject which had successfully baffled the best efforts of mathematicians. I had, however, somewhat familiarized myself with some of the methods employed by mathematicians in the treatment of this and similar problems by devoting the little leisure at my command to this subject during a number of years. The subject was therefore quite in harmony with my previous course of reading, and notwithstanding some false steps have been made in the application of a new method of analysis, the results at last obtained are both interesting and satisfactory.

In my investigations thus far I have not, however, attempted to carry on the approximation to terms of so high an order of magnitude depending on the eccentricity and inclination of the orbit, as Delaunay and others have done. I preferred rather to first satisfy myself that no systematic error among terms of the third or fourth orders of magnitude depending on these quantities had by any means found its way into the calculation; because such errors would vitiate the calculations of the terms of a still higher order. My investigation includes all the terms of perturbation arising from the fourth, and inferior powers of the eccentricity and inclination of the orbits of the sun and moon, while the investigations of Delaunay include the sixth power of these quantities. The general agreement of my work with the results of Delaunay's calculation is, on the whole, quite satisfactory, but there are a few cases in which the results are entirely at variance, even in terms of the third and fourth orders. In this discussion I shall restrict myself to the comparison of those terms in which the agreement is almost perfect, and also to those in which they are most widely different.

I will first give the value of that part of the co-efficient of the inequality which is known by the name of *variation*, which is independent of the eccentricities and inclinations of the orbits. According to my method of development this coefficient is composed of the following terms:

$$2111''\cdot841 - 5''\cdot562 - 0''\cdot030 - 0''\cdot0007 = 2106''\cdot248.$$

According to Delaunay's development, this coefficient is made up of the following terms:

$$1586''\cdot888 + 424''\cdot447 + 80''\cdot091 + 12''\cdot769 + 1''\cdot809 + 0''\cdot223 + 0''\cdot021 = 2106''\cdot248.$$

These two results are identically equal to each other. But a most important distinction between them is the convergency of the series by which they are determined. The four terms of my development are more accurate than seven terms of Delaunay's, since the seventh term of the latter series is thirty times greater than the fourth term of the former.

If we now compare the coefficients of the term whose argument is *twice* the argument of the *variation*, we shall find, according to my development—

$$8''\cdot789 - 0'\cdot056 - 0''\cdot0001 = 8''\cdot733;$$

while Delaunay gives

$$5''\cdot070 + 2''\cdot612 + 0''\cdot813 + 0''\cdot196 + 0''\cdot060 = 8''\cdot751.$$

These two coefficients, though practically equal to each other, show the same remarkable difference in the convergency of the series, the second term of my development being smaller than the fifth of Delaunay's.

For the equation whose argument is three times argument of the variation, I find $0''\cdot0493 - 0''\cdot0005 = 0''\cdot0488$, while Delaunay gives $0'\cdot0218 + 0''\cdot0167 = 0''\cdot0385$.

This coefficient of Delaunay's is about one-fourth part too small, since he has not carried the approximation to terms of so high an order as he did for the two former cases. To show, however, that my coefficient is correct, I would observe that the Monthly Notices of the Royal Astronomical Society for November, 1877, contains a paper by Prof. J. C. Adams, which purports to give the coefficients of the equations we have been comparing, with extreme accuracy. If we reduce his coefficient of $\sin 6(nt-n't)$ to seconds of arc, we obtain $0''\cdot0490$ for this coefficient, a value almost identical with my own. For the coefficient of $\sin 8(nt-n't)$ I find $0''\cdot00034$, while according to Prof. Adams it is $0''\cdot00031$.

According to my development the coefficient of the parallactic inequality is composed of the following terms:

$$84''\cdot523+26''\cdot801+10''\cdot280+3''\cdot872=125''\cdot476,$$

while Delaunay gives the following series of terms:

$$74''\cdot023+34''\cdot330+11''\cdot885+4''\cdot428+1''\cdot862+0''\cdot712+0''\cdot381 =127''\cdot621.$$

The coefficient of this inequality is one of the most troublesome to be determined by the theory, and the four terms above given are all I have yet rigorously computed. If we estimate the sum of the remaining terms, by induction from those already calculated, we should increase the preceding coefficient by $2''\cdot10$, which would make it equal to $127''\cdot58$. Delaunay's coefficient ought also to be increased for the same reason, by about $0''\cdot38$, which would make it about $128''\cdot00$. These coefficients correspond to a solar parallax of $8''\cdot75$. According to my calculations the eccentricity and inclination would diminish this coefficient by $2''\cdot11$; and if we assume the mass of the moon to be *one-eightieth* of the earth's mass the perturbations of the earth by the moon would diminish it by $2''\cdot10$ more. The theoretical coefficient for the above value of the parallax would therefore be $123''\cdot37$. Were the exact value of the coefficient of this inequality determined from observation, we might, by comparing it with the theoretical coefficient, determine the correction to our assumed solar parallax.

The preceding inequalities are the principal ones in which the coefficients of different theories are directly comparable with each other. For those inequalities in which the eccentricity and inclination enter as factors, the value of the coefficient depends, to a certain extent, on the manner in which the arguments of the different equations are measured. In most of the lunar theories the anomalies are measured on the plane of the orbit, while the longitudes are measured on the plane of the ecliptic;—a needless complication, which I have carefully avoided. However, in order to show the rapid convergency of the series which determine the principal periodic inequalities depending on the eccentricity and inclination of the orbit, I here give the two terms of the coefficient of the evection which I have computed. The first two terms depending on the first power of the eccentricity are as follows:

$$4280''\cdot9+122''\cdot0,$$

while Delaunay gives the following terms:

$$3176''\cdot4+1041''\cdot5+297''\cdot5+72''\cdot3.$$

It is evident that the first series converges about ten times as rapidly as the second.

The preceding comparison is sufficient to show the correctness and value of the method which I have employed in the problem of the moon's motion; and I shall now mention a few cases in which my results are wholly different from what other calculators have found for the same inequalities.

Before doing so, however, I would observe that there are certain fundamental and axiomatic conditions which ought to be satisfied by the results arrived at, whatever be the method of analysis which we may employ. In the present case the condition to be satisfied is simply, *That all the terms introduced into the expressions of the coördinates by the disturbing function ought to disappear when the disturbing function is put equal to nothing.* It is, however, a remarkable fact in connection with the lunar theory, that, among the *four hundred and seventy-nine* equations of the longitude given by Delaunay, there are *five*, arising from the sun's attraction, which do not disappear when the disturbing function is put equal to nothing. From this circumstance it is easy to conclude that there must be something seriously wrong in his development, notwithstanding its intricacy and refinement. The same remark is also applicable to the lunar theories of LaPlace, Plana and Pontécoulant.

The most important of these equations are those having the arguments, $2F - l$, and $D + l'$, in Delaunay's theory; or, *twice the moon's distance from the node minus the mean anomaly*, and *the moon's longitude minus the longitude of the sun's perigee.* The first of these is an inequality of pure elliptic motion, with a coefficient of $+45''\cdot4$, while the coefficient arising from perturbation amounts to $-84''\cdot8$, $28''$ only of which disappears when the disturbing function is put equal to nothing. According to my analysis, the coefficient of this inequality arising from perturbation amounts to only $0''\cdot18$, a quantity less than a *four hundredth part* of Delaunay's coefficient arising from the same cause.

The coefficient of the second equation, mentioned above, depends entirely on perturbation, and has a value of about 17″ according to Delaunay, while I find a coefficient of only 0″·03. These two equations present the most remarkable differences which I have found among the equations of short period in the moon's motion.

The inequalities of long period, or those which depend wholly on the variation of the elements of elliptical motion are also very easily computed by my method. The values of the inequalities of this kind are subject to very simple and precise laws; so that if we have computed the coefficient of an inequality arising from a given force and having a given period, we may deduce the coefficient of any other inequality arising from a different force and having a different period, directly from it. For convenience we may divide the inequalities of long period into two classes, according to the nature of the forces which produce them. We shall therefore designate the inequalities arising from the variation of the central force as class (A), and those arising from the tangential force as class (B). Then *first*, the inequalities arising from forces of class (A) are to each other as the products of the forces by the periods of their respective arguments; and *second*, the inequalities produced by forces of class (B) are to each other as the products of the forces by the squares of the periods of their arguments.

We may also easily obtain the inequalities produced by either class of forces from the inequalities produced by the other class. For example, suppose the inequalities of the central force f having a period α, is found by calculation to produce an inequality m, in the moon's longitude, and we wish to obtain the inequality produced by a tangential force f', having a period α'. If we call this second inequality m', I find the following relation exists between the two inequalities:

$$3m f' \alpha'^2 = -2m' f \alpha n.$$

This gives $m' = -\frac{3}{2} m \frac{f'}{f} \frac{\alpha'^2}{\alpha n}$, n here denoting the moon's period of revolution. If $f = f'$ and $\alpha = \alpha' = 118{\cdot}3n$, which corresponds nearly to the period of the moon's perigee, we find

$$m' = -178m,$$

whence it follows that for equal central and tangential forces having a period of about nine years, the tangential force would diminish the moon's longitude *one hundred and seventy-eight* times as much as the central force would increase it, and *vice versa.*

There are two inequalities of long period in the moon's motion which have been much discussed by astronomers. They have for arguments, *twice the difference of longitude of perigee* *and node of the lunar orbit*, and *the difference of longitudes of perigee of sun and moon*, respectively. Plana was the first to give a correct approximate solution of the problem of the first of these inequalities, which is produced wholly by the variations of the central force. By means of a laborious investigation, occupying about fifty pages of his Theory of the Moon's Motion, he has obtained a tolerably correct approximation to the value of the inequality. He obtains $+1''\cdot405$ for the sum or the elliptic and perturbed coefficient; but the elliptic coefficient is equal to $-0''\cdot932$; whence it follows that the coefficient due to perturbation amounts to about $+2''\cdot34$. I obtain, almost without labor, $+2''\cdot54$ for the value of this coefficient.

The second inequality is produced by both classes of forces, and the determination of its coefficient is more complicated than that of the inequality just mentioned. The value of the force of class (A), which produces the inequality, is about *one-fifth* of the former, but it has a period about *three times* as long. The inequality produced by this force ought to be about *three-fifths* of the former inequality, which would make it equal to $1''\cdot52$. But the tangential force is far more effective, since the inequalities produced are proportional to the squares of the periods of the arguments. I find, however, by an exact calculation that the part of the coefficient of this inequality which arises from the central force amounts to $+1''\cdot45$; while the part of it which arises from tangential force amounts to $+107''\cdot08$; thus making the coefficient of the inequality equal to $108''\cdot53$. The solutions of Plana, Pontécoulant and Delaunay, all make the coefficient equal to about $0''\cdot4$, when quantities of the same order only are included.

It is remarkable that the inequalities of long period arising from the two classes of forces which produce them should follow the same law as the acquired velocity, and space passed over, by falling bodies at the surface of the earth, the one being proportional to the time and the other to the square of the time.

If we extend the comparison to the variation of the elements, we shall find that the method which I have employed possesses the advantage of more rapid convergency. For example, I find for the first two terms of the mean motion of the perigee the following value:

$$0\cdot00419643 + 0\cdot00395575 = 0\cdot00815218,$$

while the first three terms of Delaunay's series are

$$0\cdot00419643 + 0\cdot00294279 + 0\cdot00099570 = 0\cdot00813492.$$

This comparison shows that two terms of my series are considerably more accurate than three terms of Delaunay's.

The preceding comparisons are sufficient to establish two points in regard to the lunar theory. The first is, that the general methods of computation are undoubtedly correct; and the second is, that one or more of the methods have been incorrectly applied to the investigation of particular inequalities. Now, without claiming that there are no mistakes, either systematic or accidental, in my work on the lunar theory, there are some reasons for believing that it is correct in the cases to which I have called attention. One of these reasons is the fact that all the inequalities produced by perturbation would disappear from the formulas by simply putting the disturbing function equal to nothing; whereas there are a number of inequalities which do not disappear from the formulas of previous investigators by means of the same conditions.

It might seem, however, that such large changes in the values of the coefficients of some of the equations of the moon's longitude, as my researches seem to indicate, would have a tendency to make the theory less accordant with observations than it is at present, since the present lunar tables represent the moon's place within tolerably narrow limits. But a little consideration will show that such a conclusion would not necessarily follow. In order to illustrate this point, let us suppose that we have a perfect system of elements of the moon's orbit together with a perfect theory of the perturbations. It would necessarily follow that the moon's place could be perfectly predicted, and there would be no discordances between theory and observation. Suppose, now, that we omit a number of small though important equations from the computation of our ephemeris, it would follow that there would be a series of residuals between theory and observation. It is evident that these residuals would be perfectly represented by the omitted equations; but if the equations were considered as wholly lost, the theory would be in the same condition as though they had never been found; and we might seek to make up for the imperfect theory by finding certain corrections to the elements by means of equations of condition between the variations of the elements and the observed residuals. In this way we might perhaps obtain a very good agreement between theory and observations which extend over a limited interval of time,—the errors of the theory being partially compensated by the errors of the elements. But this close agreement between theory and observation would soon cease to take place, since the corrections applied to the elements would vitiate the remaining part of the theory. The imperfect ephemeris, computed by means of the changed elements and theory, would gradually depart more and more widely from the observed place of the moon, but the residuals would furnish no information in regard to the nature of the equations to be applied in order to correct them, since the calculated places were based on imperfect elements and an imperfect theory of the perturbations.

Now it seems to me that the actual history of the lunar theory indicates a passage through just such conditions and changes. It is true, however, that it has never possessed the advantages of perfection assumed above; but through the efforts of astronomers to improve the accuracy of the elements and tables, it has been subjected to the same process of correction. Beginning with Tycho Brahe, in modern times, the elements and theory of the moon's motion were so imperfect that the observed discordances forced him to recognize the existence of two considerable inequalities which were at the time unknown to the science of Europe. The discovery of these inequalities, which have received the names of *variation* and *annual equation*, was the last great step towards the perfection of the lunar theory, which preceded the discovery of the physical cause of the inequalities. Since that memorable epoch, the researches of mathematicians have instructed observers in regard to the magnitude and laws of numerous inequalities which must necessarily affect the moon's motion. The labors of Newton and Halley reduced the errors of the theory to about the *eighth* part of a degree; while Mayer, by the aid of theory and more accurate observations, succeeded in reducing the errors to less than the *thirtieth* part of a degree. Later still, the researches of Mason and Burg, according to the authority of LaPlace, reduced the errors of the theory to less than *one-quarter of a minute of arc!* If this last degree of precision was at any time really attained, it must have been owing to the partial compensation of errors of theory by the errors of the elements; because the theory very soon began to depart more and more widely from the observations; and astronomers have been obliged to suspect that the moon's motion was affected by one or more equations of very long period, which theory is hopelessly unable to point out.

About the middle of the present century, new tables of the moon's motion were constructed by Hansen; and, considering the broad basis of observation and the elaborately developed theory of her motion, the hope was justified that the elements and theory were so perfectly known, that they would permanently represent the observations. But this hope seems to have been ill founded. In a very few years the observations unmistakably indicated a growing discordance, which has continued till the present time; and notwithstanding the laborious investigations which have been made in order to detect the laws and the cause, no satisfactory explanation has yet been attained. If we consider the nature and magnitude of the discordances which now pertain to the best tables of the moon's motion, we can hardly avoid the conclusion that they are not due to the terms of a higher order of magnitude which have been neglected in the development of the theory. They must therefore result from some systematic error among terms of more importance in the lunar theory.

In bringing to a close this account of my researches, I would repeat that my only object has been to discover if possible, by means of a new method of investigation, any false steps which may have been committed by previous investigators in the mathematical development of the lunar theory. The history of philosophy affords numerous examples of the advantages of independent methods of investigation over independent calculations by the same method. It often happens that for particular values of the known quantities of a problem some terms of the solution become infinite or indeterminate, when certain general methods of investigation are employed; whereas other methods would not be subject to complications from such a cause. It is therefore evident that the comparison of the results of different methods would serve to call attention to the particular terms affected by any such critical conditions, and by thus narrowing the field of investigation, enable astronomers to concentrate their efforts on those particular terms where further research would seem to be necessary or desirable; and it is believed that the terms to which I have called attention afford the means of a much needed improvement in the lunar theory.

Cleveland, May 25, 1880.

Moon

no. 10

ВѢКОВЫЯ НЕРАВЕНСТВА

ВЪ

ДВИЖЕНІИ ЛУНЫ.

Dolgourouki

Кн. Николая Долгорукова,

КАНДИДАТА МОСК. УНИВЕРСИТЕТА.

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САНКТПЕТЕРБУРГЪ.

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1885.

10

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Деканъ *Н. Меншуткинъ*.

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СОДЕРЖАНІЕ.

ВВЕДЕНІЕ.

Подъ именемъ вѣковыхъ неравенствъ въ движеніи планетъ разумѣютъ тѣ отклоненія отъ законовъ эллиптическаго движенія, которыя заключаются въ постепенномъ и весьма медленномъ измѣненіи элементовъ свѣтила. Эти измѣненія обнаруживаются изъ наблюденій обнимающихъ только весьма продолжительные промежутки времени и подлежатъ всегда извѣстнымъ, обыкновенно весьма тѣснымъ предѣламъ.

Движеніе Луны представляетъ три особенно замѣчательныхъ вѣковыхъ неравенства: первое изъ нихъ заключается въ ускореніи средняго движенія Луны и производитъ съ теченіемъ времени весьма значительныя измѣненія въ долготѣ земнаго спутника; второе обнаруживается въ измѣненіи положенія линіи апсидъ и наконецъ третье состоитъ въ замедленіи отступательнаго движенія линіи узловъ лунной орбиты по истинной эклиптикѣ.

Первое неравенство было открыто еще въ XVII вѣкѣ Галлеемъ изъ сравненія среднихъ скоростей движенія Луны въ разныя эпохи, но оставалось необъясненнымъ до Лапласа, которому принадлежитъ и слава открытія двухъ другихъ вѣковыхъ неравенствъ при помощи одной теоріи. Едвали существуетъ — по крайней мѣрѣ со временъ Лагранжа и Лапласа — другой вопросъ небесной механики, который возбуждалъ бы къ себѣ больше вниманія и интереса со стороны величайшихъ геометровъ нашего вѣка, и вызывалъ бы болѣе усилій для его разрѣшенія, какъ вопросъ объ опредѣленіи истинной величины вѣковаго ускоренія въ движеніи Луны.

Несмотря однако на огромную массу труда и умственныхъ усилій, потраченныхъ на изслѣдованія по этому вопросу, наука не можетъ и до сихъ поръ считать его получившимъ вполнѣ удовлетворительное разрѣшеніе.

Сравненіе наблюденій древнихъ затмѣній съ позднѣйшими представляетъ самое простое средство для приблизительнаго опредѣленія величины вѣковаго ускоренія въ движеніи Луны.

Древнѣйшее достовѣрное затмѣніе, о которомъ упоминается въ исторіи, было полное лунное затмѣніе 19 марта 720 г. до Р. Х. наблюдавшееся въ Вавилонѣ. (Альмагестъ кн. IV, гл. VI). Сравнивая наблюденіе этого затмѣнія съ другимъ, которое произошло въ 1717 г., Кассини нашелъ, что среднее движеніе Луны въ 100 лѣтъ составляетъ 1336 полныхъ круговъ $+ 307^\circ 49' 52''$. Если бы движеніе Луны было равномѣрно, то очевидно, тотъ же результатъ получался бы изъ сравненія и другихъ затмѣній между собою; но всѣ наблюденія позднѣйшихъ затмѣній даютъ для средняго движенія Луны въ 100 лѣтъ величины тѣмъ большія, чѣмъ ближе сравниваемыя затмѣнія къ нашей эпохѣ.

Если взять напримѣръ наблюденія затмѣній, сдѣланныя въ 977 и 978 гг. въ Каиро Ибнъ-Юнисомъ, то для величины вѣковаго движенія Луны получится 1336 полныхъ окружностей $+ 307^\circ 52' 28'',5$, т. е. на $2' 36'',5$ болѣе, чѣмъ даетъ первое сравненіе. Пусть x неизмѣнное среднее движеніе Луны въ теченіе одного столѣтія, и y вѣковое ускореніе этого движенія. Если среднее движеніе Луны можетъ выражено формулою $ix + i^2y$, гдѣ i число вѣковъ, протекшихъ отъ какой нибудь опредѣленной эпохи, то очевидно, что принимая за эпоху 1717 годъ и считая время назадъ отъ 1717 года съ знакомъ минусъ, мы получимъ для опредѣленія x и y слѣдующія два уравненія:

$$-(24\tfrac{1}{5})x + (24\tfrac{1}{5})^2 y = -(24\tfrac{1}{5}).(307^\circ 49' 52'')$$

$$-(7\tfrac{1}{5})x + (7\tfrac{1}{5})^2 y = -(7\tfrac{1}{5})(307^\circ 52' 28'',5), \text{ откуда}$$

$$x = 307^\circ 53' 34'',64 \text{ и } y = 9'',2.$$

Къ величинѣ x надобно еще прибавить 1336 полныхъ окружностей, и тогда получится величина средняго вѣковаго движенія, которая уже не подлежитъ никакимъ измѣненіямъ. Тотъ же результатъ можетъ быть полученъ изъ сравненія средней продолжительности синодическаго мѣсяца въ двѣ по возможности отдаленныя одна отъ другой эпохи. Свѣдѣнія о различныхъ лунныхъ періодахъ, сообщаемыя Птоломеемъ въ Алмагестѣ, даютъ намъ возможность вычислить продолжительность синодическаго мѣсяца во времена Гиппарха. На стр. 33 § 2, C. IV Almagesti[1]) читаемъ: Per observationes enim quas exposuit (Hyparchus) demonstrat, quia primus dierum numerus per quem semper tempus aeclypsium in mensibus ac motibus aequalibus revolvitur 126007 dierum et horae unius aequalis est, in quibus menses invenit absolvi. 4267. Integros vero inaequalitatis restitutiones. 4573. Circulos autem zodiacos 4612. minus 7.30 gradibus proxime, quibus et sol ad 345 circulos rursum deficit, ut restitutis ipsorum ad fixas stellas perspiciatur"...

То есть въ 126007 дняхъ и 1-мъ часѣ содержалось во времена Гиппарха 4267 синодическихъ мѣсяцевъ, 4573 аномолистическихъ и 4612 тропическихъ мѣсяцевъ безъ $7^1/_2$ градусовъ. Отсюда находимъ: продолжительность мѣсяца синодическаго въ 140 году до Р. Х. была 29 дней 12 часовъ 44 минуты 3.262 секунды, а продолжительность аномалистическаго — 27 д. 13 ч. 18 м. 34 с. 717.

Современный синодическій мѣсяцъ = 29 д. 12 ч. 44 м. 2 с. 684. Такимъ образомъ мы видимъ, что синодическій мѣсяцъ сдѣлался короче на $0^s.578$ впродолженіи почти 2000 лѣтъ протекшихъ послѣ Гиппарха. Въ это время прошло приблизительно 24737 синодическихъ мѣсяцевъ. Предполагая, что ускореніе въ движеніи Луны равномѣрно, т. е. что каждый мѣсяцъ уменьшается на одну и ту-же величину, мы очевидно получимъ для n мѣсяцевъ сокращеніе $n\frac{(n-1)}{1.2}.\ \alpha$, если черезъ α обозначимъ частное $\frac{0^s.578}{24737}$, т. е. уменьшеніе длины мѣсяца въ одно обращеніе.

Въ ста годахъ содержится 1237 синодическихъ мѣсяцевъ, поэтому въ 100 лѣтъ Луна выиграетъ $\frac{1237.1236}{2}\cdot\frac{0,578}{24737} = 17$ сек. 876, а въ теченіи 17,876 сек. Луна проходитъ по долготѣ $9'',8146$.

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[1]) Изданія 1528 г. (Венеція).

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Итакъ черезъ первые 100 лѣтъ считая отъ какой нибудь эпохи, Луна окажется впереди своего средняго мѣста, вычисленнаго по средней скорости движенія въ эпоху, — на 9",8, черезъ 200 лѣтъ почти на 39" и вообще черезъ i вѣковъ — на i^2 9",8146 впереди.

Такъ какъ среднее движеніе перигея Луны не было извѣстно древнимъ съ достаточною степенью точности, то и длина аномалистическаго мѣсяца не могла быть вѣрно опредѣлена Гиппархомъ и Птоломеемъ, а потому сравненіе длины аномалистическаго мѣсяца въ эпоху Гиппарха и въ наше время не можетъ дать намъ хотя сколько нибудь заслуживающей довѣрія величины вѣковаго замедленія въ движеніи луннаго перигея.

Что касается до измѣненія величины драконическаго мѣсяца, то оно оказывается равнымъ 0,48 секунды. На стр. 33 (глава II, IV-ая книга) Альмагеста Птоломей говоритъ: „Cum rursus distantias mensium similis quibus exquisite in omnibus et magnitudinibus et temporibus observationum aeclypses extremae centinebentur Hypparchus apposuerit. In quibus aeclypsibus nulla differentia penes inaequalitatem fiebat, ut hac ratione latitudinis quoque motus restitus videretur, hanc quoque restitutionem absolvi demonstrat in mensibus quidem. 5458. Revolutionibus vero latitudinariis 5923".

Синодическій мѣсяцъ во времена Гиппарха былъ равенъ 29 д. 12 ч. 44 м. 3°,26224, отношеніе же драконическаго мѣсяца къ синодическому равно $\frac{5458}{5923}$. Отсюда находимъ продолжительность древняго драконическаго мѣсяца 27 д. 5 ч. 5 м. 35°,330, тогда какъ современный содержитъ 27 д. 5 ч. 5 м. 35°,81 (см. напр. Neison, der Mond, 412). Замѣчая что въ 100 годахъ заключается 1342,39 драконическихъ мѣсяца, легко вычислить, что въ первые 100 лѣтъ послѣ эпохи средняя долгота выходящаго узла увеличится на 9",31. Теоретическія соображенія даютъ для коэффиціента вѣковаго неравенства долготы узла около 7" т. е. менѣе на 2",3, но само собою разумѣется, что отъ вычисленія, основаннаго на сравненіи мѣсяцевъ и нельзя было ожидать большой точности.

Вѣковыя неравенства въ движеніи Луны происходятъ отъ возмущеній, оказываемыхъ на земнаго спутника Солнцемъ, въ связи съ вѣковымъ уменьшеніемъ эксцентрицитета земной орбиты. Если принимать, какъ это обыкновенно дѣлается въ теоріи Луны, центръ Земли за неподвижную точку и разсматривать относительное движеніе Солнца вокругъ Земли, то можно употреблять выраженія „орбита Солнца“, возмущенія оказываемыя планетами на Солнце и пр., такъ какъ кажущееся годовое движеніе Солнца представляетъ полное изображеніе дѣйствительнаго движенія Земли вокругъ центральнаго тѣла нашей планетной системы. Условившись относительно истиннаго смысла этихъ выраженій, можно сказать, что причина вѣковыхъ неравенствъ Луны лежитъ въ возмущеніяхъ, производимыхъ Солнцемъ, въ свою очередь возмущаемымъ въ своемъ движеніи планетами.

Вслѣдствіе этихъ возмущеній эксцентрицитетъ земной орбиты весьма медленно, но непрерывно уменьшается изъ столѣтія въ столѣтіе, и это-то измѣненіе [1]), не составившее со временъ затмѣнія, бывшаго въ 720 г. до Р. Х., и 4-хъ минутъ въ дугѣ (въ 1000 лѣтъ оно равно 89″,51), отражаясь, если можно такъ выразиться, на движеніи Луны, производитъ ускореніе ея движенія, вслѣдствіе котораго средняя долгота этого свѣтила увеличилась въ 2600 лѣтъ приблизительно на $1^1/_2{}^\circ$.

Лапласъ ограничился первымъ приближеніемъ при вычисленіи коэффиціентовъ вѣковыхъ неравенствъ Луны, предполагая можетъ быть, что послѣдующія приближенія не измѣнятъ значительно найденныхъ имъ величинъ, которыя довольно удовлетворительно согласовались съ результатами сравненій древнихъ затмѣній съ позднѣйшими. Продолжателями изъисканій великаго геометра въ этой области небесной механики явились еще при немъ Дамуазо, Карлини и Плана, которые довели приближенія гораздо дальше, чѣмъ Лапласъ, въ особенности Плана, давшій аналитическія выраженіи координатъ Луны и вѣковыхъ неравенствъ ея элементовъ съ точностью до величинъ порядка m^7 (гдѣ $m =$ отношенію ср. скоростей движенія Солнца и Луны, приблизительно $\frac{1}{13}$), но результаты, къ которымъ они пришли въ томъ, что касается до вѣковыхъ неравенствъ Луны, и въ частности вѣковаго ускоренія, весьма мало отличались отъ выводовъ Лапласа.

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[1]) Въ 11448 году до Р. Х. эксцентрицитетъ земной орбиты достигалъ наибольшей величины и былъ равенъ 0,01964775. Послѣ этого времени онъ постоянно уменьшался и будетъ уменьшаться до 25410-го года нашей эры, когда онъ дойдетъ до своего minimum'а, а именно до 0,003928435 (см. Schubert, Astr. Th. III).

Такимъ образомъ могло казаться, что теорія вполнѣ удовлетворительно объясняетъ величину ускоренія, выведенную изъ сравненія наблюденій и что истинная величина коэффиціента этого ускоренія должна быть весьма близка къ 10″—11″.

Наконецъ вычисленія Гансена еще нѣсколько увеличили эту цифру, сначала до 11″,47, а потомъ до 12″,18. Въ такомъ положеніи было дѣло до появленія въ свѣтъ мемуара Адамса, напечатаннаго въ Philosophical Transactions за 1853 годъ. Въ немъ Адамсъ указалъ на ошибку въ изслѣдованіяхъ Плана и Дамуазо, состоявшую въ томъ, что оба эти геометра интегрировали дифференціальныя уравненія движенія Луны, принимая эксцентрицитетъ земной орбиты за величину постоянную и только по интегрированіи начинали разсматривать его какъ функцію времени. Это допущеніе могло имѣть мѣсто только въ первомъ приближеніи, на которомъ остановился Лапласъ; въ дальнѣйшихъ же вычисленіяхъ, гдѣ принимались въ разсчетъ вторая и третья степени пертурбаціонной функціи, оно не могло не привести къ невѣрнымъ выводамъ.

Вмѣстѣ съ тѣмъ Адамсъ показалъ, что съ прибавленіемъ въ выраженіяхъ параллакса и долготы Луны новыхъ періодическихъ членовъ, которые должны явиться при интегрированіи дифференціальныхъ уравненій, если разсматривать à priori эксцентрицитетъ земной орбиты какъ функцію времени, величина коэффиціента вѣковаго ускоренія значительно уменьшается.

Несмотря однако на простоту и убѣдительность соображеній Адамса, они были приняты въ наукѣ далеко не сразу.

Его выводы, основанныя на этихъ соображеніяхъ, сдѣлались обще признаннымъ достояніемъ науки только послѣ продолжительнаго спора, въ которомъ постепенно приняли участіе почти всѣ геометры, занимавшіеся въ это время теоріею Луны. Горячимъ противникомъ мнѣній Адамса, поддержанныхъ Делоне, который далъ ту же величину вѣковаго ускоренія, выступилъ прежде всѣхъ авторъ „Théorie analytique du système du monde“ Понтекуланъ, который не только не допускалъ возможности представить съ помощью предлагаемой Адамсомъ величины ускоренія древнихъ затмѣній, но и отрицалъ правильность его теоретическихъ разсужденій.

Мнѣнія раздѣлились. Съ одной стороны стояли Адамсъ и Делоне, работавшій въ это время надъ своимъ обширнымъ трудомъ

„Théorie du mouvement de la Lune“, а съ другой Понтекуланъ, Плана и отчасти Леверье, основывавшій впрочемъ свои сомнѣнія въ правильности выводовъ Адамса и Делоне не столько на своихъ собственныхъ изслѣдованіяхъ, сколько на авторитетѣ Гансена.

Вотъ сравнительная табличка величинъ коэффиціентовъ вѣковыхъ неравенствъ Луны, вычисленныхъ разными авторами.

	Ср. долгота.	Долгота перигея.	Долгота восх. узла.
Лапласъ	10″,182	30″,551	7″,488
Дамуазо	10,723	39,697	6,563
Плана	10,580	36,220	6,830
Понтекуланъ	12,24	40,611	6,731
Гансенъ	12,18	37,25	7,07
Делоне	6,176	39,499	6,778
Адамсъ	5,70		

Въ настоящее время споръ о величинѣ вѣковаго ускоренія можно считать оконченнымъ, или по крайней мѣрѣ перенесеннымъ на совершенно другую почву, и хотя вопросъ о томъ, зависитъ ли ускореніе въ движеніи Луны единственно отъ вѣковаго уменьшенія эксцентрицитета земной орбиты, или существуютъ и другія причины этого явленія, до сихъ поръ остается еще открытымъ, но не подлежитъ сомнѣнію, что если приписывать вѣковое ускореніе Луны лишь дѣйствію возмущенія Солнца, какъ единственной хорошо доказанной причинѣ этого явленія, то вычисленія Адамса и Делоне должны считаться болѣе заслуживающими довѣрія, чѣмъ выводы ихъ противниковъ.

Нѣсколько лѣтъ тому назадъ Симонъ Ньюкомбъ вычислилъ еще разъ древнія и средневѣковыя затмѣнія Луны, а также многія наблюденія покрытій Булліальда, Гассенди, Гевеліуса Флемстида, Ла-Гира и Делилля и пришелъ къ заключенію, что безъ введенія эмпирическихъ поправокъ таблицы Гансена не могутъ представить одновременно древнихъ и новѣйшихъ наблюденій затмѣній и покрытій *при какой бы то небыло величинѣ вѣковаго ускоренія*[1]). Далѣе онъ говоритъ: „Still, the correction (— 3″,36) which we have deduced for the secular acceleration is clearly indicated by the com

[1]) Researches on the motion of the Moon. 1878, p. 265.

bination of all the observations. Hansen's adopted value being 12″,17, we are led to the value 8″,8, as that which on the whole best satisfies the observations which me have discussed“.

Давно уже было высказано предположеніе, что явленіе ускоренія Луны представляетъ отчасти иллюзію, происходящую отъ измѣненія величины самой единицы времени, которою мы измѣряемъ движеніе Луны, т. е. длины звѣзднаго дня. Делоне въ 1865 г. замѣтилъ, что одною изъ причинъ нѣкотораго замедленія вращенія Земли вокругъ своей оси можетъ быть явленіе опаздыванія полной воды въ данномъ мѣстѣ относительно момента кульминаціи Луны. Вслѣдствіе этого опаздыванія, воды океана въ той части земнаго шара, которая обращена въ данный моментъ къ Лунѣ, скапливаются не на томъ меридіанѣ, черезъ который проходитъ Луна, а на другомъ, отстоящемъ отъ 1-го среднимъ числомъ на 15° къ востоку и дѣйствіе Луны на эти выпуклыя массы воды въ противуположныхъ частяхъ земной поверхности выражается парою силъ, которая стремится вращать Землю съ востока на западъ, т. е. нѣсколько замедляетъ суточное движеніе.

Мы не будемъ входить въ ближайшее изслѣдованіе этого явленія, такъ какъ весьма мало вѣроятно, чтобы происходящее отъ этой причины замедленіе вращенія земли оказалось хотя сколько нибудь чувствительнымъ.

Въ настоящемъ изслѣдованіи мы задались цѣлью опредѣлить вѣковыя измѣненія элементовъ лунной орбиты происходящія отъ возмущающаго дѣйствія Солнца.

Пользуясь въ первомъ приближеніи извѣстными дифференціальными формулами Лагранжа и Пуассона для опредѣленія измѣненій элементовъ въ функціи производныхъ пертурбаціонной функціи, мы получаемъ выраженіе этихъ измѣненій въ функціи времени съ точностью до величинъ порядка m^2, гдѣ какъ уже сказано m означаетъ отношеніе среднихъ скоростей движенія Солнца и Луны.

Затѣмъ мы переходимъ къ интегрированію дифференціальныхъ уравненій движенія Луны и изъ интеграловъ этихъ уравненій выводимъ величины коэффиціентовъ трехъ главныхъ вѣковыхъ неравенствъ въ движеніи Луны а именно: вѣковаго измѣненія въ долготѣ центра Луны, ея перигея и линіи узловъ до величинъ порядка m^3 для перигея и линіи узловъ и m^4 для ускоренія Луны въ движеніи по долготѣ.

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* Звѣздочкою обозначены тѣ сочиненія, которыми мы пользовались какъ источниками.

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ГЛАВА I.

Дифференціальныя уравненія относительнаго движенія Луны вокругъ Земли.

1. Всѣ главныя неравенства эллиптическаго движенія Луны вокругъ Земли происходятъ отъ возмущающаго дѣйствія Солнца, и потому задачу изслѣдованія этого движенія можно разсматривать просто какъ частный случай задачи о трехъ тѣлахъ, дѣйствующихъ другъ на друга по закону всеобщаго тяготѣнія. Рѣшеніе этой задачи значительно упрощается, если принимать центръ тяжести одного изъ этихъ тѣлъ за неподвижную точку и разсматривать лишь относительное движеніе двухъ другихъ тѣлъ, возмущаемаго и возмущающаго, вокругъ тѣла центральнаго. Въ теоріи Луны за это центральное тѣло всегда принимается Земля.

Пусть M масса Земли въ единицахъ массы Луны, r разстояніе центровъ тяжести этихъ двухъ тѣлъ, m' масса Солнца и r' разстоніе Солнца отъ Земли. Если примемъ за начало координатъ центръ тяжести Земли, за плоскость xy — плоскость эклиптики соотвѣтствующей какой-нибудь опредѣленной эпохѣ и обозначимъ черезъ x, y, z прямоугольныя координаты центра тяжести Луны, а черезъ x', y', z' прямоугольныя координаты центра Солнца, то возмущенное движеніе Луны можетъ быть выражено слѣдующими дифференціальными уравненіями:

$$\frac{d^2x}{dt^2}=\left(\frac{dQ}{dx}\right),\quad \frac{d^2y}{dt^2}=\left(\frac{dQ}{dy}\right) \quad \text{и} \quad \frac{d^2z}{dt^2}=\left(\frac{dQ}{dz}\right)\ldots\ldots.(1)$$

где $$\left(\frac{dQ}{dx}\right),\quad \left(\frac{dQ}{dy}\right) \quad\text{и}\quad \left(\frac{dQ}{dz}\right)$$

суть частныя производныя функціи Q, которая имѣетъ видъ

$$Q = \frac{k^2(1+M)}{r} - \frac{k^2 m'(xx'+yy'+zz')}{r'^3} + \frac{k^2 m'}{\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}} . \quad (2)$$

2. Введемъ въ ур. (1) полярныя координаты.

Нетрудно убѣдиться, что обозначая соотвѣтственно черезъ P, T и S силы, дѣйствующія на Луну въ направленіи проэкціи радіуса вектора Луны на плоскость xy, въ направленіи перпендикулярномъ къ нему и наконецъ по линіи перпендикулярной къ плоскости эклиптики, мы получаемъ

$$\left.\begin{aligned}\left(\frac{dQ}{dx}\right) &= -P\cos v - T\sin v = -P\cdot\frac{x}{\rho} - T\cdot\frac{y}{\rho}\\ \left(\frac{dQ}{dy}\right) &= -P\sin v + T\cos v = -P\cdot\frac{y}{\rho} + T\cdot\frac{x}{\rho}\end{aligned}\right\} \quad ..(3)$$

$$\left(\frac{dQ}{dz}\right) = -S$$

гдѣ v долгота Луны считаемая по постоянной эклиптикѣ отъ точки ♈ соотвѣтствующей принятой эпохѣ, а ρ проэкція радіуса вектора Луны на плоскость xy.

Представляя ур. (1) въ формѣ

$$\frac{d^2x}{dt^2} = -P\cdot\frac{x}{\rho} - T\cdot\frac{y}{\rho}$$

$$\frac{d^2y}{dt^2} = -P\cdot\frac{y}{\rho} + T\cdot\frac{x}{\rho}$$

$$\frac{d^2z}{dt^2} = -S,$$

мы легко получаемъ отсюда:

$$2\frac{dx}{dt}\cdot\frac{d^2x}{dt^2} + 2\cdot\frac{dy}{dt}\cdot\frac{d^2y}{dt^2} = -\frac{2P}{\rho}\left(x\cdot\frac{dx}{dt} + y\frac{dy}{dt}\right) + \frac{2T}{\rho}\left(x\cdot\frac{dy}{dt} - y\frac{dx}{dt}\right)$$

или

$$\frac{d}{dt}\left\{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} = \frac{d}{dt}\left\{\left(\frac{d\rho}{dt}\right)^2 + \rho^2\cdot\left(\frac{dv}{dt}\right)^2\right\} = -2P\cdot\frac{d\rho}{dt} + 2T\cdot\rho\cdot\frac{dv}{dt} \cdot (4)$$

Далѣе имѣемъ

$$x \cdot \frac{d^2y}{dt^2} - y \cdot \frac{d^2x}{dt^2} = T \cdot \frac{(x^2+y^2)}{\rho} = T \cdot \rho$$

или

$$\frac{d}{dt}\left(\rho^2 \cdot \frac{dv}{dt}\right) = T\rho \ldots\ldots\ldots\ldots\ldots \quad (5)$$

Наконецъ если обозначимъ черезъ s тангенсъ широты Луны, то получимъ $z = \rho s$ и

$$\frac{d^2(\rho s)}{dt^2} = -S \ldots\ldots\ldots\ldots\ldots \quad (6)$$

3. Вмѣсто t введемъ новую независимую перемѣнную v.

Возьмемъ сначала ур. (5).

Умножая отъ части на $\rho^2 \frac{dv}{dt}$, находимъ: $\frac{1}{2} \cdot \frac{d}{dt}\left[\rho^2 \frac{dv}{dt}\right]^2 = T\rho^3 \frac{dv}{dt}$ или $\frac{1}{2} \cdot \frac{d\left(\rho^2 \frac{dv}{dt}\right)^2}{dv} = T\rho^3$, откуда $\left[\rho^2 \frac{dv}{dt}\right]^2 = h^2 + 2\int T \cdot \rho^3 \cdot dv$, гдѣ h^2 произвольное постоянное. Извлекая квадратный корень, имѣемъ:

$$\rho^2 \cdot \frac{dv}{dt} = (h^2 + 2\int T\rho^3 dv)^{\frac{1}{2}} \ldots\ldots\ldots\ldots \quad (7)$$

Помножимъ обѣ части ур. (4) на $\frac{dt}{dv}$; тогда получится:

$$\frac{d}{dv}\left\{\left(\frac{d\rho}{dt}\right)^2 + \rho^2 \cdot \left(\frac{dv}{dt}\right)^2\right\} = -2P \cdot \frac{d\rho}{dv} + 2T \cdot \rho$$

или

$$\frac{d}{dv}\left\{\left(\frac{d\rho}{dv}\right)^2 \cdot \left(\frac{dv}{dt}\right)^2 + \rho^2\left(\frac{dv}{dt}\right)^2\right\} = \frac{d}{dv}\left\{\left[\left(\frac{d\rho}{dv}\right)^2 + \rho^2\right]\left(\frac{dv}{dt}\right)^2\right\} =$$

$$\frac{d}{dv}\left\{\left[\left(\frac{d\rho}{dv}\right)^2 + \rho^2\right]\frac{1}{\rho^4}(h^2 + 2\int T\rho^3 dv)\right\} = -2P \cdot \frac{d\rho}{dv} + 2T \cdot \rho.$$

Полагая $\rho = \frac{1}{u}$, находимъ отсюда:

$$\frac{d}{dv}\left\{\left(\frac{1}{u^4} \cdot \left(\frac{du}{dv}\right)^2 + \frac{1}{u^2}\right) \cdot u^4 \left(h^2 + 2\int T \cdot \frac{dv}{u^3}\right)\right\} =$$

$$\frac{d}{dv}\left\{\left(\left(\frac{du}{dv}\right)^2 + u^2\right)\left(h^2 + 2\int T \cdot \frac{dv}{u^3}\right)\right\} = \left(h^2 + 2\int T \cdot \frac{dv}{u^3}\right)\left[2 \cdot \frac{du}{dv} \cdot \frac{d^2u}{dv^2} + 2u \cdot \frac{du}{dv}\right] +$$

$$+ \left\{\left(\frac{du}{dv}\right)^2 + u^2\right\}\frac{2T}{u^3} = \frac{2P}{u^2} \cdot \frac{du}{dv} + \frac{2T}{u}$$

или

$$\left(\frac{d^2u}{dv^2}+u\right)\left(h^2+2\int\frac{Tdv}{u^3}\right)+\left(\left(\frac{du}{dv}\right)^2+u^2\right)\cdot\frac{dv}{du}\cdot\frac{T}{u^3}=\frac{P}{u^2}+\left(\frac{T}{u}:\frac{du}{dv}\right),$$

откуда

$$\left(\frac{d^2u}{dv^2}+u\right)\left(h^2+2\int\frac{Tdv}{u^3}\right)+\left(\frac{T}{u}:\frac{du}{dv}\right)\left\{\left[\left(\frac{du}{dv}\right)^2+u^2\right]\cdot\frac{1}{u^2}-1\right\}=\frac{P}{u^2}.$$

Но

$$\left[\left(\frac{du}{dv}\right)^2+u^2\right]\frac{1}{u^2}-1=\frac{1}{u^2}\cdot\left(\frac{du}{dv}\right)^2,\text{ слѣд.}$$

$$\left(\frac{d^2u}{dv^2}+u\right)\left(h^2+2\int\frac{Tdv}{u^3}\right)+\frac{T}{u}\cdot\frac{1}{u^2}\cdot\frac{du}{dv}=\frac{P}{u^2}\;\ldots\ldots(8)$$

Такимъ-же образомъ ур. (6) даетъ

$$\frac{d^2\left(\frac{s}{u}\right)}{dt^2}=-S,\ \frac{d\left(\frac{s}{u}\right)}{dt}=\frac{d\left(\frac{s}{u}\right)}{dv}\cdot\frac{dv}{dt}=\frac{1}{u^2}\left(u\cdot\frac{ds}{dv}-s\cdot\frac{du}{dv}\right)\cdot\frac{dv}{dt}$$

$$=\left(u\frac{ds}{dv}-s\frac{du}{dv}\right)\cdot\sqrt{h^2+2\int\frac{Tdv}{u^3}},\quad\frac{d^2\left(\frac{s}{u}\right)}{dt^2}=$$

$$=\frac{dv}{dt}\cdot\frac{d}{dv}\left\{\left(u\frac{ds}{dv}-s\frac{du}{dv}\right)\sqrt{h^2+2\int\frac{Tdv}{u^3}}\right\}=u^2\left(h^2+2\int\frac{Tdv}{u^3}\right)^{\frac{1}{2}}$$

$$\cdot\left\{\left(u\frac{ds}{dv}-s\frac{du}{dv}\right)\cdot\frac{d}{dv}\sqrt{h^2+2\int\frac{Tdv}{u^3}}+\sqrt{h^2+2\int\frac{Tdv}{u^3}}\left(u\frac{d^2s}{dv^2}-s\cdot\frac{d^2u}{dv^2}\right)\right\}$$

$$=u^2\left(h^2+2\int\frac{Tdv}{u^3}\right)^{\frac{1}{2}}\left\{\left(u\frac{ds}{dv}-s\frac{du}{dv}\right)\cdot\frac{T}{u^3\sqrt{h^2+2\int\frac{T}{u^3}dv}}+\right.$$

$$\left.+\left(h^2+2\int\frac{Tdv}{u^3}\right)^{\frac{1}{2}}\left(u\frac{d^2s}{dv^2}-s\frac{d^2u}{dv^2}\right)\right\}=\frac{T}{u}\left(u\frac{ds}{dv}-s\frac{du}{dv}\right)+$$

$$+u^2\left(h^2+2\int\frac{Tdv}{u^3}\right)\left(u\frac{d^2s}{dv^2}-s\frac{d^2u}{dv^2}\right)=-S,$$

откуда

$$u\frac{d^2s}{dv^2}-s\cdot\frac{d^2u}{dv^2}=-\frac{\left[\frac{T}{u}\left(u\frac{ds}{dv}-s\frac{du}{dv}\right)+S\right]}{u^2\left(h^2+2\int\frac{Tdv}{u^3}\right)}\;\ldots\ldots(9)$$

Ур. (8) можно представить въ видѣ

$$\frac{d^2u}{dv^2} + u - \frac{\left(\frac{P}{u^2} - \frac{T}{u^3} \cdot \frac{du}{dv}\right)}{h^2 + 2\int \frac{Tdv}{u^3}} = 0.$$

Умножая это ур. на s и складывая произведеніе съ ур. (9), находимъ

$$\frac{d^2s}{dv^2} + s + \frac{\frac{S}{u^3} - \frac{Ps}{u^3} + \frac{T}{u^3} \cdot \frac{ds}{dv}}{h^2 + 2\int \frac{Tdv}{u^3}} = 0 \ldots\ldots\ldots (10)$$

4. Въ эти ур. легко ввести частныя производныя функціи Q. Такъ какъ

$$x = \frac{\cos v}{u},\ y = \frac{\cos v}{u},\ z = \frac{s}{u},$$

и

$$dQ = dx \cdot \left(\frac{dQ}{dx}\right) + dy \cdot \left(\frac{dQ}{dy}\right) + dz \cdot \left(\frac{dQ}{dz}\right),$$

то мы получаемъ

$$dQ = -\frac{du}{u^2}\left[\left(\frac{dQ}{dx}\right)\cos v + \left(\frac{dQ}{dy}\right) \cdot \sin v + s \cdot \left(\frac{dQ}{dz}\right)\right]$$

$$-\frac{dv}{u}\left[\left(\frac{dQ}{dx}\right)\sin v - \left(\frac{dQ}{dy}\right)\cos v\right] + \frac{ds}{u} \cdot \left(\frac{dQ}{dz}\right).$$

Съ другой стороны, такъ-какъ Q можетъ быть представлено въ функціи u, v и s, то

$$dQ = du \cdot \left(\frac{dQ}{du}\right) + dv \cdot \left(\frac{dQ}{dv}\right) + ds \cdot \left(\frac{dQ}{ds}\right)$$

Сравнивая коэффиціенты при одинаковыхъ дифференціалахъ, мы находимъ:

$$\left(\frac{dQ}{du}\right) = -\frac{1}{u^2}\left[\left(\frac{dQ}{dx}\right)\cos v + \left(\frac{dQ}{dy}\right) \cdot \sin v + s \cdot \left(\frac{dQ}{dz}\right)\right]$$

$$\left(\frac{dQ}{dv}\right) = -\frac{1}{u}\left[\left(\frac{dQ}{dx}\right)\sin v - \left(\frac{dQ}{dy}\right) \cdot \cos v\right]$$

$$\left(\frac{dQ}{ds}\right) = \frac{1}{u} \cdot \left(\frac{dQ}{dz}\right)$$

Подставляя-же въ эти ур. вмѣсто $\left(\frac{dQ}{dx}\right)$, $\left(\frac{dQ}{dy}\right)$ и $\left(\frac{dQ}{dz}\right)$ ихъ величины изъ формулъ (3), получаемъ

$$\left(\frac{dQ}{du}\right)=\frac{1}{u^2}\cdot P-\frac{s}{u^2}\left(\frac{dQ}{dz}\right)=\frac{P}{u^2}+\frac{s}{u^2}\cdot S$$

$$\left(\frac{dQ}{dv}\right)=\frac{T}{u}$$

$$\left(\frac{dQ}{ds}\right)=-\frac{1}{u}\cdot S,$$

откуда

$$P=u^2\cdot\left(\frac{dQ}{du}\right)+us\cdot\left(\frac{dQ}{ds}\right),\qquad T=u\left(\frac{dQ}{dv}\right),\qquad S=-u\left(\frac{dQ}{ds}\right),$$

значитъ найденныя выше ур. 7, 8 и 10 принимаютъ видъ:

$$\left.\begin{aligned}
&\left(\frac{d^2u}{dv^2}+u\right)\left(1+\frac{2}{h^2}\int\left(\frac{dQ}{dv}\right)\cdot\frac{1}{u^2}dv\right)-\frac{1}{h^2}\left(\frac{dQ}{du}\right)-\frac{s}{h^2u}\left(\frac{dQ}{ds}\right)+\left(\frac{dQ}{dv}\right)\cdot\frac{1}{h^2u^2}\cdot\frac{du}{dv}=0\\
&\frac{dt}{dv}=\frac{1}{u^2\sqrt{h^2+2\int\left(\frac{dQ}{dv}\right)\cdot\frac{dv}{u^2}}}\\
&\left(\frac{d^2s}{dv^2}+s\right)\left(1+\frac{2}{h^2}\int\left(\frac{dQ}{dv}\right)\cdot\frac{dv}{u^2}\right)-\frac{1}{h^2u^2}\cdot\left(\frac{dQ}{ds}\right)-\frac{s}{h^2u}\cdot\left(\frac{dQ}{du}\right)-\frac{s^2}{h^2u^2}\cdot\left(\frac{dQ}{ds}\right)+\frac{dQ}{dv}\cdot\frac{1}{h^2u^2}\cdot\frac{ds}{dv}=0
\end{aligned}\right\}(11)$$

Это суть основныя дифференціальныя уравненія для опредѣленія относительнаго движенія Луны вокругъ Земли. Они даны въ этой формѣ Лапласомъ въ № 15, 2-й книги Mécanique Céléste.

ГЛАВА II.

Опредѣленіе постоянной части пертурбаціонной функціи.

5. Мы обозначили буквою Q выраженіе

$$\frac{k^2(M+1)}{r} - \frac{k^2\, m'(xx'+yy'+zz')}{r'^3} + \frac{k^2\, m'}{\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}}$$

Введемъ сюда полярныя координаты и положимъ для краткости $M+1=\mu$, $k^2=1$. Отмѣчая значкомъ сверху величины относящіяся къ Солнцу, мы имѣемъ

$$xx'+yy'+zz' = \rho\rho'\left(\frac{x}{\rho}\cdot\frac{x'}{\rho'}+\frac{y}{\rho}\cdot\frac{y'}{\rho'}+\frac{z}{\rho}\cdot\frac{z'}{\rho'}\right) = \rho\rho'(\cos(\rho,\rho')+ss^1)$$

и слѣд.

$$\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2} = \sqrt{r^2-2\rho\rho'[\cos(v-v'+ss']+r'^2}$$

$$Q = \frac{\mu}{r} - \frac{m'\rho\rho'[\cos(v-v')+ss']}{r'^3} + \frac{m'}{\sqrt{r^2-2\rho\rho'[\cos(v-v')+ss']+r'^2}}$$

Обозначимъ черезъ Ω сумму двухъ послѣднихъ членовъ, такъ-что $Q=\frac{\mu}{r}+\Omega$, и разложимъ

$$\frac{m'}{\sqrt{r^2-2\,\rho\rho'[\cos(v-v')+ss']+r'^2}}$$

по биному, введя вмѣсто ρ и ρ' дроби

$$\frac{r}{\sqrt{1+s^2}} \text{ и } \frac{r'}{\sqrt{1+s'^2}}$$

Ограничиваясь 2-ю степенью разности $\frac{r^2}{r'^2} - \frac{2r\lambda}{r'}$, гдѣ мы обозначили черезъ λ выраженіе $\frac{\cos(v-v')+ss'}{\sqrt{1+s^2}\cdot\sqrt{1+s'^2}}$, мы находимъ:

$$\Omega = -\frac{m'r\,[\cos(v-v')+ss']}{r'^2\sqrt{1+s^2}\cdot\sqrt{1+s'^2}} + \frac{m'}{r'} + \frac{m'r\,[\cos(v-v')+ss']}{r'^2\sqrt{1+s^2}\cdot\sqrt{1+s'^2}}$$

$$-\frac{m'r^2}{2r^3} + \frac{3}{8}\frac{m'}{r'}\left(\frac{r^2}{r'^2} - \frac{2r\lambda}{r'}\right)^2 - \ldots$$

или

$$\Omega = \frac{m'}{r'} - \frac{m'r^2}{2r'^3} + \frac{3}{8}\frac{m'r^4}{r'^5} - \frac{3}{2}\frac{m'r^3}{r'^4}\lambda + \frac{3}{2}\frac{m'r^2}{r'^3}\lambda^2 + \ldots$$

$$= \frac{m'}{r'} - \frac{m'r^2}{2r'^3}(1-3\lambda^2) - \ldots\ldots\ldots\ldots \quad (12)$$

Что касается до величины λ, то легко видѣть, что $\lambda = \cos(r, r')$, т. е. косинусу угла между радіусами векторами Солнца и Луны. Въ самомъ дѣлѣ, мы имѣемъ

$$\cos(v-v') + ss' = \frac{xx'+yy'+zz'}{\rho\rho'},$$

слѣдовательно

$$\lambda = \frac{\cos(v-v')+ss'}{\sqrt{1+s^2}\cdot\sqrt{1+s'^2}} = \frac{xx'+yy'+zz'}{rr'} = \cos(r, r').$$

6. Введемъ теперь вмѣсто v и v', считаемыхъ по неподвижной эклиптикѣ, долготы v_0 и v_0', считаемыя соотвѣтственно по орбитѣ Луны отъ восходящаго узла этой орбиты, и по орбитѣ Земли, т. е. по истинной эклиптикѣ. Если обозначимъ черезъ θ долготу восходящаго узла орбиты Луны и θ' долготу восходящаго узла истинной эклиптики на неподвижной, а черезъ i и i' наклонности лунной орбиты и истинной эклиптики относительно неподвижной, то по извѣстнымъ формуламъ получимъ;

$$\lambda = \frac{xx'+yy'+zz'}{rr'} = (\cos\theta\cdot\cos v_0 - \cos i\cdot\sin\theta\cdot\sin v_0)\cdot$$

$$\cdot(\cos\theta'\cdot\cos v_0' - \cos i'\cdot\sin\theta'\cdot\sin v_0') + (\sin\theta\cdot\cos v_0 + \cos i\cdot\cos\theta\cdot\sin v_0)\cdot$$

$$\cdot(\sin\theta'\cdot\cos v_0' + \cos i'\cdot\cos\theta'\cdot\sin v_0') + \sin i\cdot\sin v_0\cdot\sin i'\cdot\sin v_0' =$$

$$= \cos\theta\cdot\cos\theta'\cdot\cos v_0\cdot\cos v_0' - \cos i\sin\theta\cdot\cos\theta'\sin v_0\cdot\cos v_0'$$

$$-\cos\theta\cdot\cos v_0\cdot\cos i'\cdot\sin\theta'\cdot\sin v_0'+\cos i\cdot\cos i'\cdot\sin\theta\cdot\sin\theta'\cdot\sin v_0\cdot\sin v'_0$$
$$+\sin\theta\cdot\sin\theta'\cdot\cos v_0\cdot\cos v_0'+\cos i\cdot\cos\theta\cdot\sin\theta'\cdot\sin v_0\cdot\cos v_0'+$$
$$+\sin\theta\cdot\cos\theta'\cdot\cos v_0\cdot\sin v_0'\cdot\cos i'+\cos i\cdot\cos i'\cdot\cos\theta\cdot\cos\theta'\cdot\sin v_0\cdot\sin v_0'+$$
$$+\sin i\cdot\sin v_0\cdot\sin i'\cdot\sin v_0'.$$

Отсюда получаемъ, соединяя члены 1-й и 5-й и замѣняя вездѣ $\cos i$ черезъ $1-2\sin^2\frac{i}{2}$:

$$\lambda=\cos v_0\cdot\cos v_0'\cdot\cos(\theta-\theta')-\sin\theta\cdot\cos\theta'\cdot\sin v_0\cdot\cos v_0'$$
$$+2\sin\theta\cdot\cos\theta'\cdot\sin v_0\cdot\cos v_0'\cdot\sin^2\frac{i}{2}-\cos\theta\cdot\cos v_0\cdot\sin\theta'\cdot\sin v_0'$$
$$-2\cos\theta\cdot\cos v_0\cdot\sin\theta'\cdot\sin v_0'\cdot\sin^2\frac{i'}{2}+\cos\theta\cdot\sin\theta'\cdot\sin v_0\cdot\cos v_0$$
$$-2\cos\theta\cdot\sin\theta'\cdot\sin v_0\cdot\cos v_0'\cdot\sin^2\frac{i}{2}+\sin\theta\cdot\sin\theta'\cdot\sin v_0\cdot\sin v_0'$$
$$+4\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cdot\sin\theta\cdot\sin\theta'\cdot\sin v_0\cdot\sin v_0'-2\sin^2\frac{i}{2}\cdot\sin\theta\cdot\sin\theta'\cdot\sin v_0\cdot\sin v_0'$$
$$-2\sin^2\frac{i'}{2}\sin\theta\cdot\sin\theta'\cdot\sin v_0\cdot\sin v_0'+\sin\theta\cdot\cos\theta'\cdot\cos v_0\cdot\sin v_0'$$
$$-2\sin^2\frac{i'}{2}\cdot\sin\theta\cdot\cos\theta'\cdot\cos v_0\cdot\sin v_0'+\cos\theta\cdot\cos\theta'\cdot\sin v_0\cdot\sin v_0'$$
$$+4\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cos\theta\cdot\cos\theta'\cdot\sin v_0\cdot\sin v_0'-2\sin^2\frac{i}{2}\cos\theta\cdot\cos\theta'\cdot\sin v_0\cdot\sin v_0'$$
$$-2\sin^2\frac{i'}{2}\cos\theta\cdot\cos\theta'\cdot\sin v_0\cdot\sin v_0'+\sin i\cdot\sin i'\cdot\sin v_0\cdot\sin v_0'.$$

Въ этомъ выраженіи сумма членовъ:

1-го, 8-го и 14-го равна $+\cos(\theta-\theta').\cos(v_0-v_0')$

2-го и 12-го „ $-\sin\theta.\cos\theta'.\sin(v_0-v_0')$

4-го и 6-го „ $+\sin\theta'.\cos\theta.\sin(v_0-v_0')$

3-го и 7-го равна $+2\sin^2\frac{i}{2}\cdot\sin v_0 . \cos v_0' . \sin(\theta-\theta')$

5-го и 13-го „ $-2\sin^2\frac{i'}{2}\cdot\sin v_0' . \cos v_0 . \sin(\theta-\theta')$

9-го и 15-го „ $+4\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cdot\sin v_0 . \sin v_0' . \cos(\theta-\theta')$

10-го и 16-го „ $-2\sin^2\frac{i}{2}\cdot\sin v_0 . \sin v_0' . \cos(\theta-\theta')$

11-го и 17-го „ $-2\sin^2\frac{i'}{2}\cdot\sin v_0' . \sin v_0 . \cos(\theta-\theta')$.

Отсюда безъ труда находимъ

$$\lambda=\cos(\theta-\theta'+v_0-v_0')-2\sin^2\frac{i}{2}\cdot\sin v_0\ \sin(v_0'+\theta'-\theta)$$

$$-2\sin^2\frac{i'}{2}\cdot\sin v_0' . \sin(v_0+\theta-\theta')+4\sin^2\frac{i}{2}\sin^2\frac{i'}{2}\sin v_0\sin v_0'\cos(\theta-\theta').$$

Составляемъ далѣе $1-3\lambda^2$, пренебрегая въ λ^2 членами умноженными на $\sin^3\frac{i'}{2}$ и $\sin^4\frac{i'}{2}$:

$$1-3\lambda^2=1-3\left[\tfrac{1}{2}+\tfrac{1}{2}\cos 2(v_0-v_0'+\theta-\theta')+4\sin^4\frac{i}{2}\cdot\sin^2 v_0 . \sin^2(v_0'+\theta'-\theta)\right.$$

$$-4\sin^2\frac{i}{2}\cdot\sin v_0 . \sin(v_0'+\theta'-\theta) . \cos(v_0-v_0'+\theta-\theta')$$

$$-4\sin^2\frac{i'}{2}\cdot\sin v_0'\cdot\sin(v_0+\theta-\theta') . \cos(v_0-v_0'+\theta-\theta')$$

$$+8\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cdot\sin v_0\sin v_0' . \sin(v_0'+\theta'-\theta) . \sin(v_0+\theta-\theta')$$

$$+8\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cdot\sin v_0\sin v_0' . \cos(v_0-v_0'+\theta-\theta') . \cos(\theta-\theta')$$

$$+\sin^2 i . \sin^2 i' . \sin^2 v_0 . \sin^2 v_0'+2\sin i . \sin i' . \sin v_0 . \sin v_0'\cos(v_0-v_0'+\theta-\theta')$$

$$-4\sin i\cdot\sin^2\frac{i}{2}\cdot\sin i' . \sin^2 v_0 . \sin v_0' . \sin(v_0'+\theta'-\theta).$$

7. Замѣнимъ теперь въ предъидущемъ выраженіи произведенія синусовъ и косинусовъ дугъ, зависящихъ отъ v_0, v_0', θ и θ', синусами и косинусами суммъ и разностей дугъ, а $\sin\frac{i}{2}$ и $\sin\frac{i'}{2}$ — соотвѣтствующими дугами, причемъ отбросимъ члены, въ которыхъ i входитъ въ степени выше 4-й, а i' выше второй. Имѣя въ виду, что наклонность лунной орбиты немногимъ болѣе 5°, а i', выраженное въ угловыхъ единицахъ, составляетъ всего нѣсколько секундъ, мы достигнемъ такимъ образомъ вполнѣ достаточной степени точности.

Разлагая члены 2-й части предъидущаго равенства, мы находимъ послѣдовательно:

$$3)\quad 4\sin^4\frac{i}{2}\cdot\sin^2 v_0.\ \sin^2(v_0'+\theta'-\theta)=\frac{i^4}{16}\Big[1-\cos 2v_0$$
$$-\frac{1}{2}\cos(2v_0+2v_0'+2\theta'-2\theta)-\frac{1}{2}\cos(2v_0-2v_0'-2\theta'+2\theta)\Big].$$

$$4)-4\sin^2\frac{i}{2}\cdot\sin v_0\sin(v_0'+\theta'-\theta).\cos(v_0-v_0'+\theta-\theta')=$$
$$=-\frac{i^2}{4}[1-\cos 2v_0+\cos(2v_0'+2\theta-2\theta')-\cos(2v_0-2v_0'-2\theta-2\theta')].$$

$$5)-4\sin^2\frac{i'}{2}\cdot\sin v_0'.\sin(v_0+\theta-\theta').\cos(v_0-v_0'+\theta-\theta')$$
$$=-\frac{i'^2}{4}[1-\cos 2v_0'+\cos(2v_0-2v_0'+2\theta-2\theta')-\cos(2v_0+2\theta-2\theta')].$$

$$6)\ 8\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cdot\sin v_0.\sin v_0'.(\sin v_0'+\theta'-\theta).\sin(v_0+\theta-\theta')$$
$$=\frac{1}{4}i^2i'^2\sin v_0.\sin(v_0+\theta-\theta')[\cos(\theta-\theta')-\cos(2v_0'+\theta'-\theta)]$$
$$=\frac{1}{8}i^2.\,i'^2[\cos(\theta-\theta')-\cos(2v_0+\theta-\theta')][\cos(\theta-\theta')-$$
$$-\cos(2v_0'+\theta'-\theta)]$$
$$=\frac{1}{16}i^2i'^2[1+\cos 2(\theta-\theta')-\cos 2v_0-\cos(2v_0+2\theta-2\theta')$$
$$-\cos 2v_0'-\cos(2v_0'+2\theta'-2\theta)+\cos(2v_0+2v_0')+$$
$$+\cos(2v_0-2v_0'+2\theta-2\theta')].$$

$$7)\quad 8\sin^2\frac{i}{2}\cdot\sin^2\frac{i'}{2}\cdot\sin v_0\sin v_0'.\cos(v_0-v_0'+\theta-\theta').\cos(\theta-\theta')=$$

$$=\frac{1}{4}i^2.i'^2\sin v_0.\sin v_0'[\cos(v_0-v_0'+2\theta-2\theta')+\cos(v_0-v_0')]$$

$$=\frac{1}{8}i^2.i'^2\sin v_0[\sin(v_0+2\theta-2\theta')+\sin(2v_0'-v_0-2\theta+2\theta')$$

$$+\sin v_0+\sin(2v_0'-v_0)]=$$

$$=\frac{1}{16}i^2\cdot i'^2\ [\cos\ (2\theta-2\theta')-\cos\ (2v_0+2\theta-2\theta')+$$

$$+\cos(2v_0'-2v_0-2\theta+2\theta')-\cos(2v_0'-2\theta+2\theta')+1-\cos 2v_0$$

$$+\cos(2v_0'-2v_0)-\cos 2v_0'].$$

8) $\sin^2 i.\sin^2 i'.\sin^2 v_0.\sin^2 v_0'=\frac{i^2 i'^2}{4}(1-\cos 2v_0)(1-\cos 2v_0')=$

$$=\frac{i^2 i'^2}{4}\Big[1-\cos 2v_0-\cos 2v_0'+\frac{1}{2}\cos(2v_0+2v_0')+\frac{1}{2}\cos(2v_0-2v_0')\Big].$$

Въ двухъ послѣднихъ членахъ оставимъ пока $\sin i$ и $\sin i'$.

9) $2\sin i.\sin i'\sin v_0.\sin v_0'.\cos(v_0-v_0'+\theta-\theta')=$

$$=\sin i.\sin i'\sin v_0[\sin(v_0+\theta-\theta')+\sin(2v_0'-v_0-\theta+\theta')]$$

$$=\frac{\sin i\sin i'}{2}[\cos(\theta-\theta')-\cos(2v_0+\theta-\theta')]+$$

$$+\frac{\sin i\sin i'}{2}[\cos(2v_0'-2v_0-\theta+\theta')-\cos(2v_0'-\theta+\theta')].$$

10) $-4\sin i.\sin^2\frac{i}{2}\cdot\sin i'.\sin^2 v_0.\sin v_0'.\sin(v_0'+\theta'-\theta)=$

$$=-2\sin i.\sin^2\frac{i}{2}\cdot\sin i'.\sin^2 v_0[\cos(\theta-\theta')-\cos(2v_0'+\theta'-\theta)]$$

$$=-\sin i.\sin^2\frac{i}{2}\cdot\sin i'\ [\cos\ (\theta-\theta')-\cos\ (2v_0'+\theta'-\theta)]+$$

$$+\sin i.\sin^2\frac{i}{2}\cdot\sin i'[\cos\ (2v_0+\theta-\theta')+\cos\ (2v_0-\theta+\theta')]$$

$$-\sin i.\sin^2\frac{i}{2}\cdot\sin i'[\cos(2v_0+2v_0'+\theta'-\theta)+\cos(2v_0-2v_0'-\theta'+\theta)].$$

Всѣ полученные члены могутъ быть раздѣлены на 3 группы: первую, состоящую изъ членовъ независимыхъ отъ $2v_0$ и $2v_0'$, вторую, содержащую $2v_0$, но независимую отъ $2v_0'$, и третью, содержащую $2v_0'$ подъ знаками косинусовъ.

Соберемъ теперь члены 1-й и 2-й группы порознь, сумму-же членовъ 3-й обозначимъ просто черезъ $\Sigma A \cos(2v_0' + \alpha)$.

Соединяя члены 9-й и 10-й и оставляя безъ вниманія входящіе въ нихъ члены 3-й группы, мы получаемъ:

$$\left[\frac{\sin i.\sin i'}{2} - \sin i.\sin^2\frac{i}{2}\cdot\sin i'\right]\cos(\theta-\theta') - \left(\frac{\sin i.\sin i'}{2} - \sin i.\sin^2\frac{i}{2}\sin i'\right)$$

$$.\cos(2v_0+\theta-\theta') + \sin i.\sin^2\frac{i}{2}\cdot\sin i'.\cos(2v_0+\theta'-\theta)$$

Но $\frac{\sin i.\sin i'}{2} - \sin i.\sin^2\frac{i}{2}\cdot\sin i' = \frac{\sin i.\sin i'}{2}\left(1 - 2\sin^2\frac{i}{2}\right) =$

$= \frac{\sin i.\sin i'.\cos i}{2} = \frac{\sin.2i.\sin i'}{4} = \frac{ii'}{2} - \frac{1}{3}i^3 i'$, слѣд.

$$(9)+(10) = \left(\frac{ii'}{2} - \frac{1}{3}i^3 i'\right)\cos(\theta-\theta') - \left(\frac{ii'}{2} - \frac{1}{3}i^3 i'\right)\cos(2v_0+\theta-\theta')$$
$$+ \frac{i^3 i'}{4}\cos(2v_0 - \theta + \theta').$$

Итакъ мы имѣемъ

$$1 - 3\lambda^2 = -\left[\frac{1}{2} - \frac{3i^2}{4} - \frac{3i'^2}{4} + \frac{3i^4}{16} + \left(\frac{3}{16} + \frac{3}{16} + \frac{3}{4} = \frac{9}{8}\right)i^2\right.$$

$$\left. + \frac{3ii'}{2}\cos(\theta-\theta') - i^3 i'\cos(\theta-\theta') + \frac{3}{8}i^2 i'^2\cos 2(\theta-\theta')\right]$$

$$- \cos 2v_0\left(\frac{3i^2}{4} - \left(\frac{3}{16} + \frac{3}{16} + \frac{3}{4} = \frac{9}{8}\right)i^2 i'^2 - \frac{3i^4}{16}\right)$$

$$+ \frac{9}{8}i^2 i'^2\cos(2v_0+2\theta-2\theta') + 3\left(\frac{ii'}{2} - \frac{1}{3}i^3 i'\right)\cos(2v_0+\theta-\theta')$$

$$- \frac{3i^3 i'}{4}\cos(2v_0 - \theta + \theta') + \Sigma A\cos(2v_0' + \alpha).$$

8. Составивъ выраженіе $1 - 3\lambda^2$, мы немедленно можемъ приступить къ опредѣленію постоянной части пертурбаціонной функціи, т. е. суммѣ тѣхъ ея членовъ, въ которые не входитъ ни v_0, ни v_0'. При этомъ мы будемъ пока считать θ и ω, а также элементы, земной орбиты за величины постоянныя.

Когда постоянная часть пертурбаціонной функціи будетъ извѣстна, мы перейдемъ къ опредѣленію вѣковыхъ измѣненій элементовъ Луны по извѣстнымъ формуламъ теоріи измѣненія произвольныхъ постоянныхъ. При нахожденіи постоянной части пертурбаціонной функціи мы можемъ совершенно опустить членъ $\frac{m'}{r'}$, такъ-какъ отъ него возмущенія Луны не зависятъ.

Представимъ себѣ, что мы подставили въ выраженіе $\Omega = -\frac{m'r^2}{2r_1^3}$ $(1 - 3\lambda^2)$ вмѣсто r, r' и λ ихъ разложенія въ функціи среднихъ долготъ Луны и Солнца, $nt + \varepsilon$ и $n't + \varepsilon'$, и постоянныхъ величинъ. Мы получимъ такимъ образомъ, очевидно, выраженіе вида $M + \Sigma N$ $[\cos i(nt + \varepsilon) + i'(n't + \varepsilon') + \alpha]$, гдѣ M нѣкоторая функція, независящая отъ $n't + \varepsilon'$, N и α функціи элементовъ, а i и i' какія нибудь цѣлыя числа, отличныя отъ 0. Ясно, что интегрируя это выраженіе, умноженное на $d(n't)$, въ предѣлахъ отъ 0 до 2π, мы получимъ

$$\int_0^{2\pi} \Omega d(n't) = \int_0^{2\pi} M d(n't) = 2\pi M.$$

Чтобы получить часть пертурбаціонной функціи, независящую и отъ аргумента $nt + \varepsilon$, достаточно помножить M на $d(nt)$ и интегрировать произведеніе $M.\,d\,(nt)$ въ предѣлахъ отъ 0 до 2π. Пусть $M = \Omega_0 + \Sigma P \cos(i(nt + \varepsilon) + \beta)$. При интегрированіи въ указанныхъ предѣлахъ періодическая часть очевидно изчезаетъ, и мы получаемъ

$$\Omega_0 = \frac{1}{2\pi}\int_0^{2\pi} M\,.\,d\,(nt) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} \Omega\,.\,d\,(nt)\,.\,d\,(n't).$$

Замѣтимъ однако, что такъ-какъ выраженіе $1 - 3\lambda^2$ представлено у насъ въ функціи v_0 и v'_0, то мы можемъ скорѣе достигнуть цѣли путемъ замѣны въ предъидущемъ выраженіи дифференціаловъ $d(nt)$ и $d(n't)$ соотвѣтственно величинами $\frac{r^2 dv_0}{a^2\sqrt{1-e^2}}$ и $\frac{r'^2 dv_0'}{a'^2\sqrt{1-e'^2}}$, введя только, сообразно этой замѣнѣ, другіе предѣлы интегрированія, а именно: ω'_0 и $2\pi + \omega_0'$ при интегрированіи по dv_0', ω_0 и $2\pi + \omega_0$ при интегрированіи по dv_0, гдѣ ω_0' и ω_0 суть разстоянія перигеевъ отъ соотвѣтствующихъ узловъ. Это преобразованіе предѣловъ основано на извѣстной формулѣ:

$$v_0 - \omega_0 = nt + \varepsilon + \Sigma A_i e^i \sin(int + i\varepsilon - i\omega),$$

которая, mutatis mutandis, относится и къ Солнцу. Такимъ образомъ мы находимъ

$$M = \frac{1}{2\pi}\int_{\omega_0'}^{2\pi+\omega_0'} \Omega . \frac{r'^2\, dv_0'}{a'^2\sqrt{1-e'^2}} = -\frac{m'}{4\pi a'^2\sqrt{1-e'^2}}\int_{\omega_0'}^{2\pi+\omega_0'} \frac{r^2}{r'}(1-3\lambda^2)\, dv_0'$$

$$\Omega_0 = -\frac{m'}{8\pi^2 a^2 a'^2\sqrt{1-e^2}.\sqrt{1-e'^2}}\int_{\omega_0}^{2\pi+\omega_0}\int_{\omega_0'}^{2\pi+\omega_0'} \frac{r^4}{r'}(1-3\lambda^2)\, dv_0'\, dv_0 \ldots (13)$$

Будемъ сначала интегрировать по v_0'. Вмѣсто r' въ выраженіе $\frac{1-3\lambda^2}{r'}$ мы должны подставить эллиптическую величину радіуса вектора, т. е. $\frac{a'(1-e'^2)}{1+e'\cos(v_0'-\omega'_0)}$, но произведеніе $(1-3\lambda^2)\, e'\cos(v_0'-\omega_0')$ мы можемъ прямо отбросить, потому что, какъ видно изъ выраженія функціи $1-3\lambda^2$, всѣ члены, происходящіе отъ этого произведенія, послѣ интегрированія по v_0' совершенно изчезаютъ.

Итакъ мы имѣемъ

$$\int_{\omega_0'}^{2\pi+\omega_0'} \frac{(1-3\lambda^2)\, dv_0'}{r'} = \int_{\omega_0'}^{2\pi+\omega_0'} \frac{(1-3\lambda^2)\, dv_0'}{a'(1-e'^2)} =$$

$$-\frac{2\pi}{a'(1-e'^2)}\Big[\frac{1}{2} - \frac{3i^2}{4} - \frac{3i'^2}{4} + \frac{3i^4}{16} + \frac{9}{8}i^2 i'^2 + \frac{3}{2}ii'\cos(\theta-\theta')$$

$$- i^3 i'\cos(\theta-\theta') + \frac{3}{8}i^2 i'^2\cos 2(\theta-\theta') + \frac{3i^2}{4}\cos 2v_0\Big] \ldots (14)$$

Здѣсь мы удержали изъ членовъ 2-й группы въ выраженіи $1-3\lambda^2$ только 1-й членъ, такъ какъ остальные не дадутъ ничего замѣтнаго въ выраженіи Ω_0.

Теперь это выраженіе (13) нужно умножить на $r^4 dv_0$ и интегрировать по dv_0.

Обозначимъ 2-ую часть уравненій (14) для краткости черезъ $-\frac{2\pi}{a'(1-e'^2)}. K$.

Если представить себѣ r^4 разложеннымъ въ рядъ по косинусамъ кратныхъ дугъ истинной аномаліи, то изъ всѣхъ членовъ ряда

$$a_0 + a_1 \cos(v_0 - \omega_0) + a_2 \cos 2(v_0 - \omega_0) + a_3 \cos 3(v_0 - \omega_0)$$

только 1-й и 3-й члены дадутъ въ произведеніи Kr^4 члены независящіе отъ v_0 (ибо въ выраженіи K буква v_0 входитъ съ коэффиціентомъ 2); поэтому достаточно положить

$$r^4 = a^4(1-e^2)^4(m + n\cos 2(v_0 - \omega_0)), \text{ гдѣ } m \text{ и } n$$

неопредѣленные коэффиціенты.

Такъ-какъ $r^4 = \frac{a^4(1-e^2)^4}{[1+e\cos(v_0-\omega_0)]^4}$,

$$\frac{1}{[1+e\cos(v_0-\omega_0)]^4} = 1 - 4e\cos(v_0-\omega_0) + 10e^2\cos^2(v_0-\omega_0) + 20e^3\cos^3(v_0-\omega_0) + 35e^4\cos^4(v_0-\omega_0)$$

$$= 1 - 4e\cos(v_0-\omega_0) + 5e^2 + 5e^2\cos 2(v_0-\omega_0) - 5e^3\cos 3(v_0-\omega_0) - 15e^3\cos(v_0-\omega_0) + \frac{105}{8}e^4 + \frac{35}{2}e^4\cos 2(v_0-\omega_0),$$

то

$$m = 1 + 5e^2 + \frac{105}{8}e^4, \quad n = 5e^2 + \frac{35}{2}e^4, \text{ и слѣд.}$$

$$r^4 = a^4(1-e^2)^4\left\{1 + 5e^2 + \tfrac{105}{8}e^4 + (5e^2 + \tfrac{35}{2}e^4)\cos 2(v_0-\omega_0)\right\}.$$

Подставляя это выраженіе въ ур. (13), находимъ

$$\Omega_0 = \frac{m' . 2\pi\, a^4(1-e^2)^4}{8\pi^2 a^2 . a'^3\sqrt{1-e^2}\cdot(\sqrt{1-e'^2})^3}\int_{\omega_0}^{2\pi+\omega_0} K\left(1 + 5e^2 + \frac{105}{8}e^4 + \left(5e^2 + \frac{35}{2}e^4\right)\cos 2(v_0-\omega_0)\right)dv_0$$

$$= \frac{m' a^2(1-e^2)^{\frac{7}{2}}}{2a'^3(1-e'^2)^{\frac{3}{2}}}\cdot K_0 \cdot Y \cdots (15), \text{ гдѣ } Y = 1 + 5e^2 + \frac{105}{8}e^4 + \frac{15}{8}i^2 \cdot\left(e^2 + \frac{7}{2}e^4\right)\cos 2\omega_0, \quad K_0 = K - \frac{3i^2}{4}\cos 2v_0.$$

Коэффиціентъ

$$\frac{(1-e^2)^{\frac{7}{2}}}{(1-e'^2)^{\frac{3}{2}}}=\left(1-\frac{7}{2}e^2+\frac{35}{8}e^4-\cdots\right)\left(1+\frac{3}{2}e'^2+\frac{15}{8}e'^4+\cdots\right)$$

$$=1-\frac{7}{2}e^2+\frac{35}{8}e^4+\frac{3}{2}e'^2+\frac{15}{8}e'^4-\frac{21}{4}e^2e'^2+\frac{105}{16}e'^2e^4-\frac{105}{16}e^2e'^4=S.$$

Составимъ теперь произведеніе YK_0, вѣрное до членовъ 6-й степени включительно.

Мы имѣемъ

$$\left(1+5e^2+\frac{105}{8}e^4+\frac{15}{8}i^2\left(e^2+\frac{7}{2}e^4\right)\cos 2\omega_0\right)\left[\frac{1}{2}-\frac{3i^2}{4}-\frac{3i'^2}{4}+\frac{3i^4}{16}\right.$$

$$\left.+\frac{9}{8}i^2i'^2+\frac{3}{2}ii'\cos(\theta-\theta')-i^3i'\cos(\theta-\theta')+\frac{3}{8}i^2i'^2\cos 2(\theta-\theta')\right]$$

$$=\frac{1}{2}-\frac{3}{4}i^2+\frac{3}{16}i^4-\frac{3}{4}i'^2+\frac{9}{8}i^2i'^2+\frac{3}{2}ii'.\cos(\theta-\theta')-i^3i'\cos(\theta-\theta')$$

$$+\frac{5}{2}e^2-\frac{15}{4}e^2i^2+\frac{15}{16}e^2i^4-\frac{15}{4}e^2i'^2+\frac{45}{8}e^2i^2i'^2+\frac{105}{16}e^4-\frac{315}{32}e^4i^2$$

$$-\frac{315}{32}e^4i'^2+\frac{15}{2}e^2.ii'.\cos(\theta-\theta')+\frac{15}{16}i^2e^2\cos 2\omega_0+\frac{3}{8}i^2i'^2\cos 2(\theta-\theta').$$

Произведеніе $\frac{1}{2}YK_0S$ равно

$$\frac{1}{4}\left[1-\frac{3}{2}i^2-\frac{3}{2}i'^2+\frac{3}{2}e^2+\frac{3}{2}e'^2+\frac{3}{8}i^4+\frac{9}{4}e^2e'^2\right.$$

$$+\frac{9}{4}i^2i'^2-\frac{9}{4}e^2i^2-\frac{9}{4}e'^2i^2-\frac{9}{4}e^2i'^2-\frac{9}{4}e'^2i'^2+\frac{15}{8}e'^4$$

$$+\left(\frac{105}{8}-\frac{105}{8}=0\right)e^4+\frac{9}{16}i^4e^2+\frac{9}{16}i^4e'^2-\frac{27}{8}e^2e'^2i^2$$

$$-\frac{27}{8}e^2e'^2i'^2+\frac{27}{8}e^2i^2i'^2+\frac{27}{8}e'^2i^2i'^2-\frac{45}{16}i^2e'^4+\frac{45}{16}e^2e'^4$$

$$-\left(\frac{315}{32}+\frac{105}{32}-\frac{105}{8}=0\right)e^4i^2-\left(\frac{315}{32}+\frac{105}{32}-\frac{105}{8}=0\right)e^4i'^2$$

$$+\left(\frac{315}{32}+\frac{105}{32}-\frac{105}{8}=0\right)e^4e'^2+3ii'.\cos(\theta-\theta')+\left(15-\frac{21}{2}=\frac{9}{2}\right)e^2ii'\cos(\theta-\theta')$$

$$\left.+\frac{15}{4}e^2i^2\cos 2\omega_0+\frac{3}{16}i^2i'^2\cos 2(\theta-\theta')-2i^3i'\cos(\theta-\theta')\right]\ldots(16_1)$$

Чтобы получить Ω_0, это выраженіе нужно еще помножить, согласно ур. (15), на $\frac{m'a^2}{a'^3}$ или на $\frac{m^2\mu}{a}$, такъ-какъ $\frac{m'a^2}{a'^3}=\frac{a'^3.n'^2.a^2.n^2}{a'^3.n^2}$ $=\frac{m^2\ a^3n^2}{a}=\frac{m'\mu}{a}$, гдѣ мы обозначили черезъ m отношеніе среднихъ скоростей годоваго движенія Солнца и Луны; величина m приблизительно равна $\frac{1}{13}$.

Итакъ

$$\Omega_0=\frac{1}{2}\cdot\frac{m^2\mu}{a}\cdot YK_0S\ldots\ldots\ldots\ldots\ldots(16_2)$$

Замѣтимъ, что предпослѣдній членъ предъидущаго выраженія можетъ быть совершенно опущенъ, безъ ущерба для точности, членъ-же $\frac{15}{4}\ e^2 i^2.\ \cos 2\omega_0$ легко представить въ другой формѣ. Если обозначимъ черезъ $\tilde{\omega}$ долготу перигея Луны, считаемую по неподвижной эклиптикѣ, то очевидно получимъ

$$\operatorname{tg}(\tilde{\omega}-\theta)=\cos i.\operatorname{tg}\omega_0\,,$$

откуда

$$\omega_0=\tilde{\omega}-\theta+\operatorname{tg}^2\frac{i}{2}\cdot\sin 2\omega_0-\cdots,\text{слѣд.}\ \frac{15}{4}e^2i^2\cos 2\omega_0=\frac{15}{4}e^2i^2\cos(2\tilde{\omega}-2\theta)+\ldots$$

ГЛАВА III.

9. Для опредѣленія вѣковыхъ измѣненій элементовъ Луны, производимыхъ возмущающею силою Солнца, мы воспользуемся для перваго приближенія извѣстными формулами, дающими производныя элементовъ по времени въ зависимости отъ частныхъ производныхъ пертурбаціонной функціи.

Мы имѣемъ

$$\left.\begin{aligned}
\frac{da}{dt} &= \frac{2na^2}{\mu}\cdot\frac{d\Omega_0}{d\varepsilon} \\
\frac{d\varepsilon}{dt} &= -\frac{2a^2n}{\mu}\cdot\frac{d\Omega_0}{da} - \frac{an\sqrt{1-e^2}}{\mu e}\left[\sqrt{1-e^2}-1\right]\frac{d\Omega_0}{de} + \frac{an}{\sqrt{1-e^2}}\cdot\operatorname{tg}\frac{i}{2}\cdot\frac{d\Omega_0}{di} \\
\frac{de}{dt} &= -\frac{na}{e}\cdot\frac{\sqrt{1-e^2}}{\mu}\cdot\frac{d\Omega_0}{d\tilde{\omega}} + \frac{na}{\mu e}(1-e^2)\frac{d\Omega_0}{d\varepsilon} \\
\frac{d\tilde{\omega}}{dt} &= \frac{an\sqrt{1-e^2}}{\mu e}\cdot\frac{d\Omega_0}{de} + \frac{an}{\mu\sqrt{1-e^2}}\cdot\operatorname{tg}\frac{i}{2}\cdot\frac{d\Omega_0}{di} \\
\frac{d\theta}{dt} &= \frac{an}{\mu\sqrt{1-e^2}.\sin i}\cdot\frac{d\Omega_0}{di} \\
\frac{di}{dt} &= \frac{an}{\mu\sqrt{1-e^2}}\operatorname{ctg} i\,\frac{d\Omega_0}{d\tilde{\omega}} - \frac{an}{\mu\sqrt{1-e^2}}\operatorname{cos\acute{e}c} i\cdot\frac{d\Omega_0}{d\theta}
\end{aligned}\right\}\quad(17)$$

Здѣсь $\mu = 1 + M$, ε ср. долгота эпохи, $\tilde{\omega}$ — долгота перигея. Прежде всего найдемъ варьяціи элементовъ θ и i.

Дифференцируя функцію Ω_0 по i, мы имѣемъ

$$\frac{d\Omega_0}{di}=\frac{3m^2\mu.i}{4a}\Big\{-1-\frac{3}{2}e^2-\frac{3}{2}e'^2-\frac{9}{4}e^2e'^2+\frac{1}{2}i^2+\frac{3}{2}i'^2-\frac{15}{8}e'^4+\frac{3}{4}i'^2.e'^2$$
$$-2ii'\cos(\theta-\theta')+\frac{ii'.\cos(\theta-\theta')}{i^2}\Big\}$$

Такъ-какъ $\frac{d\Omega_0}{\sin i.di}=\frac{d\Omega_0}{idi}(1+\frac{i^2}{6}+\ldots)$, то ограничиваясь пока только первымъ и послѣднимъ членами предъидущаго выраженія для $\frac{d\Omega_0}{di}$, мы получаемъ

$$\frac{d\theta}{dt}=-\frac{3m^2n}{4}\left(1-\frac{ii'\cos(\theta-\theta')}{i^2}\right)\ldots\ldots\ldots \quad (18)$$

Далѣе находимъ, пренебрегая членомъ, происходящимъ изъ $\frac{d\Omega_0}{d\varpi}$, и ограничиваясь главными членами въ $\frac{d\Omega_0}{d\theta}$,

$$\frac{di}{dt}=-\frac{an}{\mu\sqrt{1-e^2}}\cdot\cos \acute{e}ci\cdot\frac{d\Omega_0}{d\theta}=\frac{3}{4}\frac{m^2n\,\mathrm{cos\acute{e}c}\,i}{\sqrt{1-e^2}}\cdot ii'\cdot\sin(\theta-\theta')$$
$$=\frac{3}{4}m^2n\,.\,i'\,.\sin(\theta-\theta')\ldots\ldots\ldots\ldots \quad (19)$$

Введемъ теперь двѣ новыя перемѣнныя p и q, связанныя съ θ и i уравненіями

$$p=i.\sin\theta,\qquad q=i.\cos\theta$$

и продифференцировавъ эти выраженія, подставимъ въ уравненіе

$$\frac{dp}{dt}=\frac{di}{dt}\cdot\sin\theta+i\cdot\cos\theta\cdot\frac{d\theta}{dt}\ \text{и}\ \frac{dq}{dt}=\frac{di}{dt}\cdot\cos\theta-i\cdot\sin\theta\cdot\frac{d\theta}{dt}$$

сейчасъ найденныя приблизительныя величины производныхъ $\frac{di}{dt}$ и $\frac{d\theta}{dt}$. мы находимъ

$$\frac{dp}{dt}=\frac{3}{4}m^2n\cdot i'\cdot\sin(\theta-\theta')\cdot\sin\theta-\frac{3}{4}m^2ni\cdot\cos\theta+\frac{3}{4}m^2n\cdot i'\cdot\cos\theta\cdot\cos(\theta-\theta')$$

$$\frac{dq}{dt}=\frac{3}{4}m^2n\cdot i'\cdot\sin(\theta-\theta')\cdot\cos\theta+\frac{3}{4}m^2n\cdot i\cdot\sin\theta-\frac{3}{4}m^2n\cdot i'\cdot\sin\theta\cdot\cos(\theta-\theta')$$

или $\frac{dp}{dt}=\frac{3}{4}m^2ni'\cdot\cos\theta'-\frac{3}{4}m^2ni\cdot\cos\theta$

$$\frac{dq}{dt}=-\frac{3}{4}m^2n\cdot i'\cdot\sin\theta'+\frac{3}{4}m^2n\cdot i\cdot\sin\theta.$$

Обозначая для краткости $i'\cos\theta'$ и $i'\sin\theta'$ соотвѣтственно черезъ p' и q', а $\frac{3}{4}m^2n$ черезъ α, получаемъ слѣдующія два совокупныя ур. для опредѣленія p и q:

$$\frac{dp}{dt}+\alpha q=\alpha p' \text{ и } \frac{dq}{dt}-\alpha p=-\alpha q' \ldots\ldots\ (20)$$

Мы рѣшимъ эти ур. по способу Д'Аламбера.

Помножимъ 2-е ур. на x и сложимъ съ 1-мъ:

$$\frac{dp}{dt}+x\cdot\frac{dq}{dt}+\alpha(q-px)=\alpha(p'-q'x).$$

Если положимъ $z=p+qx$, откуда $p=z-qx$, $px=zx-x^2q$, $\frac{dp}{dt}+x\cdot\frac{dq}{dt}=\frac{dz}{dt}-q\cdot\frac{dx}{dt}$, то получится

$$\frac{dz}{dt}-q\cdot\frac{dx}{dt}+\alpha q-\alpha zx+\alpha x^2q=\alpha(p'-q'x).$$

Приравнивая нулю коэффиціентъ при q, находимъ

$$\frac{dx}{dt}-\alpha(1+x^2)=0 \quad \text{и} \quad \frac{dz}{dt}-\alpha xz=\alpha(p'-q'x)$$

Послѣднее ур. даетъ

$$z=e^{\alpha\int xdt}\left\{C+\alpha\int(p'-q'x)\,.\,e^{-\alpha\int xdt}dt\right\}$$

Ур. $\frac{dx}{dt}-\alpha(1+x^2)=0$ очевидно удовлетворяется величинами $x_1=+\sqrt{-1}$ и $x_2=-\sqrt{-1}$, слѣдовательно

$$z_1=e^{\alpha t\sqrt{-1}}\left\{C_1+\alpha\int(p'-q'\sqrt{-1})e^{-\alpha t\sqrt{-1}}dt\right\}$$

$$z_{11}=e^{-\alpha t\sqrt{-1}}\left\{C_{11}+\alpha\int(p'+q'\sqrt{-1})e^{+\alpha t\sqrt{-1}}dt\right\}$$

Замѣняя здѣсь $e^{\pm\alpha t\sqrt{-1}}$ черезъ $\cos(\alpha t)\pm\sqrt{-1}\,.\,\sin(\alpha t)$ мы получаемъ:

$$z_1=(\cos(\alpha t)+\sqrt{-1}\,.\sin(\alpha t))\left\{C_1+\alpha\int(p'-q'\sqrt{-1})(\cos(\alpha t)-\sqrt{-1}\sin(\alpha t))\right\}$$

$$z_{11} = (\cos(\alpha t) - \sqrt{-1}\,.\sin(\alpha t)).\{C_{11} + \alpha\int(p' + q'\sqrt{-1})(\cos(\alpha t) + \sqrt{-1}\sin(\alpha t))$$

откуда

$$z_1 = C_1(\cos(\alpha t) + \sqrt{-1}\,.\ \sin(\alpha t)) + \alpha\ \{\cos(\alpha t)\int p'\cos(\alpha t).\ dt - \cos(\alpha t)\int q'.\sin(\alpha t).dt\}$$

$$+ \alpha\sin(\alpha t)\{\int p'.\sin(\alpha t)dt + \int q'.\cos(\alpha i).dt\}$$

$$- \alpha\sqrt{-1}\,.\cos(\alpha t)\{\int p'.\sin(\alpha t)dt + \int q'\cos(\alpha t).dt\}$$

$$+ \alpha\sqrt{-1}\,.\sin(\alpha t)\{\int p'.\cos(\alpha t)\ dt - \int q'.\sin(\alpha t).dt\}$$

или

$$z_1 = C_1(\cos\alpha t) + \sqrt{-1}\,.\sin(\alpha t)) +$$

$$+ \alpha\{\cos(\alpha t)\int[p'\cos(\alpha t) - q'\sin(\alpha t)]\,dt + \sin(\alpha t)\int(p'\sin(\alpha t) + q'\cos(\alpha t))\,dt\}$$

$$-\alpha\sqrt{-1}\{\cos(\alpha t)\int[p'\sin(\alpha t) + q'\cos(\alpha t)]dt - \sin(\alpha t)\int p'\cos(\alpha t) - q'.\sin(\alpha t))\,dt\}$$

Точно также получимъ

$$z_{11} = C_{11}(\cos(\alpha t)\,1 - \sqrt{-1}\,.\sin(\alpha t) +$$

$$+ \alpha\ \{\ \cos(\alpha t)\int[p'\cos(\alpha t) - q'\sin(\alpha t']\,dt + \sin(\alpha t)\int[p'\sin(\alpha t) + q'\cos(\alpha t)\,]\,dt\ \}$$

$$+ \alpha\sqrt{-1}\ \{\cos(\alpha t)\int[p'\sin(\alpha t) + q'\sin(\alpha t']dt - \sin(\alpha t)\int(p'\cos(\alpha t) - q'\sin(\alpha t)).\ dt\ \}$$

Вслѣдствіе ур. $z = p + qx$ мы находимъ

$$z_1 = p + q\sqrt{-1}\ ,\ z_{11} = p - q\sqrt{-1}\ ,\ \text{слѣд.}$$

$$p = \frac{1}{2}(z_1 + z_{11})\ ,\ q = \frac{z_1 - z_{11}}{2\sqrt{-1}}\ ,\ \text{и потому}$$

$$p = \frac{1}{2}\cos(\alpha t)\,[C_1 + C_{11}] + \frac{(C_1 - C_{11})\sqrt{-1}}{2}.\ \sin(\alpha t) +$$

$$+\alpha\{\cos(\alpha t)\int(p'\cos(\alpha t)-q'\sin(\alpha t))\,dt+\sin(\alpha t)\int(p'\sin(\alpha t)+q'.\cos(\alpha t))\,dt$$

$$q=\frac{C_1-C_{11}}{2\sqrt{-1}}\cos(\alpha t)+\frac{C_1+C_{11}}{2}\sin(\alpha t)-$$

$$-\alpha\{\cos(\alpha t)\int(p'\sin(\alpha t)+q'\cos(\alpha t))\,dt-\sin(\alpha t)\int(p'\cos(\alpha t)-q'.\sin(\alpha t))\,dt$$

Положимъ

$$\frac{C_1-C_{11}}{2\sqrt{-1}}=i_1.\cos\theta_1\ ,\ \frac{C_1+C_{11}}{2}=i_1.\sin\theta_1\ ,$$

тогда получимъ:

$$p=\cos(\alpha t).i_1.\sin\theta_1-\sin(\alpha t).i_1.\cos\theta_1+$$

$$+\alpha\{\cos(\alpha t).S+\sin(\alpha t)S'.\}$$

$$q=\cos(\alpha t).i_1.\cos\theta_1+\sin(\alpha t).i_1.\sin\theta_1$$

$-\alpha\{\cos(\alpha t)S'-\sin(\alpha t).S\}$, гдѣ мы обозначили

черезъ S и S' интегралы $\int(p'\cos(\alpha t)-q'\sin(\alpha t))\,dt$

и

$$\int(p'\sin(\alpha t)+q'.\cos(\alpha t))\,dt.$$

Изъ теоріи движенія Земли извѣстно, что величины p' и q', равныя соотвѣтственно $i'\cos\theta'$ и $i'\sin\theta'$, могутъ быть представлены въ видѣ $\Sigma N\cos(kt+\beta)$ и $\Sigma N\sin(kt+\beta)$, гдѣ коэффиціенты N и k суть величины порядка i', слѣд.

$$S=\int dt\,(p'\cos(\alpha t)-q'.\sin(\alpha t))=\int dt\,\Sigma N\cos(kt+\alpha t+\beta)$$
$$=\frac{\Sigma N\sin(kt+\alpha t+\beta)}{\alpha+k}$$

$$S'=\int dt\,(p'\sin(\alpha t)+q'.\cos(\alpha t))=\int dt.\Sigma N\sin(kt+\alpha t+\beta)$$
$$=-\frac{\Sigma N\cos(kt+\alpha t+\beta)}{\alpha+k}$$

Но величиною k можно смѣло пренебречь въ дѣлителѣ по сравненію съ α. Въ самомъ дѣлѣ наибольшій изъ коэффиціентовъ k не превышаетъ $50''.1$ (См. напр. Schubert, Astr. Théor. III, p. 510), между тѣмъ какъ $\alpha = \frac{3}{4} m^2 n =$ приблизительно $971982''$, если принимать за единицу времени Юліанскій годъ.

Итакъ мы получаемъ:

$$p = i_1 \sin(\theta_1 - \alpha t) + \cos(\alpha t) . \Sigma N \sin(kt + \alpha t + \beta)$$

$$- \sin(\alpha t) . \Sigma N \cos(kt + \alpha t + \beta) = i_1 \sin(\theta_1 - \alpha t) + \Sigma N \sin(kt + \beta)$$

$$q = i_1 . \cos(\theta_1 - \alpha t) + \cos(\alpha t) \Sigma N \cos(kt + \alpha t + \beta) +$$

$$+ \sin(\alpha t) . \Sigma N \sin(kt + \alpha t + \beta) = i_1 \cos(\theta_1 - \alpha t) + \Sigma N \cos(kt + \beta)$$

или

$$\left.\begin{aligned} i \sin\theta &= i_1 . \sin(\theta_1 - \tfrac{3}{4} m^2 nt) + i' . \sin\theta' \\ i \cos\theta &= i_1 . \cos(\theta_1 - \tfrac{3}{4} m^2 nt) + i' . \cos\theta' \end{aligned}\right\} \ldots\ldots\ldots (21)$$

Отсюда легко находимъ

$$\left.\begin{aligned} i^2 &= i_1^2 + 2 i_1 . i' . \cos(\theta_1 - \alpha t - \theta') + i'^2 \\ ii' . \cos(\theta - \theta') &= i'^2 + i_1 . i' . \cos(\theta_1 - \alpha t - \theta') \end{aligned}\right\} \ldots\ldots\ldots (22)$$

10. Пусть J уголъ, образуемый плоскостями лунной орбиты и истинной эклиптики для даннаго момента времени. Мы имѣемъ:

$$\cos J = \cos i' . \cos i + \sin i . \sin i' . \cos(\theta - \theta') \quad \text{или}$$

$\cos J = 1 - \frac{i^2}{2} - \frac{i'^2}{2} + ii' \cos(\theta - \theta')$, если пренебречь 3-й степенью i. Подставляя сюда вмѣсто $ii' \cos(\theta - \theta')$ и $\frac{i^2}{2}$ выраженія этихъ членовъ по ур. (22), находимъ

$$\cos J = 1 + i'^2 + i_1 . i' . \cos(\theta_1 - \alpha t - \theta') - \frac{i_1^2}{2} - \frac{i'^2}{2} - i_1 i' \cos(\theta_1 - \alpha t - \theta') - \frac{i'^2}{2}$$

$$= 1 - \frac{i'^2}{2} = \cos i_1, \text{ или } J = i_1.$$

Итакъ съ точностью до величинъ 3-го порядка относительно i можно считать J или наклонность лунной орбиты относительно истинной эклиптики величиною постоянною.

Мы докажемъ теперь, что и долгота и широта Луны, измѣряемыя относительно истинной эклиптики, независятъ отъ вѣковаго измѣненія наклонности этой плоскости.

Обозначимъ черезъ v_1 и β_1 соотвѣтственно истинныя долготу и широту Луны и представимъ себѣ сферическій треугольникъ, вершины котораго E, E_1, и M суть полюсы 3-хъ большихъ круговъ: неподвижной эклиптики, истинной эклиптики и орбиты ☾. Углы E и E_1 въ этомъ треугольникѣ равны соотвѣтственно

$$\frac{\pi}{2} + v - \theta' \text{ и } \frac{\pi}{2} - (v_1 - \theta');$$

а стороны: $EE_1 = i'$, $EM = 90° - \beta$, $E_1 M = 90 - \beta_1$.

Мы имѣемъ: $\sin\beta_1 = \cos i' . \sin\beta - \sin i' . \cos\beta . \sin(v - \theta')$...(a)

$$\cos\beta_1 \ \sin(v_1 - \theta') = \sin i' . \sin\beta + \cos\beta . \cos i' . \sin(v - \theta')$$

$$\cos(v_1 - \theta') . \cos\beta_1 = \cos(v - \theta') . \cos\beta.$$

Дѣля второе ур. на третье, находимъ

$$\operatorname{tg}(v_1 - \theta') = \frac{\sin i' . \sin\beta + \cos\beta . \cos i' . \sin(v - \theta')}{\cos(v - \theta') . \cos\beta}$$

или, обозначая по прежнему черезъ s тангенсъ широты Луны относительно неподвижной эклиптики,

$$\operatorname{tg}(v_1 - \theta') = \frac{s . \sin i' + \cos i' . \sin(v - \theta')}{\cos(v - \theta')}$$

Замѣняемъ здѣсь $\cos i'$ черезъ $1 - 2\sin^2\frac{i'}{2}$:

$$\operatorname{tg}(v_1 - \theta') = \frac{s . \sin i'}{\cos(v - \theta')} + \operatorname{tg}(v - \theta') - 2\sin^2\frac{i'}{2} \cdot \operatorname{tg}(v - \theta')$$

или

$$\operatorname{tg}(v_1 - \theta') - \operatorname{tg}(v - \theta') = \frac{s . \sin i'}{\cos(v - \theta')} - 2\sin^2\frac{i'}{2} \cdot \operatorname{tg}(v - \theta')$$

Пользуясь формулой $\operatorname{tg}\alpha - \operatorname{tg}\beta = \frac{\sin(\alpha - \beta)}{\cos\alpha . \cos\beta}$, находимъ отсюда:

$$\sin(v_1 - v) = \cos(v_1 - \theta') . \cos(v - \theta')\left\{\frac{s . \sin i'}{\cos(v - \theta')} - 2\sin^2\frac{i'}{2} tg(v - \theta')\right\}$$

$$= s . \sin i' . \cos(v_1 - \theta') - 2\sin^2\frac{i'}{2} \cdot \sin(v - \theta') . \cos v_1 - \theta')$$

По незначительности дуги $v_1 - v$ можно принять:

$$v_1 = v + s . \sin i' . \cos (v - \theta') - \sin^2 \frac{i'}{2} \cdot \sin (2v - 2\theta')$$

или

$$v_1 = v + s . i' . \cos (v - \theta') - \frac{i'^2}{4} \cdot \sin (2v - 2\theta').$$

Но такъ какъ

$$i' \sin \theta' = \Sigma N . \sin (kt + \beta)$$

$$i' . \cos \theta' = \Sigma N . \cos (kt + \beta),$$

то

$$i' . \cos (v - \theta') = \Sigma N \cos (v - kt - \beta).$$

Съ другой стороны

$$s = \operatorname{tg} i . \sin (v - \theta) =$$

$$\sin v . i . \cos \theta - \cos v . i . \sin \theta$$

Подставляя сюда вмѣсто $i \cos \theta$ и $i \sin \theta$ ихъ величины изъ ур. (21), находимъ:

$$s = \sin v (i_1 . (\cos (\theta_1 - \alpha t) + i' . \cos \theta') - \cos v (i_1 . \sin (\theta_1 - \alpha t) + i' \sin \theta),$$

$$= i_1 \sin (v - \theta_1 + \alpha t) + \Sigma N \sin (v - kt - \beta), \text{ слѣд.}$$

$$s . i' . \cos (v - \theta') = [i_1 \sin (v - \theta_1 + \alpha t) + \Sigma N \sin (v - kt - \beta] \Sigma N \cos (v - kt - \beta)$$

$$= -\frac{1}{2} i_1 \Sigma N \sin (\theta_1 - \alpha t - kt - \beta) + \frac{1}{2} i_1 \Sigma N . \sin (2v - \theta_1 + \alpha t - kt - \beta)$$

и потому, пренебрегая періодич. членами

$$\frac{1}{2} i_1 \Sigma N \sin (2v - \theta_1 + \alpha t - kt - \beta) \text{ и } -\frac{i'^2}{4} \sin (2v - 2\theta'),$$

имѣемъ:

$$v_1 = v - \frac{1}{2} i_1 \Sigma N \sin (\theta_1 - \alpha t - kt - \beta).$$

Вслѣдствіе незначительности коэффиціентовъ $i_1 N$ неравенство большаго періода, выражаемое совокупностью членовъ

$$-\tfrac{1}{2} i_1 \Sigma N \sin (\theta_1 - \alpha t - kt - \beta)$$

оказывается совершенно нечувствительнымъ.

Мы скоро увидимъ, что и это неравенство изчезаетъ въ выраженіи средней долготы Луны, выраженной въ функціи долготы ея, измѣряемой по истинной эклиптикѣ.

Возьмемъ теперь ур. (a) и обозначивъ $\operatorname{tg}\beta_1$ черезъ s_1, напишемъ его въ видѣ:

$$\sin\beta_1 = \frac{s_1}{\sqrt{1+s_1^2}} = \cos\beta\left\{\cos i'.s - \sin i'.\sin(v-\theta')\right\} = \frac{s.\cos i' - \sin i'.\sin(v-\theta')}{\sqrt{1+s^2}}$$

или

$$\frac{s_1}{\sqrt{1+s_1^2}} = \frac{s - \sin v.\cos\theta'.\sin i' + \cos v.\sin\theta'.\sin i'}{\sqrt{1+s^2}}$$

Но такъ какъ

$$\sin i'.\cos\theta' = \Sigma N\cos(kt+\beta)$$

$$\sin i'.\sin\theta' = \Sigma N\sin(kt+\beta),$$

а выше мы нашли

$$s = i_1.\sin(v-\theta_1+\alpha t) + \Sigma N\sin(v-kt-\beta),$$

то получается:

$$\frac{s_1}{\sqrt{1+s_1^2}} = \frac{i_1\sin(v-\theta_1+\alpha t)+\Sigma N\sin(v-kt-\beta)-\sin v.\Sigma N\cos(kt+\beta)+\cos v\,\Sigma N\sin(kt+\beta)}{\sqrt{1+s^2}}$$

или

$$\frac{s_1}{\sqrt{1+s_1^2}} = \frac{i_1\sin(v-\theta_1+\alpha t)}{\sqrt{1+s^2}} = i_1\sin(v-\theta_1+\alpha t) - \frac{1}{2}i_1.\sin(v-\theta_1+\alpha t).s^2$$

$$= i_1\sin(v-\theta_1+\alpha t) - \frac{1}{2}i_1.\sin(v-\theta_1+\alpha t).2i_1.\sin(v-\theta_1+\alpha t).\Sigma N.$$
$$\sin(v-kt-\beta)+\ldots$$

$$= i_1\sin(v-\theta_1+\alpha t) - \frac{i_1^2}{2}\Sigma N\sin(v-kt-\beta)\ldots$$

Отбрасывая совершенно нечувствительные періодическіе члены съ коэффиціентами $\frac{i_1^2N}{2}$, мы получаемъ просто: $s_1 = i_1\sin(v-\theta_1+\alpha t)$, откуда и заключаемъ, что широта Луны относительно истинной эклиптики не зависитъ отъ перемѣнной величины i'.

Такимъ образомъ можно считать доказаннымъ съ вполнѣ достаточною степенью точности, что Луна участвуетъ въ вѣковомъ перемѣщеніи плоскости земной орбиты. Положеніе истинной эклиптики измѣняется непрерывно по отношеніи къ неподвижной и, какъ извѣстно изъ теоріи Земли, средняя наклонность этихъ 2-хъ плоскостей увеличивается въ одинъ Юліанскій годъ на 0."47929, но возмущающее дѣйствіе Солнца постоянно приводитъ орбиту Луны къ одной и той-же средней наклонности относительно плоскости истинной эклиптики для той-же эпохи.

Это постоянство взаимной наклонности лунной орбиты и эклиптики давно уже замѣчено изъ наблюденій и представляетъ собою одно изъ интереснѣйшихъ явленій въ теоріи вѣковыхъ неравенствъ Луны. Въ теоріи планетъ мы встрѣчаемся съ аналогичнымъ явленіемъ, которое заключается въ неизмѣняемости средней наклонности плоскостей орбитъ двухъ взаимно-возмущаемыхъ планетъ, но причины этихъ явленій совершенно различны.

Постоянство угла, образуемаго плоскостями орбитъ двухъ планетъ, возмущающихъ одна другую, есть прямой результатъ ихъ взаимодѣйствія по закону всемірнаго тяготѣнія, въ движеніи-же Луны неизмѣняемость средней наклонности орбиты Луны относительно плоскости истинной эклиптики обусловливается не непосредственными возмущеніями, оказываемыми Солнцемъ на Луну, а дѣйствіемъ планетъ на Землю и ея реакціей на своего спутника. Прямое дѣйствіе Солнца здѣсь не играетъ никакой роли. При этомъ нужно замѣтить однако, что, какъ доказалъ Гансенъ, орбита Луны для даннаго времени сохраняетъ постоянную наклонность не относительно эклиптики по которой въ это время движется Земля, но относительно той плоскости, съ которой совпадала орбита Земли около 3-хъ лѣтъ тому назадъ. Этотъ феноменъ «запаздыванія передачи дѣйствія» встрѣчаетъ себѣ также нѣкоторую аналогію въ явленіяхъ возмущеній моря, а именно въ несовпаденіи момента кульминаціи Луны съ временемъ полной воды въ данномъ мѣстѣ прибрежья Океана.

11. Мы выведемъ теперь болѣе подробныя выраженія долготы восходящаго узла и наклонности лунной орбиты относительно неподвижной эклиптики.

Возьмемъ уравненіе

$$\frac{di}{dt} = \frac{3}{4} m^2 n . i' . \sin(\theta - \theta') \dots\dots\dots\dots (a)$$

$$\frac{d\theta}{dt} = -\frac{3}{4}\frac{m^2 n}{\sqrt{1-e^2}} + \frac{3}{4} m^2 n . \frac{\cos(\theta-\theta')}{i} \dots\dots\dots (в)$$

Пренебрегая членомъ $\frac{3}{4} m^2 n \frac{i'.\cos(\theta-\theta')}{i}$ и квадратомъ эксцентрицитета, мы находимъ изъ 2-го ур. по интегрированіи: $\theta = \theta_1 - \frac{3}{4} m^2 nt$.

Итакъ мы видимъ, что долгота узла постоянно уменьшается, или другими словами, узлы лунной орбиты отступаютъ по эклиптикѣ со среднею скоростью $\frac{3}{4} m^2 n$.

Подставляя во 2-ую часть уравненія (*b*) вмѣсто θ приблизительную величину $\theta_1 - \frac{3m^2 n}{4} t$, а вмѣсто i постоянную i_1, и обозначая $\frac{3m^2 n}{4}$ по прежнему черезъ α, мы получаемъ

$$d\theta = -\frac{\alpha dt}{\sqrt{1-e^2}} + \frac{\alpha . i'}{i_1} \cos(\theta_1 - \alpha t - \theta') . dt$$

$$= -\alpha dt + \frac{\alpha i'}{i_1}[\cos\theta' . \cos(\theta_1 - \alpha t) + \sin\theta' . \sin(\theta_1 - \alpha t)]\, dt$$

Изъ теоріи Земли извѣстно, что производныя $\frac{d(i'\sin\theta')}{dt}$ и $\frac{d(i'\cos\theta')}{dt}$ могутъ быть представлены въ видѣ:

$$\frac{d(i'\sin\theta')}{dt} = a_1 + 2a_2 t \quad \text{и} \quad \frac{d(i'\cos\theta')}{dt} = b_1 + 2b_2 t$$

гдѣ коэффиціенты a_2 и b_2 настолько незначительны, что ими можно вполнѣ пренебречь. Въ IV томѣ Annales de l'observatoire de Paris Леверрье даетъ слѣдующія величины для коэффиціентовъ a_1, b_1, a_2 и b_2

$$a_1 = 0''.05888 \qquad a_2 = 0''.00001964$$

$$b_1 = -0''.47566 \qquad b_2 = 0''.00000568$$

Итакъ, полагая a_2 и b_2 равными 0 и замѣняя a_1 и b_1 новыми перемѣнными i'' и θ'', введенными посредствомъ уравненій $i''\sin\theta'' = a_1$ и $i''\cos\theta'' = b_1$, мы имѣемъ по интегрированіи:

$$\theta=\theta_1-\alpha t-\frac{i'}{i_1}\cdot\cos\theta'.\sin(\theta_1-\alpha t)+\frac{d(i'.\cos\theta')}{dt}\cdot\frac{\cos(\theta_1-\alpha t)}{\alpha i_1}$$

$$+\frac{i'}{i_1}\cdot\sin\theta'.\sin(\theta_1-\alpha t)+\frac{d(i'.\sin\theta')}{dt}\cdot\frac{\sin(\theta_1-\alpha t)}{\alpha i_1}$$

или

$$\theta=\theta_1-\alpha t-\frac{i'}{i_1}\sin(\theta_1-\alpha t-\theta')+\frac{i''}{\alpha i_1}\cdot\cos(\theta_1-\alpha t-\theta'')$$

Подставляемъ это выраженіе въ формулу для di:

$$di=\alpha i'\sin\left[\theta_1-\alpha t-\theta'-\frac{i'}{i_1}\sin(\theta_1-\alpha t-\theta')+\frac{i''}{\alpha i_1}\cdot\cos(\theta_1-\alpha t-\theta'')\right]dt$$

$$=\alpha i'\cdot\sin(\theta_1-\alpha t-\theta')\cos\left[\frac{i'}{i_1}\cdot\sin(\theta_1-\alpha t-\theta')-\frac{i''}{\alpha i_1}\cdot\cos(\theta_1-\alpha t-\theta'')\right]dt$$

$$-\alpha i'.\cos(\theta_1-\alpha t-\theta').\sin\left[\frac{i'}{i_1}\cdot\sin(\theta_1-\alpha t-\theta')-\frac{i''}{\alpha i_1}\cos(\theta_1-\alpha t-\theta'')\right]dt$$

Полагая

$$\cos\left[\frac{i'}{i_1}\cdot\sin(\theta_1-\alpha t-\theta')-\frac{i''}{\alpha i_1}\cos(\theta_1-\alpha t-\theta'')\right]=1,$$

и замѣняя во 2-мъ членѣ синусъ той-же дуги самою дугою, имѣемъ:

$$di=\alpha i'.\sin(\theta_1-\alpha t-\theta')dt-\alpha i'\cos(\theta_1-\alpha t-\theta')\left[\frac{i'}{i_1}\sin(\theta_1-\alpha t-\theta')\right.$$
$$\left.-\frac{i''}{\alpha i'}\cos(\theta_1-\alpha t-\theta'')\right]dt,$$

а отсюда, пренебрегая членами, умноженными на i'^3 и $i'^2 i''$, находимъ

$$di=dt\left[\alpha i'.\sin(\theta_1-\alpha t-\theta')-\frac{1}{2}\frac{\alpha i'^2}{i_1}\sin 2(\theta_1-\alpha t-\theta')\right.$$

$$\left.+\frac{1}{2}\cdot\frac{i'.i''}{i'}\cdot\cos(\theta'-\theta'')+\frac{1}{2}\cdot\frac{i'.i''}{i_1}\cos(2\theta_1-2\alpha t-\theta'-\theta'')\right]$$

Интегралъ 1-го члена $\alpha\int i'.\sin(\theta_1-\alpha t-\theta').dt=i'\cos(\theta_1-\alpha t-\theta')-$

$$-\int\cos(\theta_1-\alpha t-\theta').\frac{di'}{dt}=i'.\cos(\theta_1-\alpha t-\theta')+\frac{1}{\alpha}\cdot\frac{di'}{dt}\cdot\sin(\theta_1-\alpha t-\theta')$$

Далѣе

$$\frac{1\alpha}{2i_1}\int i'^2.\sin 2(\theta_1-\alpha t-\theta')dt=\frac{1}{4}\cdot\frac{i'^2}{i_1}\cos(2\theta_1-2\alpha t-2\theta')$$

$$-\frac{1}{4i_1}\int\cos(2\theta_1-2\alpha t-2\theta')\,.\,\frac{di'^2}{dt}\cdot dt=\frac{i'^2}{4i_1}\cos(2\theta_1-2\alpha t-2\theta')$$

$$+\frac{i'}{8\alpha}\cdot\frac{d(i'^2)}{dt}\cdot\sin 2(\theta_1-\alpha t-\theta')$$

Но

$$\frac{d(i'^2)}{dt}=\frac{d\,[i'^2\cos^2\theta'+i'^2\sin^2\theta']}{dt}=2i'\cos\theta'\,.\,\frac{d(i'.\cos\theta')}{dt}+2i'\sin\theta'\,.\,\frac{d(i'\sin\theta')}{dt}$$

$=2i'.\cos\theta'.i''.\cos\theta''+2i'.\sin\theta'.i''.\sin\theta''=2i'i''.\cos(\theta'-\theta'')$, слѣд.

$$\frac{i'}{8\alpha}\cdot\frac{d(i'^2)}{dt}\cdot\sin 2(\theta_1-\alpha t-\theta')=\frac{i'^2.i''}{4\alpha}\cdot\cos(\theta'-\theta'').\sin 2(\theta_1-\alpha t-\theta')=$$

$$=\frac{i'^2 i''}{8\alpha}\sin(2\theta_1-2\alpha t-\theta'-\theta'')+\frac{i'^2 i''}{8\alpha}\cdot\sin(2\theta_1-2\alpha t-3\theta'+\theta'')$$

Этими членами можно пренебречь. Такимъ образомъ мы получаемъ для величины втораго интеграла

$$-\frac{1}{2}\cdot\frac{\alpha}{i_1}\int i'^2.\sin(2\theta_1-2\alpha t-2\theta').dt=-\frac{i'^2}{4i_1}\cos(2\theta_1-2\alpha t-2\theta')$$

Третій членъ равенъ $\frac{1}{2i_1}\int i'.i''.\cos(\theta'-\theta'').\,dt$.

Подставляя сюда вмѣсто $i''.\cos\theta''$ и $i''.\sin\theta''$ производныя $\frac{d(i'\cos\theta')}{dt}$ и $\frac{d(i'\sin\theta')}{dt}$,

находимъ:

$$\frac{1}{2i_1}\int i'.i''.\cos(\theta'-\theta'').dt=\frac{1}{2i_1}\int(i'i''.\cos\theta'.\cos\theta''+i'i''.\sin\theta'.\sin\theta'')dt=$$

$$=\frac{1}{2i_1}\int\left[i'.\cos\theta'.\frac{d(i'\cos\theta')}{dt}+i'.\sin\theta'\frac{d(i'.\sin\theta')}{dt}\right]dt=$$

$$=\frac{1}{4i_1}\int\frac{d}{dt}(i'^2\cos^2\theta'+i'^2\sin^2\theta').dt=\frac{i'^2}{4i_1}.$$

Наконецъ 4-й членъ дастъ по интегрированіи:

$$\frac{1}{2i^1}\int i'.i''.\cos(2\theta_1-2\alpha t-\theta'-\theta'')dt=-\frac{i'i''}{4\alpha i_1}\sin(2\theta_1-2\alpha t-\theta'-\theta'')$$

$$+\frac{i''}{4\alpha i_1}\int\sin(2\theta_1-2\alpha t-\theta'-\theta'')\frac{di'}{dt}\cdot dt$$

$$= -\frac{i'i''}{4\alpha i_1}\cdot\sin(2\theta_1-2\alpha t-\theta'-\theta'')+\frac{i''}{8\alpha^2 i_1}\cdot\frac{di'}{dt}\cos(2\theta_1-2\alpha t-\theta'-\theta'')$$

Собирая полученные члены, находимъ

$$i=i_1+i'.\cos(\theta_1-\alpha t-\theta')+\frac{1}{\alpha}\cdot\frac{di'}{dt}\cdot\sin(\theta_1-\alpha t-\theta')$$

$$-\frac{i'^2}{4i_1}\cos 2(\theta_1-\alpha t-\theta')+\frac{i'^2}{4i_1}-\frac{i'i''}{4\alpha i_1}\sin(2\theta_1-2\alpha t-\theta'-\theta'')$$

$$+\frac{i''}{8\alpha^2 i_1}\cdot\frac{di'}{di}\cos(2\theta_1-2\alpha t-\theta'-\theta'')\ldots\ldots(23)$$

Таково выраженіе наклонности лунной орбиты относительно неподвижной эклиптики.

12. Перейдемъ теперь къ опредѣленію вѣковыхъ членовъ въ выраженіи долготы восходящаго узла.

Мы имѣемъ:

$$\frac{d\theta}{dt}=\frac{an}{\mu\sqrt{1-e^2}}\cdot\frac{d\Omega_0}{\sin i di}=\frac{an}{\mu\sqrt{1-e^2}}\cdot\frac{d\Omega_0}{idi}\left(1+\frac{i^2}{6}+\ldots\right)$$

$$=\frac{3}{4}m^2n\left(1+\frac{1}{2}e^2+\ldots\right)\left(1+\frac{i^2}{6}+\ldots\right)\Big\{-1-\frac{3}{2}e^2-\frac{3}{2}e'^2+$$

$$+\frac{1}{2}i^2+\frac{3}{2}i'^2+\frac{3}{4}i^2e'^2-\frac{9}{4}e^2e'^2+\frac{9}{4}e'^2i'^2$$

$$-\frac{15}{8}e'^4-2ii'\cos(\theta-\theta')+\frac{ii'\cos(\theta-\theta')}{i^2}\Big\}=$$

$$=\frac{3}{4}m^2n\Big\{-1-2e^2-\frac{3}{2}e'^2+\frac{1}{3}i^2+\frac{3}{2}i'^2-\frac{15}{8}e'^4-2ii'\cos(\theta-\theta')$$

$$-3e^2e'^2+\left(\frac{3}{4}-\frac{3}{12}=\frac{1}{2}\right)i^2e'^2+$$

$$+\frac{ii'\cos(\theta-\theta')}{6}+\frac{ii'\cos(\theta-\theta')}{i^2}\Big\}$$

Подставляя сюда вмѣсто i^2 и $ii'\cos(\theta-\theta')$ изъ величины по уравненію (22) и отбрасывая періодическій членъ $\frac{ii'\cos(\theta-\theta)}{i^2}$, находимъ, что i'^2 изчезаетъ изъ выраженія $\frac{d\theta}{dt}$. Въ самомъ дѣлѣ

$$\frac{3}{2}i'^2+\frac{1}{3}i^2-\frac{11}{6}ii'\cos(\theta-\theta')=\frac{3}{2}i'^2+\frac{1}{3}i_1{}^2+\frac{2}{3}i_1 i'\cos(\theta_1-\alpha t-\theta')+\frac{1}{3}i'^2$$

$$-\frac{11}{6}i'^2-\frac{11}{6}i_1 i'\cos(\theta_1-\alpha t-\theta')=$$

$$=\frac{1}{3}i_1{}^2+\left(\frac{3}{2}+\frac{1}{3}-\frac{11}{6}=0\right)i'^2-\frac{7}{6}i_1 i'\cos(\theta_1-\alpha t-\theta')$$

Итакъ

$$\frac{d\theta}{dt}=-\frac{3}{4}m^2n\left\{1+2e^2+\frac{3}{2}e'^2-\frac{1}{3}i_1{}^2+\frac{7}{6}\,i_1 i'\cos(\theta_1-\alpha t-\theta')\right.$$
$$\left.+3e^2e'^2-\frac{1}{2}i^2e'^2+\frac{15}{8}e'^4\right\}$$

Изъ теоріи Земли извѣстно, что эксцентрицитетъ земной орбиты, подобно наклонности, не есть величина постоянная.

Пусть

$$e'^2=E'^2+At+Bt^2 \text{ и } e'^4=E'^4+ct$$

Подставляя эти величины вмѣсто e'^2 и E'^4 въ предъидущее уравненіе, и потомъ замѣняя снова At^2+Bt^2 и ct соотвѣтственно черезъ $(e'^2-E'^2)$ и $(e'^4-E'^4)$ находимъ по интегрированіи

$$\theta=\theta_1-\frac{3}{4}m^2nt\left(1+2e^2-\frac{1}{3}i_1{}^2+\frac{3}{2}E'^2+\frac{15}{8}E'^4\right)-\frac{9}{8}m^2n\int(e'^2-E'^2)\,dt$$

$$-\left(\frac{9}{4}e^2-\frac{3}{8}i^2\right)m^2n\int(e'^2-E'^2)dt-\frac{45}{32}m^2n\int(e'^2-E'^4)dt\ldots(24)$$

Мы скоро увидимъ, что и среднее движеніе n есть въ строгомъ смыслѣ величина перемѣнная, поэтому напишемъ интегралы во 2-й части въ формѣ:

$$-\frac{9}{8}m^2\int(e'^2-E'^2)\,ndt \text{ и } -\frac{45}{32}m^2\int(e'^4-E'^4)\,ndt.$$

Ниже мы приведемъ числовыя величины этихъ вѣковыхъ членовъ, теперь же перейдемъ къ изслѣдованію варьяцій другихъ элементовъ.

13. Возьмемъ формулу

$$\frac{d\tilde{\omega}}{dt} = \frac{an\sqrt{1-e^2}}{\mu e} \cdot \frac{d\Omega_0}{de} \cdot + \frac{an}{\mu\sqrt{1-e^2}} \operatorname{tg} \frac{i}{2} \cdot \frac{d\Omega_0}{di}$$

Вычисляя производную $\frac{d\Omega_0}{de}$, находимъ

$$\frac{d\Omega_0}{de} = \frac{3m^2\mu}{4a} \cdot e\left\{1 + \frac{3}{2}e'^2 - \frac{3}{2}i^2 - \frac{3}{2}i'^2 + 3ii'\cos(\theta-\theta') + \frac{15}{8}e'^4 - \frac{9}{4}e'^2(i^2+i'^2)\right\}$$

На стр. 20 мы нашли выраженіе $\frac{d\Omega_0}{di}$; легко видѣть что

$$\operatorname{tg}\frac{1}{2} \cdot \frac{d\Omega_0}{di} = \left(\frac{i}{2} + \frac{i^3}{24} + \cdots\right)\frac{d\Omega_0}{di} = \frac{i^2}{2} \cdot \frac{d\Omega_0}{idi} +$$

$$= \frac{3}{4}\frac{m^3\mu}{a}\left\{-\frac{1}{2}i^2 + \frac{1}{2}ii'.\cos(\theta-\theta') - \frac{3}{4}i^2e'^2\right\}$$

Остальными членами очевидно можно пренебречь.

Такимъ образомъ получаемъ:

$$\frac{d\tilde{\omega}}{dt} = \frac{3}{4}m^2n\left(1 - \frac{1}{2}e^2 + \cdots\right)\left(1 + \frac{3}{2}e'^2 - \frac{3}{2}i^2 - \frac{3}{2}i'^2 + \frac{15}{8}e'^4\right.$$

$$\left. + 3ii'.\cos(\theta-\theta') - \frac{9}{4}e'^2(i^2+i'^2)\right)$$

$$+ \frac{3}{4}m^2n\left\{-\frac{1}{2}i^2 + \frac{1}{2}i.i'.\cos(\theta-\theta') - \frac{3}{4}i^2e'^2\right\}$$

$$= \frac{3}{4}m^2n\left\{1 - \frac{1}{2}e^2 + \frac{3}{2}e'^2 - 2i^2 - \frac{3}{2}i'^2 + \frac{7}{2}ii'\cos(\theta-\theta') + \right.$$

$$\left. \frac{15}{8}e'^4 - \frac{3}{4}e^2e'^2 - \left(\frac{9}{4} + \frac{3}{4} = 3\right)i^2e'^2 - \frac{9}{4}e'^2i'^2\right\}$$

Пользуясь уравненіями (22), составляемъ:

$$-2i^2 - \frac{3}{2}i'^2 + \frac{7}{2} \cdot ii'\cos(\theta-\theta') = -2i_1^{\,2} - \left(\frac{3}{2} + 2 - \frac{7}{2} = 0\right)i'^2 -$$

$$\frac{1}{2}i_1 i'.\cos(\theta_1 - \alpha t - \theta')$$

Такимъ образомъ мы опять видимъ, что члены, которые могли-бы дать вѣковое уравненіе долготы перигея, зависящее отъ i', взаимно сокращаются; остается одинъ періодическій членъ, который по своей незначительности всегда можетъ быть отброшенъ.

Подставляя по предъидущему вмѣсто e'^2 и e'^4 въ уравненіе для $\frac{d\tilde{\omega}}{dt}$ ихъ величины въ функціи времени и опять замѣняя при этомъ $At + Bt^2$ черезъ $e'^2 - E'^2$ и ct черезъ $e'^4 - E'^4$, находимъ по интегрированіи:

$$\tilde{\omega} = \tilde{\omega}_1 + \frac{3}{4} m^2 nt \left[1 - \frac{1}{2} e^2 - 2i_1{}^2 + \frac{3}{2} E'^2 + \frac{15}{8} E'^4\right]$$

$$+ \left(\frac{9}{8} - \frac{9}{16} e^2 - \frac{9}{4} i^2\right) m^2 \int (e'^2 - E'^2)\, ndt + \frac{45}{32} m^2 \int (e'^4 - E'^4)\, ndt \ldots (25)$$

Сравнивая это выраженіе съ уравненіемъ (24), мы видимъ, что получаемыя въ первомъ приближеніи величины угловыхъ скоростей наступательнаго движенія и обратнаго движенія линіи лунныхъ узловъ оказываются равными между собою. Вѣковыя уравненія движенія перигея и узловъ также равны по величинѣ и противоположны по знаку.

Такъ-какъ эксцентрицитетъ земной орбиты съ теченіемъ столѣтій уменьшается, то $\int (e'^2 - E'^2)\, ndt$ есть величина отрицательная, и потому линія узловъ, вслѣдствіе вѣковаго уравненія, медленнѣе отступаетъ, а линія апсидъ медленнѣе подвигается впередъ сравнительно съ движеніями этихъ линій, соотвѣтствующими какой-нибудь опредѣленной предшествующей эпохѣ.

Что касается до величины средней скорости и вѣковаго уравненія линіи узловъ, то достигнутая нами степень точности даетъ результаты въ этомъ случаѣ уже довольно близкіе къ наблюденіямъ.

Нельзя того-же сказать относительно движенія перигея. Первое приближеніе оказывается здѣсь совершенно недостаточнымъ. Ограничиваясь главными членами въ выраженіяхъ скорости движенія перигея и узловъ, мы находимъ для абсолютной величины угловаго движенія перигея и узловъ въ теченіе одного года (т. е. для $nt = \frac{360^\circ}{m}$) приблизительно: $\frac{3}{4} m^2 \left(\frac{360}{m}\right) = \frac{3}{4} . 0{,}074801 . 360^\circ = 20^\circ 11{,}'7$, истинныя-же величины средняго годоваго движенія линіи узловъ и перигея равны соотвѣтственно $19^\circ\ 20'\ 29''{,}76$ и $40^\circ 41' 25''{,}54$.

Такимъ образомъ дѣйствительная скорость движенія перигея почти вдвое болѣе того, что даетъ намъ первое приближеніе.

Теорія показываетъ, что скорость поступательнаго движенія линіи апсидъ выражается весьма медленно сходящимся рядомъ, первый членъ котораго, какъ мы нашли, равенъ $\frac{3}{4}m^2$, 2-й равняется $\frac{225}{32}m^3$ или почти $\frac{1}{2}m^2$, 3-й членъ немного менѣе $\frac{1}{5}m^2$, и т. д., такъ-что отъ прибавочныхъ членовъ даваемая первымъ приближеніемъ величина скорости движенія почти удваивается. Когда Клеро впервые вычислилъ первый членъ этого ряда и нашелъ такое разительное несогласіе полученнаго результата съ наблюденіями, то онъ пришелъ сначала къ заключенію, что движеніе перигея лунной орбиты не можетъ быть объяснено теоріею Ньютона и что законъ всемірнаго тяготѣнія долженъ выражаться болѣе сложною формулою. Дальнѣйшія вычисленія скоро привели однако Клеро къ выводу гораздо болѣе согласному съ дѣйствительностью, и такимъ образомъ еще разъ оправдали тѣ принципы, на которыхъ основана современная физическая астрономія.

14. Перейдемъ теперь къ опредѣленію измѣненій эпохи.

Мы имѣемъ

$$\frac{d\varepsilon}{dt} = -\frac{2a^2n}{\mu}\cdot\frac{d\Omega_0}{da} - \frac{an\sqrt{1-e^2}}{\mu e}\left[\sqrt{1-e^2}-1\right].\frac{d\Omega_0}{de} + \frac{an}{\sqrt{1-e^2}}\cdot tg\,\frac{i}{2}\cdot\frac{d\Omega_0}{di}$$

Прежде чѣмъ дифференцировать Ω_0 по a, возстановимъ коэффиціентъ $\frac{m^2\mu}{4a}$ въ первоначальномъ видѣ, т. е. возьмемъ

$$\Omega_0 = \frac{1}{4}\cdot\frac{m'a^2}{a'^3}.Y\,K_0\,S$$

Отсюда $-\frac{2a^2n}{\mu}\cdot\frac{d\Omega_0}{da} = -\frac{m'a^3n}{a'^3.\mu}.\ Y.K_0 S$

$$= -m^2n\Big[1-\frac{3}{2}i^2-\frac{3}{2}i'^2+\frac{3}{2}e^2+\frac{3}{2}e'^2+\frac{15}{8}e'^4+3\,ii'\cos(\theta-\theta') + \frac{9}{4}e^2e'^2-\frac{9}{4}e'^2i^2.$$

Замѣчая далѣе, что $\sqrt{1-e^2}\left[\sqrt{1-e^2}-1\right] = -\frac{1}{2}e^2-\frac{1}{8}e^3+\ldots$, мы можемъ ограничиться въ разложеніи $\frac{d\Omega_0}{ede}$ (см. стр. 34) только первыми 2-мя членами, т. е. положить $\frac{an\sqrt{1-2^2}}{\mu e}[\sqrt{1-e^2}-1]\frac{d\Omega_0}{de}$ $= + \frac{3}{8}m^2 n\left(e^2 + \frac{3}{2}e'^2 e^2\right)$

Наконецъ

$$\frac{an}{\sqrt{1-e_2}}\cdot \operatorname{tg}\frac{i}{2}\cdot\frac{d\Omega_0}{di}=\frac{3}{4}m^2n\left\{-\frac{1}{2}i^2+\frac{1}{2}ii'^2\cdot\cos(\theta-\theta')-\frac{3}{4}i^2e'^2\right\}$$

Собирая полученные члены, находимъ:

$$\frac{d\varepsilon}{dt}=-m^2n\Big[1-\frac{9}{8}i^2-\frac{3}{2}i'^2+\frac{21}{8}ii'^2.\cos(\theta-\theta')+\frac{9}{8}e^2+$$
$$\frac{3}{2}e'^2+\frac{15}{8}e'^4-\frac{27}{16}e^2e'^2+\frac{27}{16}i^2e'^2\Big]$$

Но по уравненію (22)

$$\frac{9}{8}i^2+\frac{3}{2}i'^2-\frac{21}{8}ii'.\cos(\theta-\theta')=\frac{3}{8}[3i^2+4i'-7ii'\cos(\theta-\theta')]$$

$$=\frac{3}{8}[3i_1{}^2+6i_1i'\cos(\theta_1-\alpha t-\theta')+3i'^2+4i'^2-7i'^2-7i_1i'.\cos(\theta_1-\alpha t-\theta')]$$

$$=\frac{9}{8}i_1{}^2-\frac{3}{8}i_1i'.\cos(\theta_1-\alpha t-\theta'),$$

слѣдовательно и долгота эпохи, подобно долготамъ перигѣя и восходящаго узла, не зависитъ отъ вѣковаго измѣненія наклонности эклиптики.

Затѣмъ, по предъидущему, получаемъ:

$$\varepsilon=\varepsilon_0-m^2nt\Big[1+\frac{9}{8}e^2-\frac{9}{8}i_1{}^2+\frac{3}{8}i_1i'.\cos(\theta_1-\alpha t-\theta')+$$
$$\frac{3}{2}E'^2+\frac{15}{8}E'^4\Big]-\frac{3}{8}i_1.i'.\frac{m^2n}{\alpha}\sin(\theta_1-\alpha t-\theta')$$
$$-\left(\frac{3}{2}-\frac{27}{16}e^2+\frac{27}{16}i^2\right)m^2\int(e'^2-E'^2)ndt-\frac{15}{8}m^2\int(e'^4-E'^4)ndt\ldots(26)$$

Такъ-какъ ε всегда входитъ въ разложенія радіуса вектора, истинной долготы и пр. не иначе какъ вмѣстѣ съ nt, то очевидно измѣненіе эпохи можно приписать измѣненію средней скорости невозмущеннаго движенія Луны. Пусть n_1 среднее движеніе въ единицу времени въ эллиптической орбитѣ Луны, т. е. въ той, которую Луна описывала-бы вокругъ Земли, еслибы не было возмущеній. Составляя выраженіе $n_1 t + \varepsilon$, мы находимъ, что члены пропорціональные времени въ выраженіи ε соединяются съ n_1 и такимъ образомъ получается величина n, которую даютъ наблюденія и которая называется среднимъ движеніемъ Луны въ единицу времени. Такимъ образомъ оказывается, что возмущающее дѣйствіе Солнца уменьшаетъ скорость движенія Луны. Это можно видѣть также изъ слѣдующихъ простыхъ соображеній.

Если обозначимъ по предъидущему черезъ Q функціи $\frac{\mu}{r} + \Omega$, гдѣ Ω пертурбаціонная функція, то центральная сила P, дѣйствующая на Луну въ направленіи радіуса вектора, можетъ быть выражена, какъ извѣстно, уравненіемъ: $P = -\frac{dQ}{dr}$

На стр. 8 мы нашли $\Omega = \frac{m'}{r'} - \frac{m' r^2}{2r'^3}(1 - 3\lambda^2)$

Обращая вниманіе на значеніи функіи λ и пренебрегая величинами s и s', мы легко находимъ отсюда:

$$\Omega = \frac{m'}{r'} + \frac{m'r^2}{4r'^3}[1 + 3\cos 2(v - v')] - \ldots$$

и слѣдовательно $P = \frac{\mu}{r^2} - \frac{d\Omega}{dr} = \frac{\mu}{r^2} - \frac{m'r}{2r'^3}[1 + 3\cos 2(v - v')]$, т. е. возмущающее дѣйствіе Солнца уменьшаетъ силу притяженія оказываемаго Землею на Луну приблизительно на величину $\frac{m'r}{2r'^3}[1 + 3\cos 2(v - v')]$

Отношеніе величины этого уменьшенія къ силѣ притяженія Луны Землею равно $\frac{m'r}{2r'^3} : \frac{\mu}{r^2}$ или приблизительно $\frac{m^2}{2}$, т. е. около $\frac{1}{357}$.

Среднее разстояніе Луны отъ Земли увеличивается очевидно въ томъ-же отношеніи, вслѣдствіе же увеличенія средняго разстоянія средняя скорость обращенія въ силу уравненія $a^3 n^2 = \mu$ должна уменьшаться, и это-то уменьшеніе средней скорости выражается въ уравненіи (26) членомъ $- m^2 n$.

Если обозначимъ черезъ $-\Delta P$ среднее уменьшеніе притяженія Луны Землею, происходящее отъ возмущающаго дѣйствія Солнца, то изъ формулы $P = \frac{\mu}{r^2} - \frac{m'r}{2r'^3}[1 + 3\cos 2(v - v')]$ легко видѣть, что абсолютная величина ΔP увеличивается, когда Земля находится въ перигеліи и уменьшается когда она проходитъ черезъ афелій. Обозначимъ уменьшеніе притяженія въ 1-мъ случаѣ черезъ — $(\Delta P + \Delta^2 P_\pi)$, а во 2-мъ черезъ — $(\Delta P - \Delta^2 P_\alpha)$, тогда полусумма уменьшеній притяженія въ перигеліи и афеліи выразится формулою

$$-\Delta P + \frac{\Delta^2 P_\alpha - \Delta^2 P_\pi}{2}$$

Но вслѣдствіе уменьшенія e', при неизмѣнности большой оси, съ вѣками уменьшается и разность между наибольшимъ и наименьшимъ радіусами векторами земной орбиты, слѣдовательно $\Delta^2 P_\alpha$ возрастаетъ, а $\Delta^2 P_\pi$ убываетъ, и потому притяженіе Луны Землею съ теченіемъ времени немного увеличивается, а съ увеличеніемъ притяженія и среднее движеніе Луны получаетъ вѣковое ускореніе.

Вѣковое ускореніе въ движеніи Луны по долготѣ впервые замѣчено было Галлеемъ, открытіе-же истинной причины этого явленія, такъ-же какъ и вѣковыхъ уравненій въ движеніи перигея и линіи узловъ принадлежитъ Лапласу и относится къ 1787 году.

15. Вычислимъ теперь величины первыхъ членовъ въ найденныхъ нами аналитическихъ выраженіяхъ вѣковыхъ уравненій перигея, линіи узловъ и средняго движенія.

Предварительно намъ нужно найти выраженіе интеграла $\int (e'^2 - E'^2)\, ndt$ въ функціи времени. Принимая за эпоху 1-е января 1850 г., Леверрье въ IV томѣ Анналовъ Парижской Обсерваторіи даетъ слѣдующее выраженіе для эксцентрицитета земной орбиты:

$$e' = 0.016770464 - 0''.08951\, t - 0''.00000282\, t^2,$$

гдѣ за единицу времени принятъ Юліанскій годъ. Выражая секунды дуги въ частяхъ радіуса, мы находимъ отсюда:

$$e'^2 - E'^2 = -0.0000014553.\left(\frac{t}{100}\right) - 0.00000000270245\left(\frac{t}{100}\right)^2$$

$$\int (e'^2 - E'^2)\, dt = -0.000727765.\left(\frac{t}{100}\right)^2 - 0.000000090082\left(\frac{t}{100}\right)^3$$

Годовое движеніе Луны въ секундахъ дуги (т. е. величина n) $= 17325593''.54$, такъ что

$\int (e'^2 - E'^2)\, ndt = -1263''.962\, i^2 - 1''.56\, i^3$, гдѣ $i = 100$ годамъ

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Отсюда:

$$\frac{3}{2} m^2 \int (e'^2 - E'^2)\, ndt = \frac{3}{2} (0.0748013)^2 \int (e'^2 - E'^2)\, ndt$$

$$= - 10''.58227\, i^2 - 0''.013098\, i^3$$

$$\frac{9}{8} m^2 \int (e'^2 - E'^2)\, ndt = - 7''.93688\, i^2 - 0''.00982\, i^3$$

Теорія Луны Делоне, въ которой приближеніе доведено до величинъ порядка m^6, даетъ для коэффиціентовъ вѣковыхъ уравненій слѣдующія числа: [1])

Средняя долгота..........	$+ 6''.176$
Линія апсидъ.............	$- 39''.499$
Линія узловъ.............	$+ 6''.778$

Мы видимъ, что первое приближеніе не даетъ и четверти полнаго коэффиціента вѣковаго уравненія линіи апсидъ, для двухъ другихъ приближеніе оказывается гораздо большимъ, но все-таки недостаточнымъ.

Что касается до другихъ членовъ въ коэффиціентахъ при $\int (e'^2 - E'^2)\, ndt$, то эти члены, заключая въ себѣ весьма малые множители i^2 и e^2, не даютъ ничего замѣтнаго, по крайней мѣрѣ по сравненіи съ первыми членами. Тоже самое можно сказать относительно велины интеграла $\int (e'^4 - E'^4)\, ndt$. Мы приведемъ ниже величины этихъ членовъ.

16. Теперь мы докажемъ предложеніе, о которомъ упомянули на стр. 30, т. е. что средняя долгота Луны, выраженная въ функціи истинной долготы ея, не заключаетъ ни вѣковыхъ, ни періодическихъ членовъ большаго періода, зависящихъ отъ наклонности орбиты Земли относительно неподвижной эклиптики, порядка ii'.

Въ самомъ дѣлѣ, средняя долгота Луны можетъ быть выражена въ функціи v посредствомъ уравненія вида

$$nt + \varepsilon = v - \Sigma A e^k i^m \sin j (v - \beta)$$

(см. напр. Méc. Cel. II, § 22)

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[1]) Comptes Rendus des séances de l'Ac. des Sc. de Paris, t. XLIX, 1859.

Но на стр. 26 мы нашли:

$$v = v_1 + \frac{1}{2} i_1 \Sigma N \sin(\theta_1 - \alpha t - kt - \beta)$$

или

$$v = v_1 + \frac{1}{2} i_1 \sin(\theta_1 - \alpha t) \Sigma N \cos(kt + \beta) - \frac{1}{2} i_1 . (\sin\theta_1 - \alpha t) . \Sigma N \sin(kt + \beta)$$

$$= v_1 + \frac{1}{2} i_1 i' . \sin(\theta_1 - \alpha t - \theta') ,$$

потому-что

$$\Sigma N \frac{\sin}{\cos}(kt + \beta) = i' \frac{\sin}{\cos} \cdot (\theta')$$

Съ другой стороны, замѣчая, что въ выраженіе долготы эпохи ε входитъ членъ $\frac{3}{8} \frac{i_1 i'}{\alpha} m^2 n . \sin(\theta_1 - at - \theta')$, равный $\frac{1}{2} i_1 . i' . \sin(\theta_1 - at - \theta')$, (такъ какъ $\alpha = \frac{3}{4} m^2 n$), мы находимъ по подстановкѣ въ уравненіе $nt + \varepsilon = v - \Sigma A e^k i^m \sin j(v - \beta)$ предъидущаго выраженія v и ε по уравненію 26, что члены

$$\frac{1}{2} i_1 i' . \sin(\theta_1 - dt - \theta')$$

взаимно сокращаются, — что и требовалось доказать.

17. Полученныя выше выраженія $\tilde{\omega}$ и θ мы должны еще дополнить небольшими постоянными членами, происходящими отъ члена $\frac{15}{16} \mu \frac{m^2}{a} i^2 e^2 \cos(2\tilde{\omega} - 2\theta)$, который входитъ въ выраженіе функціи Ω_0 и которымъ мы до сихъ поръ пренебрегали.

Нетрудно видѣть, что отъ присоединенія этого члена выраженія производныхъ $\frac{d\tilde{\omega}}{dt}$ и $\frac{d\theta}{dt}$ получаютъ слѣдующіе дополнительные члены:

$$\frac{15}{8} m^2 n i^2 . \cos(2\tilde{\omega} - 2\theta) \text{ и } \frac{15}{8} m^2 n e^2 \cos(2\tilde{\omega} - 2\theta)$$

Оставляя безъ вниманія прочіе члены, мы можемъ написать

$$\left.\begin{aligned} \frac{d\tilde{\omega}}{dt} &= \frac{3}{4} m^2 n + \frac{15}{8} m^2 n i^2 \cos(2\tilde{\omega} - 2\theta) \\ \frac{d\theta}{dt} &= -\frac{3}{4} m^2 n + \frac{15}{8} m^2 n e^2 \cos(2\tilde{\omega} - 2\theta) \end{aligned}\right\} \ldots\ldots (a)$$

Подставляя теперь во 2-я части этихъ уравненій приблизительныя величины $\tilde{\omega}$ и θ, равныя соотвѣтственно $\tilde{\omega}_1 + \frac{3}{4} m^2nt$ и $\theta_1 - \frac{3}{4} m^2nt$, мы находимъ по интегрированіи

$$\tilde{\omega} = \tilde{\omega}_1 + \frac{3}{4} m^2 nt + \frac{5}{4} \sin\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right)$$

$$\theta = \theta_1 - \frac{3}{4} m^2 nt + \frac{5}{4} e^2 . \sin\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right)$$

Новая подстановка даетъ:

$$\frac{d\tilde{\omega}}{dt} = \frac{3}{4} m^2 n + \frac{15}{8} m^2 n\, i^2 \cos\left[2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt + \frac{5}{4}(i^2 - e^2) \sin\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right)\right]$$

$$\frac{d\theta}{\lambda t} = -\frac{3}{4} m^2 n + \frac{15}{8} m^2 n\, e^2 \cos\left[2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt + \frac{5}{4}(i^2 - e^2) \sin\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right)\right]$$

Но

$$\cos\left[(2\tilde{\omega}_1 - 2\theta_1) + \frac{3}{2} m^2 nt + \frac{5}{4}(i^2 - e^2) \sin(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right] =$$

$$\cos\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right) - \frac{5}{4}(i^2 - e^2) . \sin^2\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right) =$$

$$\cos\left(2\tilde{\omega}_1 - 2\theta_1 + \frac{3}{2} m^2 nt\right) - \frac{5}{8}(i^2 - e^2) - \ldots, \text{ слѣд.}$$

$$\frac{d\tilde{\omega}}{dt} = -\frac{3}{4} m^2 n - \frac{75}{64} m^2 n\, i^2 (i^2 - e^2) - \ldots$$

$$\frac{d\theta}{dt} = -\frac{3}{4} m^2 n - \frac{75}{64} m^2 n\, e^2 (i^2 - e^2) - \ldots$$

Итакъ разсмотрѣнный членъ вводитъ въ выраженія долготы перигея и восходящаго узла новые члены равные соотвѣтственно $-\frac{75}{64} m^2 n\, i^2 (i^2 - e^2) t$ и $-\frac{75}{64} m^2 n\, e^2 (i^2 - e^2) t$.

18. Намъ остается теперь только найти варьяціи большой полуоси эксцентрицитета.

Такъ какъ $\frac{d\Omega_0}{d\varepsilon} = \frac{d\Omega_0}{ndt}$, то 1-е изъ уравненій (17) можетъ быть представлено въ видѣ

$$\frac{da}{dt} = \frac{2na^2}{\mu} \cdot \frac{d\Omega_0}{ndt} = \frac{1}{2} \cdot an\, m^2 \cdot \frac{d\Omega_0}{ndt} = \frac{3}{2}\, am^2 \cdot e' \frac{de'}{dt}$$

Такъ какъ $\frac{de'}{dt} = -0''.08951$ или въ частяхъ радіуса

-0.00000047146, а $e' = 0.016770464$, $\lg m = 8.8739091$,

то отношеніе $\frac{da}{dt}$ къ a оказывается менѣе $\frac{1}{10^{11}}$ и слѣдовательно варьяція большой полуоси есть величина сама по себѣ совершенно незамѣтная.

Обозначая черезъ a_0 величину большой полуоси при $t = 0$, мы находимъ:

$$a = a_0\,(1 - 0.0000000000006646.\ t)$$

Отсюда легко вычислить, что постоянная величина горизонтальнаго параллакса Луны въ 100.000 лѣтъ увеличивается только на $0''.023$.

Изъ ур. 3-го системы 17-й получаемъ:

$$\frac{de}{dt} = \frac{na}{\mu e}(1 - e^2)\frac{d\Omega_0}{ndt} = \frac{3}{4}\,\frac{m^2(1-e^2)}{e} \cdot \frac{e'\,de'}{dt}$$

Переводя это выраженіе въ числа, находимъ, что измѣненіе эксцентрицитета также совершенно незначительно. Въ 100 лѣтъ e уменьшается только на 0.00000006.

ГЛАВА IV.

Интегрированіе дифференціальныхъ уравненій движенія Луны.

19. Чтобы перейти къ слѣдующей степени приближенія, мы моглибы продолжить далѣе разложеніе пертурбаціонной функціи и затѣмъ опредѣлить измѣненія элементовъ по тѣмъ же формуламъ Лагранжа. Но этотъ способъ далеко уступаетъ въ удобствѣ тому, который заключается въ изслѣдованіи періодическихъ членовъ, входящихъ въ выраженіе полярныхъ координатъ Луны и который почти исключительно употреблялся всѣми геометрами, писавшими о теоріи Луны до временъ Гансена и Делоне. Въ сущности мы и здѣсь должны будемъ прибѣгнуть къ разложенію пертурбаціонной функціи, но рѣшеніе дифференціальныхъ уравненій, опредѣляющихъ координаты Луны, приведетъ насъ къ цѣли путемъ болѣе естественнымъ, изслѣдованіе-же интеграловъ этихъ уравненій покажетъ намъ связь существующую между измѣненіями элементовъ орбиты Луны и коэффиціентами различныхъ періодическихъ членовъ, выражающихъ главныя неравенства въ движеніи земнаго спутника.

Если пренебречь возмущающимъ дѣйствіемъ Солнца, т. е. положить $Q = O$, то уравненіе (11) получаютъ видъ:

$$\frac{d^2u}{dv^2} + u - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} = 0\,, dt = \frac{dv}{hu^2}\,, \frac{d^2s}{dv^2} + s = 0.$$

Послѣднее уравненіе интегрируется непосредственно и даетъ $s = \gamma \sin(v - \theta)$ гдѣ γ и θ произвольныя постоянныя, значеніе которыхъ опредѣляется очень просто. Въ самомъ дѣлѣ, изъ сферическаго треугольника, образуемаго эклиптикою, орбитою Луны и кругомъ широты, находимъ, обозначая долготу восходящаго узла черезъ Ω:

$$\operatorname{tg}\beta = \operatorname{tg} i \,.\, \sin(v - \Omega)$$

Но по опредѣленіи $s = \operatorname{tg}\beta$, слѣдовательно $\gamma = \operatorname{tg} i$, т. е. тангенсу узла, образуемаго плоскостями орбиты Луны и эклиптики, и $\theta = \Omega$. Кромѣ того изъ уравненія $s = \gamma \sin(v - \theta)$ можно заключить, что лунная орбита есть большой кругъ.

Интегралъ 1-го уравненія безъ послѣдняго члена имѣетъ видъ:

$$u = c \,.\, \sin v + c' \,.\, \cos v$$

То-же выраженіе можетъ удовлетворять и уравненію съ послѣднимъ членомъ, если считать c и c' за функціи v. Мы получаемъ для опредѣленія c и c' два условныя уравненія:

$$\frac{dc}{dv} \cdot \sin v + \frac{dc'}{dv} \cos v = 0 \quad \text{и} \quad \frac{dc}{dv} \cdot \cos v - \frac{dc'}{dv} \sin v = \pi,$$

гдѣ черезъ Π мы обозначили $\dfrac{\mu}{\gamma^2 (1 + s^2)^{\frac{3}{2}}}$

Изъ этихъ уравненій легко находимъ

$c = \int \Pi \,.\, \cos v \,.\, dv + c''$, $c' = -\int \Pi \,.\, \sin v\, dv + c'''$, слѣдовательно $u = c'' \sin v + c''' \cos v + \sin v \,.\, \int \Pi \,.\, \cos v \,.\, dv - \cos v \int \Pi \,.\, \sin v dv$. Чтобы вычислить $\int \Pi \cos v dv$ и $\int \Pi \sin v dv$, надобно выразить $\sin v dv$ и $\cos v\, dv$ въ функціи s.

Такъ какъ

$s = \gamma \,.\, \sin(v - \theta) = \gamma \sin v \,.\, \cos\theta - \gamma \,.\, \cos v \,.\, \sin\theta$, то $\cos(v - \theta) = \cos v \cos\theta +$

$$+ \sin v \,.\, \sin\theta = \sqrt{1 - \frac{s^2}{\gamma^2}} = \frac{\sqrt{\gamma^2 - s^2}}{\gamma},$$

откуда

$$\cos v = \frac{\cos\theta}{\gamma} \cdot \sqrt{\gamma^2 - s^2} - \frac{s}{\gamma} \cdot \sin\theta \,, \quad \sin v = \frac{\sin\theta}{\gamma} \cdot \sqrt{\gamma^2 - s^2} + \frac{s}{\gamma} \cdot \cos\theta$$

Дифференцируя эти выраженія, находимъ:

$$\sin v\,dv = \frac{\sin\theta}{\gamma}\cdot ds + \frac{s.\cos\theta\,ds}{\gamma.\sqrt{\gamma^2-s^2}},\quad \cos v\,.\,dv = \frac{\cos\theta}{\gamma}\cdot ds - \frac{s.\sin\theta}{\gamma\sqrt{\gamma^2-s^2}}\,ds,$$

и потому

$$u = c''\sin v + c'''\cos v - \left(\frac{\cos\theta}{\gamma}\cdot\sqrt{\gamma^2-s^2} - \frac{s}{\gamma}\cdot\sin\theta\right).$$

$$\int\frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}\left(\frac{\sin\theta}{\gamma} + \frac{s.\cos\theta}{\gamma\sqrt{\gamma^2-s^2}}\right)ds + \left(\frac{\sin\theta}{\gamma}\cdot\sqrt{\gamma^2-s^2} + \frac{s}{\gamma}\cdot\cos\theta\right)$$

$$\int\frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}\left(\frac{\cos\theta}{\gamma} - \frac{s.\sin\theta}{\gamma\sqrt{\gamma^2-s^2}}\right)ds$$

$$= c''\sin v + c'''\cos v + \frac{\sin^2\theta}{\gamma^2}\cdot\frac{\mu s}{h^2}\cdot\int\frac{ds}{(1+s^2)^{\frac{3}{2}}} - \frac{\mu}{h^2\gamma^2}\cdot\cos^2\theta\,.\sqrt{\gamma^2-s^2}$$

$$\int\frac{s\,ds}{(1+s^2)^{\frac{3}{2}}\sqrt{\gamma^2-s^2}}$$

$$+ \frac{\cos^2\theta}{\gamma^2}\cdot\frac{\mu s}{h^2}\int\frac{ds}{(1+s^2)^{\frac{3}{2}}} - \frac{\mu}{h^2\gamma^2}\cdot\sin^2\theta\,.\sqrt{\gamma^2-s^2}\,.\int\frac{s\,ds}{(1+s^2)^{\frac{3}{2}}\cdot\sqrt{\gamma^2-s^2}}$$

$$= c''\sin v + c'''\cos v + \frac{\mu s}{h^2\gamma^2}\int\frac{ds}{(1+s^2)^{\frac{3}{2}}} - \frac{\mu}{h^2\gamma^2}\sqrt{\gamma^2-s^2}\int\frac{s\,ds}{(1+s^2)^{\frac{3}{2}}\cdot\sqrt{\gamma^2-s^2}}.$$

Положимъ здѣсь

$$c'' = \frac{\mu e}{h^2(1+\gamma^2)}\sin\varpi,\quad c''' = \frac{\mu e}{h^2(1+\gamma^2)}\cos\varpi,$$

т. е. введемъ вмѣсто c'' и c''' двѣ новыя произвольныя постоянныя e и $\sin\varpi$; тогда получится

$$c''\sin v + c'''\cos v = \frac{\mu e}{h^2(1+\gamma^2)}\cdot\cos(v-\varpi)$$

Что касается до 3-го члена, то мы имѣемъ непосредственно:

$$\frac{\mu s}{h^2\gamma^2}\int\frac{ds}{(1+s^2)^{\frac{3}{2}}} = \frac{\mu s^2}{h^2\gamma^2(1+s^2)^{\frac{1}{2}}}$$

Чтобы найти интегралъ послѣдняго члена, замѣтимъ, что подъинтегральную функцію въ немъ можно представить въ формѣ

$$\frac{sds}{(1+\gamma^2)(\gamma^2-s^2)^{\frac{1}{2}}(1+s^2)^{\frac{1}{2}}}(1+s^2+\gamma^2-s^2)=\frac{sds}{(1+\gamma^2)\sqrt{\gamma^2-s^2}\sqrt{1+s^2}}+\frac{\sqrt{\gamma^2-s^2}\,.\,sds}{(1+\gamma^2)\,.\,(1+s^2)^{\frac{3}{2}}}$$

$$=-\frac{1}{1+\gamma^2}\left\{\frac{1}{\sqrt{1+s^2}}\cdot d\sqrt{\gamma^2-s^2}+\sqrt{\gamma^2-s^2}\cdot d\left(\frac{1}{\sqrt{1+s^2}}\right)\right\}=-\frac{1}{1+\gamma^2}\cdot d\left(\frac{\sqrt{\gamma^2-s^2}}{\sqrt{1+s^2}}\right),$$

откуда

$$\int\frac{sds}{(1+s^2)^{\frac{3}{2}}\cdot\sqrt{\gamma^2-s^2}}=-\frac{1}{1+\gamma^2}\cdot\frac{\sqrt{\gamma^2-s^2}}{\sqrt{1+s^2}}$$

Такимъ образомъ находимъ:

$$u=\frac{\mu e}{h^2(1+\gamma^2)}\cos(v-\tilde{\omega})+\frac{\mu}{h^2\gamma^2(1+s^2)^{\frac{1}{2}}}+\frac{\mu}{h^2\gamma^2}\frac{(\gamma^2-s^2)}{(1+\gamma^2)\sqrt{1+s^2}}=$$

$$=\frac{\mu e}{h^2(1+\gamma^2)}\cdot\cos(v-\tilde{\omega})+\frac{\mu\sqrt{1+s^2}}{h^2(1+\gamma^2)},$$

или

$$u=\frac{\mu}{h^2(1+\gamma^2)}\left\{e\cos(v-\tilde{\omega})+\sqrt{1+s^2}\right\}\ldots\ldots.(27)$$

Нетрудно показать, что это есть уравненіе эллипса, представляющаго проэкцію на плоскость эклиптики того эллипса, по которому двигалась-бы Луна еслибы не было возмущеній.

Обозначая по прежнему черезъ r радіусъ векторъ, Луны имѣемъ

$$r=\frac{\sqrt{1+s^2}}{u}=\frac{\frac{h^2(1+\gamma^2)}{\mu}}{1+\frac{e\cos(v-\tilde{\omega})}{\sqrt{1+s^2}}}$$

Пусть v_0 аргументъ широты Луны, β широта ея; изъ прямоугольнаго сферическаго треугольника, составленнаго орбитою Луны, эклиптикою и кругомъ широты, мы находимъ

$$\sin\beta=\frac{s}{\sqrt{1+\gamma^2}}=\sin i\,.\sin v_0$$

$$\operatorname{tg}(v-\theta)=\cos i \,.\, \operatorname{tg} v_0$$

$$\operatorname{tg}\beta=s=\operatorname{tg} i \,.\, \sin(v-\theta).$$

Изъ 2-го уравненія имѣемъ

$$\cos(v-\theta)=\frac{\cos v_0}{\sqrt{\cos^2 v_0+\cos^2 i \,.\, \sin^2 v_0}}=\frac{\cos v_0}{N}$$

$$\sin(v-\theta)=\frac{\cos i \,.\, \sin v_0}{\sqrt{\cos^2 v_0+\cos^2 i \,.\, \sin^2 v_0}}=\frac{\cos i \,.\, \sin v_0}{N},$$

гдѣ черезъ N мы обозначили $\sqrt{\cos^2 v_0+\cos^2 i \,.\, \sin^2 v_0}$.

Далѣе имѣемъ $\cos(v-\tilde{\omega})=\cos(v-\theta-\tilde{\omega}+\theta)$

$$=\cos(v-\theta) \,.\, \cos(\tilde{\omega}-\theta)+\sin(v-\theta) \,.\, \sin(\tilde{\omega}-\theta)=$$

$$=\frac{\cos v_0}{N} \,.\, \cos(\tilde{\omega}-\theta)+\frac{\cos i \sin v_0}{N}\sin(\tilde{\omega}-\theta) \ldots\ldots (a)$$

По подстановкѣ выраженія $\sin(v-\theta)$ въ 3-е уравненіе находимъ:

$$s=\frac{\sin i \,.\, \sin v_0}{N}, \text{ откуда } \frac{1}{\sqrt{1+s^2}}=\frac{\sqrt{N^2+\sin^2 i \,.\, \sin^2 v_0}}{N}=$$

$$=\frac{N}{\sqrt{\cos^2 v_0+\cos^2 i \,.\, \sin^2 v_0+\sin^2 i \,.\, \sin^2 v_0}}=N$$

и потому на основаніи уравненія (a)

$$r=\frac{h^2(1+\gamma^2)}{\mu(1+Ne\cos(v-\tilde{\omega}))}=\frac{h^2(1+\gamma^2)}{\mu[1+e\cos v_0 \,.\, \cos(\tilde{\omega}-\theta)+e\cos i \,.\, \sin v_0 \sin(\tilde{\omega}-\theta)]}$$

Съ другой стороны, если орбита Луны есть эллипсъ, то должно быть: $r=\frac{a_1(1-e^2)}{1+e_1\cos(v_0-\tilde{\omega}_0)}$, и произвольныя постоянныя должны опредѣляться изъ уравненій:

$$\left.\begin{aligned} h^2(1+\gamma^2)&=\mu a_1(1-e^2) \\ e_1\cos\tilde{\omega}_0&=e \,.\, \cos(\tilde{\omega}-\theta) \\ e_1\sin\tilde{\omega}_0&=e \,.\, \cos i \,.\, \sin(\tilde{\omega}-\theta) \end{aligned}\right\} \ldots\ldots\ldots\ldots (b)$$

Очевидно, что эти уравненія всегда допускаютъ рѣшеніе, соотвѣтствующее эллипсу.

Подставляя теперь въ уравненіе 27 вмѣсто $h^2 (1 + \gamma^2)$ величину $\mu a_1 (1 - e^2)$, а вмѣсто $\sqrt{1 + s^2}$ разложеніе этого радикала, находимъ:

$$u = \frac{1}{a_1(1-e^2)}\left\{e\cos(v-\bar\omega) + 1 + \frac{\gamma^2}{4} - \frac{1}{4}\gamma^2.\cos(2v-2\theta) + \ldots\right\}$$

$$= \frac{1}{a_1}\left\{1 + e^2 + \frac{1}{4}\gamma^2 + e\cos(v-\bar\omega) - \frac{1}{4}\gamma^2\cos(2v-2\theta) + \ldots\right\} \ldots (28)$$

Чтобы проинтегрировать ур. $dt = \frac{dv}{hu^2}$, нужно предварительно разложить функцію $\frac{1}{u^2}$. Мы имѣемъ:

$$\frac{1}{u^2} = a_1^2\left\{1 - 2e\cos(v-\bar\omega) - 2e^2 + 3e^2\cos^2(v-\bar\omega) - \frac{\gamma^2}{2} + \frac{1}{2}\gamma^2\cos(2v-2\theta)\right\}$$

$$= a_1^2\left\{1 - 2e\cos(v-\bar\omega) - \frac{1}{2}e^2 + \frac{3e^2}{2}\cdot\cos(2v-2\bar\omega) - \frac{\gamma^2}{2} + \frac{1}{2}\gamma^2.\cos(2v-2\theta) + \ldots\right\}$$

Отсюда $\frac{1}{hu^2} = \frac{(1+\gamma^2)^{\frac{1}{2}}}{a_1^{\frac{1}{2}}\sqrt{\mu}(1-e^2)^{\frac{1}{2}}}\cdot\frac{1}{u^2} =$

$$= \frac{a_1^{\frac{3}{2}}}{\sqrt{\mu}}\left\{1 - 2e\cos(v-\bar\omega) + \frac{3}{2}e^2.\cos(2v-2\bar\omega) + \frac{1}{2}\gamma^2\cos(2v-2\theta) + \ldots\right\}$$

Умножая обѣ части этого равенства на ndv и замѣчая, что $\frac{na_1^{\frac{3}{2}}}{\sqrt{\mu}} = 1$, находимъ по интегрированіи:

$$nt + \varepsilon = v - 2e\sin(v-\bar\omega) + \frac{3}{4}e^2\sin(2v-2\bar\omega) + \frac{1}{4}\gamma^2\sin(2v-2\theta) + \ldots\ldots\ldots (29)$$

Для Солнца получаемъ подобныя-же выраженіе:

$$n't + \varepsilon' = v' - 2e'.\sin(v'-\bar\omega) + \frac{3}{4}e'^2\sin(2v'-2\bar\omega') + \frac{1}{4}\gamma'^2\sin(2v'-2\theta')\ldots\ldots\ldots (30)$$

Если считать начало времени съ того момента когда ☉ и ☾ были въ среднемъ соединеніи, или когда $nt + \varepsilon = n't + \varepsilon'$, то можно принять $\varepsilon = \varepsilon' = 0$. Исключая t изъ (29) и (30) ур. и отбрасывая члены, умноженные на γ^2, легко находимъ:

$$v' - 2e'.\sin(v' - \tilde{\omega}') + \frac{3e'^2}{4}.\sin(2v' - 2\tilde{\omega}') = mv - 2me\sin(v - \tilde{\omega}) + \frac{3me^2}{4}.\sin(2v - 2\tilde{\omega})\ldots(31)$$

Чтобы опредѣлить отсюда v' въ функціи v нужно прибѣгнуть къ формулѣ Лагранжа.

Но прежде чѣмъ примѣнять ее, сдѣлаемъ одно весьма важное замѣчаніе. Перигей Луны, какъ мы видѣли, движется настолько быстро, что необходимо принять въ разсчетъ это движеніе въ предъидущемъ выраженіи для средней долготы Луны и въ ур. (31).

Если обозначимъ черезъ c сумму ряда

$$1 - \left(\frac{3m^2}{4} + b_0 m^3 + c_0 m^4 + \ldots\right),$$

въ которыхъ коэффиціенты b_0 и c_0 остаются пока неопредѣленными, то можно принять $\tilde{\omega} = \tilde{\omega}_1 + (1 - c)v$, гдѣ $\tilde{\omega}_1$ та величина $\tilde{\omega}$, которая соотвѣтствуетъ $v = 0$ и слѣд. $t = 0$ по нашему условію относительно долготы эпохи. Итакъ

$v - \tilde{\omega} = v - \tilde{\omega}_1 - (1 - c)v = cv - \tilde{\omega}_1$, а вмѣсто этого напишемъ просто $cv - \tilde{\omega}$, считая здѣсь $\tilde{\omega}$ уже за величину постоянную.

Обратимся теперь къ ур. (31), введемъ въ аргументы синусовъ во 2-й части вмѣсто $v - \tilde{\omega}$ исправленную величину $cv - \tilde{\omega}$ и обозначимъ 2-ую часть буквою z, а разность $2e'\sin(v' - \tilde{\omega}') - \frac{3e'^2}{4}\sin(2v' - 2\tilde{\omega}')$ черезъ $\varphi(v')$. Тогда мы получимъ ур. (31) въ видѣ $v' = z + \varphi(v')$, откуда

$$v' = z + \varphi(z) + \frac{1}{2}\cdot\frac{d(\varphi z)^2}{dz} + \ldots$$

Пренебрегая 3-й степенью e' и произведеніями ee'^2, e^2e', мы находимъ:

$$\varphi(z) = 2e'.\sin(mv - 2me\sin(cv - \tilde{\omega}) - \tilde{\omega}') - \frac{3e'^2}{4}.\sin(2mv - 2\tilde{\omega}')$$

$$= 2e'.\sin(mv-\tilde\omega') - 4mee'.\sin(cv-\tilde\omega).\cos(mv-\tilde\omega') =$$

$$= 2e'\sin(mv-\tilde\omega') - emee'.\sin(cv-\tilde\omega-mv-\tilde\omega') - 2mee'.\sin(cv-mv-\tilde\omega-\tilde\omega)$$

$$-\frac{3e'^2}{4}\cdot\sin(2mv-2\tilde\omega')$$

$$[\varphi(z)]^2 = 4e'^2\sin^2(mv-\tilde\omega') = 2e'^2 - 2e'^2\cos(2mv-2\tilde\omega')$$

$$\frac{1}{2}\frac{d(\varphi z)^2}{dz} = \frac{1}{2}\cdot\frac{d(\varphi z)^2}{dv}\cdot\frac{dv}{dz}. = \frac{1}{2}\cdot\frac{d(\varphi z)^2}{dv}\cdot\frac{1}{m-2me\cos(cv-\tilde\omega)} =$$

$$= 2e'^2\sin(2mv-2\tilde\omega')(1+2e\cos(cv-\tilde\omega)+\cdots) = 2e'^2\sin(2mv-2\tilde\omega')+\ldots$$

Итакъ

$$v' = mv - 2\,me\sin(cv-\tilde\omega) + \frac{3me^2}{4}.\sin(2cv-2\tilde\omega)$$

$$+2e'.\sin(mv-\tilde\omega') - 2mee'.\sin(cv+mv-\tilde\omega-\tilde\omega') + 2mee'.\sin(cv-mv-\tilde\omega+\tilde\omega')$$

$$+\left(2-\frac{3}{4}=\frac{5}{4}\right)e'^2\sin(2mv-2\tilde\omega')\,\ldots\ldots\ldots(32)$$

Намъ нужно еще получить u' въ функціи v.

Такъ-какъ u' выражается формулою подобною ур. (27), только съ замѣною a_1, e, γ, v, ω, θ, тѣми-же буквами со знаками, то для полученія u' нужно составить $\cos(v'-\tilde\omega')$

Отбросимъ также въ u' члены умноженные на γ'^2.

Обозначимъ на время черезъ z сумму періодическихъ членовъ въ выраженіи v', т. е. положимъ $v' = vm + z$.

Мы имѣемъ $\cos(v'-\tilde\omega') = \cos z.\cos(mv-\tilde\omega') - \sin z.\sin(mv-\tilde\omega') = (1-\frac{1}{2}z^2)\cos(mv-\tilde\omega') - z.\sin(mv-\tilde\omega')$.

Но $z^2\cos(mv-\tilde\omega')$ очевидно не содержитъ ни одного члена того порядка, который мы удерживаемъ; далѣе

$$-e'z.\sin(mv-\tilde\omega') = 2mee'.\sin(cv-\tilde\omega).\sin(mv-\tilde\omega') -$$

$$-2e'^2\sin^2(mv-\tilde\omega') = -e'^2(1-\cos(2mv-2\tilde\omega') - mee'.\cos(cv-mv-\tilde\omega+\tilde\omega')$$

$-mee'.\cos(cv+mv-\tilde\omega-\tilde\omega')$, слѣд.

$$u' = \frac{1}{a'}\{1 + e'\cos(mv-\tilde\omega') + e'^2\cos(2mv-2\tilde\omega') +$$

$$+ mee'.\cos(cv - mv - \tilde{\omega} + \tilde{\omega}') - mee'.\cos(cv + mv - \tilde{\omega} - \tilde{\omega}')\} \cdots (33)$$

Такимъ образомъ мы выразили v' и u' въ функціи угла v.

21. Приступимъ теперь къ преобразованію ур. (11)

На стр. 7 мы нашли для функціи Q слѣдующее выраженіе:

$$Q = \frac{\mu}{r} - \frac{m'\rho\rho'.[\cos(v - v') + ss']}{r'^3} + \frac{m'}{\sqrt{r^2 - 2\rho.\rho'[\cos(v - v') + ss'] + r'^2}}$$

Если мы примемъ теперь за основную плоскость проэкцій истинную эклиптику вмѣсто неподвижной, то это выраженіе и всѣ вытекающія изъ него разложенія значительно упростятся. Мы видѣли, что истинная долгота v_1 выражается по v, или по долготѣ, измѣряемой по неподвижной эплиптикѣ, ур. $v_1 = v - \frac{1}{2} i_1 \Sigma N \sin(\theta_1 - \alpha t - kt - \beta)$ т. е. v отличается отъ v_1 только на величину порядка $i_1.N$. Условившись пренебрегать въ ур. для u квадратомъ наклонности, мы тѣмъ болѣе имѣемъ право отбросить періодическіе члены вида $i_1 N \sin(\theta_1 - \alpha t - kt - \beta)$ съ коэффиціентами изчезающей малости. Что касается до средней долготы, то она, какъ мы видѣли, не заключаетъ въ своемъ выраженіи даже и этихъ незначительныхъ членовъ. Мы доказали также, что и истинная широта Луны не зависитъ отъ вѣковаго перемѣщенія плоскости эклиптики.

Полагая такимъ образомъ $s' = 0$, $\rho' = \frac{1}{u'} = r'$, и замѣняя въ выраженіи Q величину ρ черезъ $\frac{1}{u}$, мы легко находимъ:

$$Q = \frac{\mu u}{(1 + s^2)^{\frac{1}{2}}} - \frac{m' u'^2 \cos(v - v')}{u} + m' \left\{ \frac{(1 + s^2)}{u^2} - \frac{2\cos(v - v')}{uu'} + \frac{1}{u'^2} \right\}^{-\frac{1}{2}}$$

Послѣдній членъ равенъ

$$m' u' \left\{ 1 - \frac{2u' \cos(v - v')}{u} + \frac{u'^2(1 + s^2)}{u^2} \right\}^{-\frac{1}{2}}$$

$$= m' u' \left[\left\{ 1 + \frac{u' \cos(v - v')}{u} - \frac{u'^2(1 + s^2)}{2u^2} \right\} + \frac{3}{8} \left\{ \frac{2\cos(v - v') u'}{u} - \frac{u'^2(1 + s^2)}{u^2} \right\}^2 \right.$$

$$-\frac{5}{16}\left\{\frac{2\cos(v-v')u'}{u}-\frac{u'^2(1+s^2)}{u^2}\right\}^3+\ldots\Bigg]$$

$$=m'u'+\frac{m'u'^2\cos(v-v')}{u}-\frac{m'u'^3(1+s^2)}{2u^2}+\frac{3}{4}\frac{m'u'^3}{u^2}+\frac{3}{4}\frac{m'u'^3\cos 2(v-v')}{u^2}+\ldots,$$

слѣдовательно

$$Q=\frac{\mu u}{(1+s)^{\frac{1}{2}}}+m'u'+\frac{m'u'^3}{4u^2}\{1+3\cos(2v-2v')-2s^2\}+\ldots$$

Отсюда мы легко получаемъ

$$\frac{dQ}{du}=\frac{\mu}{(1+s^2)^{\frac{1}{2}}}-\frac{m'u'^3}{2u^3}[1+3\cos 2(v-v')-2s^2]$$

$$\frac{dQ}{dv}=-\frac{3m'u'^3}{2u^2}\cdot\sin 2(v-v')$$

$$\frac{dQ}{ds}=-\frac{\mu us}{(1+s^2)^{\frac{3}{2}}}-\frac{m'u'^3s}{u^2}$$

$$\frac{s}{u}\cdot\frac{dQ}{ds}=-\frac{\mu s^2}{(1+s^2)^{\frac{3}{2}}}-\frac{m'u'^3s^2}{u^3}$$

Подставляя эти выраженія частныхъ производныхъ функціи Q въ ур. (11) стр. 6, находимъ:

$$\left(\frac{d^2u}{dv^2}+u\right)\left(1-\frac{3m'}{h^2}\int\frac{u'^3}{u^4}.\sin 2(v-v')dv\right)-\frac{\mu}{h^2(1+s^2)^{\frac{1}{2}}}$$

$$+\frac{m'u'^3}{2h^2u^3}[1+3\cos 2(v-v')-2s^2]+\frac{\mu s^2}{h^2(1+s^2)^{\frac{3}{2}}}+\frac{m'u'^3s^2}{h^2u^3}+\ldots=0$$

$$\frac{dt}{dv}=\frac{1}{u^2}\left(h^2-\frac{3m'}{h^2}\int\frac{u'^3}{u^4}\cdot\sin 2(v-v')dv\right)^{-\frac{1}{2}}=$$

$$=\frac{1}{hu^2}+\frac{3m'}{2h^3u^2}\int\frac{u'^3dv}{u^4}.\sin 2(v-v')+\frac{27}{8}\cdot\frac{m'^2}{h^5h^2}\left[\int\frac{u'^3}{u^4}.\sin(2v-2v')dv\right]^2$$

$$\left(\frac{d^2s}{dv^2}+s\right)\left(1-\frac{3m'}{h^2}\int\frac{u'^3}{u^4}.\sin(2v-2v')dv\right)+\frac{3m'u'^3s}{2h^2u^4}+$$

$$+\frac{3m'u'^3s}{2h^2u^4}.\cos 2(v-v')-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{ds}{dv}.\sin 2(v-v')+\ldots=0$$

$$
(34)\begin{cases}
\frac{d^2 u}{dv^2} + u - \frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} + \frac{m' u'^3}{2h^2 u^3} + \frac{3m' u'^3}{2h^2 u^3} \cdot \cos 2(v-v') \\
-\frac{3m' u'^3}{2h^2 u^4} \cdot \frac{du}{dv} \cdot \sin 2(v-v') - \frac{3m'}{h^2}\left(\frac{d^2u}{dv^2}+u\right)\int \frac{u'^3 dv}{u^4} \cdot \sin 2(v-v') + \cdots = 0 \\
\frac{dt}{dv} = \frac{1}{hu^2} + \frac{3m'}{2h^3 u^2}\int \frac{u'^3 dv}{u^4} \cdot \sin 2(v-v') + \frac{27}{8} \cdot \frac{m'^2}{h^5 u^2}\left[\int \frac{u'^3 dv}{u^4} \sin(2v-2v')\right]^2 \\
\frac{d^2 s}{dv^2} + s + \frac{3m' u'^3 . s}{2h^2 u^4} + \frac{3m' u'^3 s}{2h^2 u^4} \cdot \cos 2(v-v') - \frac{3m' u'^3}{2h^2 u^4} \cdot \frac{ds}{dv} \sin 2(v-v') \\
-\frac{3m'}{h^2}\left(\frac{d^2 s}{dv^2} + s\right)\int \frac{u'^3 dv}{u^4} \cdot \sin(2v-2v') + \ldots = 0.
\end{cases}
$$

Уравненія (28), (32) и (33) даютъ выраженія величинъ u, v' и u' въ функціи истинной долготы Луны; подставивъ эти выраженія въ ур. (34), мы получимъ эти ур. въ видѣ:

$$
\frac{d^2 u}{dv^2} + u + f(u, v, s) = 0, \quad \frac{dt}{dv} = f_1(u, v, s) \text{ и}
$$

$$
\frac{d^2 s}{dv^2} + s + f_2(u, v, s) = 0.
$$

Интегрировать эти ур. въ конечной формѣ до сихъ поръ еще не удалось; чтобы рѣшить ихъ надобно прибѣгнуть или къ способу послѣдовательныхъ подстановокъ или къ способу неопредѣленныхъ коэффиціентовъ. Интегрированіе 1-го ур. значительно упростится, если мы прямо положимъ въ немъ $s = 0$. Это будетъ значить, что мы пренебрежемъ наклонностью лунной орбиты.

ГЛАВА V.

Интегрированіе дифференціальнаго уравненія для обратной величины проэкціи радіуса вектора.

22. Изъ разсмотрѣнія членовъ, входящихъ въ составъ 1-го изъ ур. (34) не трудно убѣдиться, что по разложеніи этихъ членовъ и надлежащемъ приведеніи это ур. можетъ быть приведено къ виду:

$$\frac{d^2u}{dv^2} + u - u_0 - \Sigma N \cos(jv \pm j'mv + \alpha) = 0,$$

гдѣ u_0, N и α постоянныя величины, зависящія отъ e, e', a, и пр., т. е. не содержащія явно буквы v. Интегралъ этого ур. очевидно равенъ

$$u = u_0 - \Sigma \frac{N}{1-(j \pm j'm)^2} \cos(jv \pm j'mv + \alpha)$$

Особеннаго вниманія въ этомъ выраженіи заслуживаютъ тѣ члены, въ аргументахъ которыхъ $j = 1$, (при этомъ j' можетъ равняться какому угодно цѣлому числу, но во всякомъ случаѣ не нулю). Дѣйствительно, если $j = 1$ [1]), то разность $1-(j \pm j'm)$ будетъ величиною 1-го порядка малости, т. е. порядка величины m, а слѣд. порядокъ соотвѣтствующаго коэффиціента N на единицу понизится. Это обстоятельство необходимо имѣть въ виду при разборѣ членовъ съ коэффиціентами одного и того-же порядка, когда требуется оставить въ

---

[1]) или отличается отъ 1 на величину порядка m^2.

$$
(34)\begin{cases}
\frac{d^2 u}{dv^2} + u - \frac{\mu}{h^2 (1+s^2)^{\frac{3}{2}}} + \frac{m' u'^3}{2h^2 u^3} + \frac{3m' u'^3}{2h^2 u^3} \cdot \cos 2(v-v') \\
- \frac{3m' u'^3}{2h^2 u^4} \cdot \frac{du}{dv} . \sin 2(v-v') - \frac{3m'}{h^2}\left(\frac{d^2 u}{dv^2} + u\right) \int \frac{u'^3\, dv}{u^4} \cdot \sin 2(v-v') + \cdots = 0 \\
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\frac{d^2 s}{dv^2} + s + \frac{3m' u'^3 . s}{2h^2 u^4} + \frac{3m' u'^3 s}{2h^2 u^4} . \cos 2(v-v') - \frac{3m' u'^3}{2h^2 u^4} \cdot \frac{ds}{dv} \sin 2(v-v') \\
- \frac{3m'}{h^2}\left(\frac{d^2 s}{dv^2} + s\right) \int \frac{u'^3\, dv}{u^4} . \sin(2v-2v') + \ldots = 0.
\end{cases}
$$

Уравненія (28), (32) и (33) даютъ выраженія величинъ u, v' и u' въ функціи истинной долготы Луны; подставивъ эти выраженія въ ур. (34), мы получимъ эти ур. въ видѣ:

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$$

$$
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$$

Интегрировать эти ур. въ конечной формѣ до сихъ поръ еще не удалось; чтобы рѣшить ихъ надобно прибѣгнуть или къ способу послѣдовательныхъ подстановокъ или къ способу неопредѣленныхъ коэффиціентовъ. Интегрированіе 1-го ур. значительно упростится, если мы прямо положимъ въ немъ $s = 0$. Это будетъ значить, что мы пренебрежемъ наклонностью лунной орбиты.

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гдѣ u_0, N и α постоянныя величины, зависящія отъ e, e', a, и пр., т. е. не содержащія явно буквы v. Интегралъ этого ур. очевидно равенъ

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[1]) или отличается отъ 1 на величину порядка m^2.

интегралѣ только члены съ наибольшими коэффиціентами даннаго типа.

Сообразно этому замѣчанію, разлагая послѣдовательно всѣ члены перваго изъ ур. (34), мы удержимъ изъ числа членовъ, умноженныхъ на e и ee', только члены съ слѣдующими аргументами: $v - 2mv + \tilde{\omega}$, $v - mv + \tilde{\omega} - \tilde{\omega}'$, $v - 3mv + \tilde{\omega} + \tilde{\omega}'$, $cv + mv - \tilde{\omega} - \tilde{\omega}'$ и $cv - mv - \tilde{\omega} + \tilde{\omega}'$. Кромѣ того мы оставимъ въ ур. тѣ члены, коэффиціенты которыхъ суть 1, m^2, e' и e'^2, а также члены съ аргументомъ $(cv - \tilde{\omega})$, совокупность которыхъ выражаетъ, какъ извѣстно, главное эллиптическое неравенство — уравненіе центра.

23. Приступимъ-же къ разложенію членовъ уравненія для u. Мы обозначили выше черезъ $\frac{1}{a_1}$ постоянную часть функціи u, при допущеніи, что пертурбаціонная функція равна нулю. Положимъ теперь эту постоянную часть въ возмущенной орбитѣ равною $\frac{1}{a}$, тогда

$$u = \frac{1}{a}(1 + e\cos(cv - \tilde{\omega}) + \ldots) \qquad (28\,b)$$

По ур. (33) имѣемъ

$$u'^3 = \frac{1}{a'^3}\Big\{1 + 3e'.\cos(mv - \tilde{\omega}') + \frac{3}{2}e'^2 + \left(3 + \frac{3}{2}\right)e'^2\cos(2mv - 2\tilde{\omega}')$$

$$+ 3mee'.\cos(cv - mv - \tilde{\omega} + \tilde{\omega}') - 3mee'\cos(cv + mv - \tilde{\omega} - \tilde{\omega}')\Big\}$$

Такъ какъ мы условились пренебрегать квадратами e и γ, то вмѣсто u^3 возьмемъ просто $a^3(1 - 3e\cos(cv - \tilde{\omega}))$, а вмѣсто h^2, μa_1.

Такимъ образомъ, замѣчая, что $\frac{m'a^3}{2a'^3\mu a_1} = \frac{n'^2\, a^3}{2\mu . a_1}$ и полагая $n'^2 a^3 = \overline{m}^2 . \mu$, находимъ:

$$\frac{m'\, u'^3}{2h^2 u^3} = \frac{\overline{m}^2}{2a_1}\Big\{1 - 3e\left(1 + \frac{3}{2}e'^2\right)\cos(cv - \tilde{\omega}) + \frac{3}{2}e'^2 + 3e'.\cos(mv - \tilde{\omega}')$$

$$+ \frac{9}{2}e'^2\cos(2mv - 2\tilde{\omega}') - \frac{3}{2}(3 - 2m)\cos(cv - mv - \tilde{\omega} + \tilde{\omega}')$$

$$- \frac{3}{2}(3 + 2m)\cos(cv + mv - \tilde{\omega} - \tilde{\omega}')\Big\} \qquad (35)$$

Составимъ теперь $\cos(2v-2v')$ и $\cos(2v-2v')\,u'^3$. Мы имѣемъ по ур. (32):

$$2(v-v')=(2v-2mv)+4\,me.\sin(cv-\tilde\omega)-4e'.\sin(mv-\tilde\omega')$$

$$-\frac{5}{2}e'^2\sin(2mv-2\tilde\omega')+4mee'.\sin(cv+mv-\tilde\omega-\tilde\omega')$$

$$-4\,mee'.\sin(cv-mv-\tilde\omega+\tilde\omega').$$

Обозначивъ для краткости 2 — 6 члены второй части послѣдняго равенства соотвѣтственно черезъ $f_1 f_2 \ldots f_5$, мы находимъ:

$$\cos 2(v-v')=\cos(2v-2mv).\cos(f_1+f_2+f_3+f_4+f_5)$$

$$-\sin(2v-2mv).\sin(f_1+f_2+f_3+f_4+f_5)=\cos(2v-2mv)$$

$$-\frac{1}{2}\cos(2v-2mv).(f_1+f_2+f_3+f_4+f_5)^2-\sin(2v-2mv).$$

$$.(f_1+f_2+f_3+f_4+f_5)$$

Но $(f_1+f_2+f_3+f_4+f_5)^2=-32mee'\sin(cv-\tilde\omega).\sin(mv-\tilde\omega')$

$$+16e'^2\sin^2(mv-\tilde\omega')+\ldots=-16\,mee'\cos(cv-mv-\tilde\omega+\tilde\omega')$$

$$+16mee'\cos(cv+mv-\tilde\omega-\tilde\omega')+8e'^2-8e'^2\cos(2mv-2\tilde\omega')+\ldots,$$

слѣд. $-\frac{1}{2}\cos(2v-2mv).(f_1+f_2+f_3+f_4+f_5)^2=$

$$4\,mee'\cos(v-mv+\tilde\omega-\tilde\omega')-4mee'.\cos(v-3mv+\tilde\omega+\tilde\omega')$$

$$-4e'^2\cos(2v-2mv)+2e'^2\cos(2v-2\tilde\omega')+2e'^2\cos(2v-4mv+2\tilde\omega')$$

Члены съ аргументами $2v-2\tilde\omega'$ и $2v-4mv+2\tilde\omega'$ мы также будемъ отбрасывать.

Далѣе $-\sin(2v-2mv)(f_1+f_2+\ldots+f_5)=$

$$=2e'.\cos(2v-3mv+\tilde\omega')-2e'.\cos(2v-mv-\tilde\omega')-2me\cos(v-2mv+\tilde\omega)$$

$$-2mee'.\cos(v-3mv+\tilde\omega+\tilde\omega')+2mee'.\cos(v-mv+\tilde\omega-\tilde\omega')+\ldots$$

Такимъ образомъ получаемъ

$$\cos 2(v-v')=\cos(2v-2mv)(1-4e'^2)+4mee'.\cos(v-mv+\varpi-\varpi')$$

$$-4mee'\cos(v-3mv+\varpi+\varpi')-2me.\cos(v-2mv+\varpi)$$

$$+2e'\cos(2v-3mv+\varpi')-2e'.\cos(2v-mv-\varpi')$$

$$-2mee'.\cos(v-3mv+\varpi+\varpi')-2mee'.\cos(v-mv+\varpi-\varpi')=$$

$$=\cos(2v-2mv)(1-4e'^2)+2mee'.\cos(v-mv+\varpi-\varpi')$$

$$-6mee'.\cos(v-3mv+\varpi+\varpi')-2me.\cos(v-2mv+\varpi)$$

$$+2e'.\cos(2v-3mv+\varpi')-2e'.\cos(2v-mv-\varpi')=Y.$$

Для составленія произведенія $m'.u'^3\cos(2v-2v')$, умножаемъ полученное выраженіе на

$$\frac{m'}{a'^3}\Big[1+3e'.\cos(mv-\varpi')+\frac{3}{2}e'^2+\frac{9}{2}e'^2\cos(2mv-2\varpi')$$

$$+3mee'.\cos(cv-mv-\varpi+\varpi')-3mee'.\cos(cv+mv-\varpi-\varpi')\Big].$$

Мы имѣемъ $m'u'^3.\cos(2v-2v')=$

$$\frac{m'}{a'^3}\Big[Y+\Big(3e'.\cos(mv-\varpi')+\frac{3}{2}e'^2+3mee'.\cos(cv-mv-\varpi+\varpi')$$

$$-3mee'.\cos(cv+mv-\varpi-\varpi')\Big)\cos(2v-2mv)-6mee'.\cos(mv-\varpi')$$

$$.\cos(v-2mv+\varpi)-6e'^2\cos(mv-\varpi').\cos(2v-mv-\varpi')+$$

$$+6e'^2.\cos(mv-\varpi').\cos(2-3mv+\varpi')\Big]$$

$$=\frac{m'}{a'^2}\Big[Y+\frac{3}{2}e'.\cos(2v-mv-\varpi')+\frac{3}{2}e'.\cos(2v-3mv+\varpi')$$

$$+\frac{3}{2}e'^2\cos(2v-2mv)+\frac{3}{2}mee'.\cos(v-mv+\varpi-\varpi')$$

$$-\frac{3}{2}mee'.\cos(v-3mv+\varpi+\varpi')-3mee'.\cos(v-3mv+\varpi+\varpi')$$

$$-8mee'.\cos(v-mv+\varpi-\varpi')-3e'^2\cos(2v-2mv)+3e'^2(2v-2mv)\Big]$$

$$=\frac{m'}{a'^3}\Big[\cos(2v-2mv)-\Big(4-\frac{3}{2}=\frac{5}{2}\Big)e'^2\cos(2v-2mv)$$

$$+\frac{1}{2}mee'.\cos(v-mv+\varpi-\varpi')-\frac{21}{2}mee'.\cos(v-3mv+\varpi+\varpi')$$

$$-2me.\cos(v-2mv+\varpi)-\frac{1}{2}e'.\cos(2v-mv-\varpi')+$$

$$+\frac{7}{2}e'.\cos(2v-3mv+\varpi')\Big].$$

Изъ предъидущаго выраженія легко получить $\frac{3}{2}\cdot\frac{m'u'^3}{h^2u^3}\cdot\cos(2v-2v')$.

Мы имѣемъ: $\frac{3}{2h^2u^3}=\frac{3}{2}\frac{a^3}{a_1}(1-3e\cos(cv-\varpi))$; очевидно, отъ умноженія $-3e.\cos(cv-\varpi)$ на $u'^3.\cos(2v-2v')$ войдутъ въ произведеніе $\frac{3}{2}\cdot\frac{m'u'^3}{h^2u^3}\cdot\cos(2v-2v')$ только 3 новыхъ члена: — $-\frac{3}{2}e\left(1-\frac{5}{2}e'^2\right)\cos(v-2mv+\varpi)-\frac{21}{4}ee'\cos(v-3mv+\varpi+\varpi')$ $+\frac{3}{4}ee'.\cos(v-mv-\varpi'+\varpi)$, такъ-что мы получимъ:

$$\frac{3}{2}\cdot\frac{m'u'^3}{h^2u^3}\cdot\cos(2v-2v')=\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\Big[\left(1-\frac{5}{2}e'^2\right)\cos(2v-2mv)$$

$$+\frac{7}{2}e'.\cos(2v-3mv+\varpi')-\frac{1}{2}e'.\cos(2v-mv-\varpi')$$

$$-\frac{1}{2}(3+4m)\left(1-\frac{5}{2}e'^2\right)\cos(v-2mv+\varpi)-\frac{21}{4}(1+2m)ee'.\cos(v-3mv+\varpi+\varpi')$$

$$+\frac{1}{4}(3+2m)ee'.\cos(v-mv+\varpi-\varpi')\Big]\ldots\ldots\ldots\ldots(36)$$

Чтобы получить выраженіе $\frac{3}{2}\cdot\frac{m'u'^3}{h^2u^3}\sin(2v-2v')$, достаточно взять въ предъидущемъ уравненіи вмѣсто $2v$ во всѣхъ аргументахъ $2v+90°$, оставляя углы mv, cv безъ перемѣны. Такъ-какъ три послѣдніе аргумента составлены изъ соединенія $2v-2mv$ съ аргументами $cv-\varpi$ и $mv-\varpi'$, то и въ нихъ, какъ въ 3-хъ первыхъ замѣна $2v$ угломъ $2v+90°$ произведетъ только перемѣну знаковъ передъ членами и преобразованіе cos въ sin.

Такимъ образомъ получимъ:

$$\frac{3m'u'^3}{2h^2u^3}\sin(2v-2v')=\frac{3\overline{m}^2}{2a_1}\Big[(1-\frac{5}{2}e'^2)\sin(2v-2mv)+\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')$$

$$-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')-\frac{1}{2}(3+4m)\Big(1-\frac{5}{2}e'^2\Big)e.\sin(v-2mv-\tilde{\omega})$$

$$-\frac{21}{4}(1+2m)ee'.\sin(v-3mv+\tilde{\omega}+\tilde{\omega}')+\frac{1}{4}(3+2m)ee'.\sin(v-mv+\tilde{\omega}-\tilde{\omega}')\Big]\ldots(37a)$$

Теперь составимъ выраженіе

$$-\frac{3m'.u'^3}{2h^2u^4}.\frac{du}{dv}.\sin(2v-2v')$$

Возьмемъ $\frac{1}{u}.\frac{du}{dv}=-e\sin(cv-\tilde{\omega})+\ldots$ и помножимъ $-e.\sin(cv-\omega)$ на предъидущее выраженіе $\frac{3m'u'^3}{2h^2u^3}\sin(2v-2v')$.

Мы легко находимъ:

$$-\frac{3m'u'^3}{h^2u^4}.\frac{du}{dv}.\sin(2v-2v')=\frac{3\overline{m}^2}{4a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big)e\cos(v-2mv+\tilde{\omega})$$

$$+\frac{7}{2}ee'.\cos(v-3mv+\tilde{\omega}+\tilde{\omega}')+\frac{1}{2}ee'.\cos(v-mv+\tilde{\omega}-\tilde{\omega}')\Big]\,..(37b)$$

Намъ остается только вычислить

$$-\frac{3m'}{h^2}\Big(\frac{d^2u}{dv^2}+u\Big)\int\frac{u'^3\,dv}{u^4}.\sin(2v-2v')$$

Помножимъ обѣ части равенства $(37a)$ соотвѣтственно на $\frac{2}{u}$ и на $+2a(1-e\cos(cv-\tilde{\omega}))$.

Обозначая для краткости $\frac{3m'u'^3}{2h^2u^3}\sin(2v-2v')$ черезъ $\frac{3\overline{m}^2}{2a}.S$, мы получаемъ

$$\frac{3m'u'^3}{h^2u^4}.\sin(2v-2v')=-3\overline{m}^2\Big[\Big(S-e(1-\frac{5}{2}e'^2\Big)\cos(cv-\tilde{\omega}).\sin(2v-2mv)$$

$$-\frac{7}{2}ee'.\cos(cv-\varpi).\sin(2v-3mv+\varpi')+\frac{1}{2}ee'.\cos(cv-\varpi).\sin(2v-mv-\varpi')\Big]$$

$$=-3\overline{m}^2\Big[S-\frac{1}{2}.e\Big(1-\frac{5}{2}e'^2\Big).\sin(v-2mv+\varpi)-\frac{7}{4}ee'.\sin(v-3mv+\varpi+\varpi')$$

$$+\frac{1}{4}ee'.\sin(v-mv+\varpi-\varpi')\Big]=-3\overline{m}^2\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)$$

$$+\frac{7}{2}e'.\sin(2v-3mv+\varpi')-\frac{1}{2}e'.\sin(2v-mv-\varpi')-\frac{7}{4}ee'.\sin(v-3mv+\varpi+\varpi')$$

$$+\frac{1}{4}(4+2m)ee'.\sin(v-mv+\varpi-\varpi')-\frac{1}{2}(4+4m)e\Big(1-\frac{5}{2}e'^2\Big)\sin(v-2mv+\varpi)$$

$$-\frac{21}{4}(1+2m)ee'.\sin(v-3mv+\varpi+\varpi')\Big].$$

Интегрируя, находимъ

$$-\frac{3m'}{h^2}\int\frac{u'^3\,dv}{u^4}\sin(2v-2v')=3\overline{m}^2\Big[\frac{(1-\frac{5}{2}e'^2)}{2-2m}\cos(2v-2mv)+$$

$$+\frac{7e'}{2(2-3m)}\cos(2v-3mv+\varpi')-\frac{e'}{2(2-m)}\cos(2v-mv-\varpi')$$

$$-\frac{7(2+3m)}{2(1-3m)}ee'\cos(v-3mv+\varpi+\varpi')+\frac{(4+2m)}{4(1-m)}ee'\cos(v-mv+\varpi-\varpi')$$

$$-\frac{(4+4m)e(1-\frac{5}{2}e'^2)}{2(1-2m)}\cos(v-2mv+\varpi)\;\ldots\ldots(38a)$$

Это нужно еще помножить на $\frac{d^2u}{dv^2}+u$, или на

$$\frac{1}{a}\{1+(1-c^2)e.\cos(cv-\varpi)+\ldots\}$$

Прибавочные члены отъ умноженія на $(1-c^2)e\cos(cv-\varpi)$ не дають ничего замѣтнаго, такъ-какъ $1 - c^2$ есть величина порядка m^2, поэтому умножаемъ (38 a) просто на $\frac{1}{a_1}$. Итакъ можно принять:

$$-\frac{3m'}{h^2}\left(\frac{d^2u}{dv^2}+u\right)\int\frac{u'^3\,dv}{u^4}.\sin(2v-2v') = -\frac{3m'}{a_1.h^2}\int\frac{u'^3\,dv}{u^4}.$$
$$\sin(2v-2v')\ldots\ (38b)$$

Въ первое изъ уравненій (34) входитъ еще членъ $-\frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}$.

Такъ-какъ мы положили $s=0$, то этотъ членъ даетъ только $-\frac{1}{a_1}$.

24. Окончивъ разложеніе членовъ, мы могли-бы уже теперь приступить къ приведенію полученныхъ выраженій и затѣмъ къ интегрированію нашего уравненія, но по отношенію къ коэффиціенту при $\cos(cv-\varpi)$ достигнутая степень приближенія оказывается недостаточною. Намъ нужно получить коэффиціентъ при $\cos(cv-\varpi)$ съ точностью до величинъ $m^3e'^2$, и для этого необходимо дополнить полученныя выраженія варіаціями всѣхъ членовъ 1-го уравненія.

Такъ-какъ въ полномъ выраженіи интеграла этого уравненія каждому члену уравненія будетъ соотвѣтствовать членъ съ тѣмъ-же аргументомъ и также подъ знакомъ косинуса, то очевидно мы можемъ представить полную величину u, которая получится по интегрированіи нашего ур., въ видѣ $u=\frac{1}{a}(1+e\cos(cv-\varpi)+\ldots)+\delta u$, гдѣ δu означаетъ совокупность періодическихъ членовъ, выражающихъ неравенства u, которыя происходятъ отъ возмущающаго дѣйствія Солнца; подъ символомъ a мы здѣсь, какъ уже упомянуто, разумѣемъ постоянную часть возмущенной величины радіуса вектора, которая становится равною a_1, если исключить возмущенія. Положимъ

$$a\delta u = A\cos(2v-2mv)+Be\cos(v-2mv+\varpi)+Ce'.\cos(2v-mv-\varpi')$$
$$+De'.\cos(2v-3mv+\varpi')+E.e'.\cos(mv-\varpi')+Fee'.\cos(v-mv+\varpi-\varpi')$$
$$+Gee'.\cos(v-3mv+\varpi+\varpi')+Hee'.\cos(cv+mv-\varpi-\varpi')+$$
$$+J.ee'.\cos(cv-mv-\varpi+\varpi')\ldots(39)$$

и вычислимъ члены съ аргументомъ $(cv-\varpi)$, которые произойдутъ отъ подстановки въ члены 1-го дифференціальнаго уравненія вмѣсто u величины $u + \delta u$, а вмѣсто v', $v' + \delta v'$, гдѣ $\delta v'$ измѣненіе v', соотвѣтствующее измѣненію δu.

Порядокъ коэффиціентовъ A, B, C... легко можетъ быть опредѣленъ à priori.

Въ самомъ дѣлѣ, возьмемъ уравненіе

$$u = \frac{1}{a}(1 + e\cos(cv - \tilde{\omega})) + \delta u \ldots\ldots\ldots (a)$$

умножимъ обѣ части на c^2 (c^2 равняется приблизительно, какъ мы видѣли, $1 - 3m^2$) и сложимъ почленно съ ур.

$$\frac{d^2u}{dv^2} = -\frac{ec^2}{a}\cos(cv - \tilde{\omega}) + \frac{d^2\delta u}{dv^2} \ldots\ldots\ldots (b)$$

которое получилось изъ (a) по двукратномъ дифференцированіи его. Такимъ образомъ получимъ:

$$\frac{d^2u}{dv^2} + uc^2 = \frac{d^2\delta u}{dv^2} + c^2\delta u - \frac{c^2}{a} \ldots\ldots\ldots (c)$$

Сравнивая это уравненіе съ даннымъ дифференціальнымъ уравненіемъ для u, которое имѣетъ видъ $\frac{d^2u}{dv^2} + u = u_0 + \Sigma N\cos(kv + \alpha)$, мы заключаемъ, что вторая часть уравненія (c) можетъ быть представлена въ видѣ суммы $\Sigma N'\cos(kv + \alpha)$, гдѣ коэффиціенты N' суть того-же порядка, какъ и N. Періодическая часть интеграла уравненія $\frac{d^2\delta u}{dv^2} + c^2\delta u = \frac{c^2}{a} + \Sigma N'\cos(kv + \alpha)$ состоитъ очевидно изъ суммы членовъ вида $\frac{N'}{c^2 - k^2}\cos(kv + \alpha)$, а такъ какъ коэффиціенты N въ тѣхъ членахъ, которые зависятъ отъ возмущающей силы Солнца, всѣ суть порядка $\bar{m}^2$, то и порядокъ коэффиціентовъ въ выраженіи $a\delta u$ можетъ быть опредѣленъ изъ формулы $\frac{m^2}{1 - 3m^2 - k^2}$.[1]) Отсюда слѣдуетъ, что коэффиціенты B, F, G, H и J суть 1-го порядка, A, C, D и E — втораго, т. е. порядка m^2, и потому въ предстоящемъ опредѣленіи членовъ съ аргументомъ $cv - \tilde{\omega}$ мы можемъ совершенно отбрасывать тѣ члены, въ коэффиціенты которыхъ войдутъ множители A, C, D, и E (ибо мы предположили вычислить коэффиціентъ при $\cos(cv - \tilde{\omega})$ только до величинъ m^3 и $m^3e'^2$ включительно).

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[1]) Мы докажемъ ниже, что m^2 отличается отъ $\bar{m}^2$ только на величину порядка m^4.

Перейдемъ къ опредѣленію варіацій послѣдовательныхъ членовъ уравненія для u.

Варьяція члена $\frac{3m' u'^3}{2h^2 u^3}$ равняется $-\frac{3m'.u'^3.\delta u}{2h^2 u^4}$ или $\frac{3}{au}\cdot\frac{m' u'^3}{2h^2 u^3}\cdot a\delta u$;

но по (35) и (28b) $\frac{3}{au}\cdot\frac{m' u'^3}{2h^2 u^3} = -\frac{3\bar{m}^2}{2a_1}\Big[1+\frac{3}{2}e'^2 - 4e\cos(cv-\tilde{\omega})+$

$$+3e'.\cos(mv-\tilde{\omega}')$$

$$+3(2+m)ee'.\cos(cv+mv-\tilde{\omega}-\tilde{\omega}')-3(2-m)ee'.\cos(v-mv-\tilde{\omega}+\tilde{\omega}')\Big].$$

Умножая это уравненія на $a\delta u$, мы получимъ, какъ легко видѣть, аргументъ $cv - \tilde{\omega}$ только изъ комбинаціи аргументовъ $(cv+mv-\tilde{\omega}-\tilde{\omega}')$ и $(cv - mv - \tilde{\omega} + \tilde{\omega}')$ въ (39) съ $(mv - \tilde{\omega}')$ въ выраженіи $\frac{3.m'u'^3}{au.2h^2u^3}$.

Такимъ образомъ мы находимъ:

$$-\frac{3m'u'^3.\delta u}{2h^2 u^4} = -\frac{3\bar{m}^2}{2a_1}\left[\frac{3}{2}(H+J)\,e'^2.e\cos(cv-\tilde{\omega})\right]\ldots.(40)$$

Далѣе, варьяція члена $\frac{3m' u'^3}{2h^2u^3}\cdot\cos(2v-2v')$ равняется

$$-\frac{9m'.u'^3}{2h^2 u^4}\cdot\delta u.\cos(2v-2v')+\frac{3m' u'^3}{h^2 u^3}\cdot\delta v'.\sin(2v-2v').$$

Если мы продифференцируемъ выраженіе $-\frac{3m'}{h^2}\int\frac{u'^3\,dv}{u^4}\cdot\sin(2v-2v')$ по dv и помножимъ производную на $\frac{3}{4a}$, то получимъ въ первой части $-\frac{9m' u'^3}{4h^2u^4a}\cdot\sin(2v-2v')$, а во 2-й должны перемѣнить множитель $3\bar{m}^2$ (см. ур. 38) на $-\frac{9\bar{m}^2}{4a_1}$, cos на sin, а также отбросить въ знаменателяхъ дѣлителей $(2-2m)$ и пр. Затѣмъ останется только подставить вмѣсто $2v$ во всѣ аргументы косинусовъ $2v+90^\circ$, и мы получимъ выраженіе

$$-\frac{9\bar{m}^2}{4}\cdot\frac{u'^4}{h^2u^4a_1}\cdot\cos(2v-2v')$$

Дифференцируя обѣ части уравненія (38), имѣемъ, по умноженіи на $\frac{3}{4a}$:

$$-\frac{9}{4}\cdot\frac{m'\,u'^3}{h^2u^4a}\cdot\sin(2v-2v')=-\frac{9\overline{m}^2}{4a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)$$

$$-2(1+m)e.\sin(v-2mv+\tilde{\omega})+\frac{7e'}{2}\cdot\sin(2v-3mv+\tilde{\omega}')-$$

$$-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')$$

$$-\frac{7(2+3m)}{2}ee'.\sin(v-3mv+\tilde{\omega}+\tilde{\omega}')+\frac{(2+m)ee'}{2}\sin(v-mv+\tilde{\omega}-\tilde{\omega}')\Big]$$

Теперь мѣняемъ $2v$ на $2v+90°$:

$$-\frac{9}{4}\cdot\frac{m'\,u'^3}{h^2u^4a}\cdot\cos(2v-2v')=-\frac{9\overline{m}^2}{4a}\Big[\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)$$

$$-2(1+m)e.\cos(v-2mv+\tilde{\omega})+\frac{7e'}{2}\cdot\cos(2v-3mv+\tilde{\omega}')-$$

$$-\frac{e'}{2}\cdot\cos(2v-mv-\tilde{\omega}')$$

$$-\frac{7(2+3m)ee'}{2}\cdot\cos(v-3mv+\tilde{\omega}+\tilde{\omega}')+\frac{(2+m)ee'}{2}\cos(v-mv+\tilde{\omega}-\tilde{\omega}')\Big]$$

Это выраженіе нужно умножить на $a\delta u$.

Мы отберемъ въ обоихъ выраженіяхъ только тѣ члены, аргументы которыхъ, комбинируясь между собою, могутъ дать $cv-\tilde{\omega}$. Такимъ образомъ составляемъ:

$$-\frac{9}{4}\cdot\frac{m'\,u'^3}{h^2u^4a}\cdot\cos(2v-2v').a\delta u=-\frac{9}{2}\cdot\frac{\overline{m}^2}{a_1}\Big\{\Big(1-\frac{5}{2}e'^2\Big)\cdot\cos(2v-2mv).$$

$$Be.\cos(v-2mv+\tilde{\omega})$$

$$+\frac{7e'}{2}\cdot\cos(2v-2mv+\tilde{\omega}').\,Gee'.\cos(v-3mv+\tilde{\omega}+\tilde{\omega}')-$$

$$-\frac{1}{2}e'.\cos(2v-mv-\tilde{\omega}')\,Fee'.\cos(v-mv+\tilde{\omega}-\tilde{\omega}')\Big\}$$

$$=-\frac{9}{4}\cdot\frac{\overline{m}^2}{a_1}\Big[Be\Big(1-\frac{5}{2}e'^2\Big)+\frac{7}{2}\cdot Gee'^2-\frac{1}{2}F.ee'^2\Big]\cos(cv-\tilde{\omega})\cdots(41)$$

Второй членъ варьяціи даетъ только одинъ членъ съ требуемымъ аргументомъ. Такъ-какъ приблизительно

$$\frac{dt}{dv} = \frac{1}{hu^2} \text{ или } \frac{dt}{dv} = \frac{1}{h}(u + \delta u)^{-2} = \frac{1}{hu^2}\left(1 - \frac{2\delta u}{u}\right) = \frac{1}{hu^2}(1 - 2\,a\delta u),$$

то какой-нибудь членъ вида $k \cos(iv + \beta)$, входящій въ выраженіе δu, даетъ въ предъидущемъ дифференціальномъ уравненіи для t членъ вида $-2k.\cos(iv + \beta)$, а слѣдовательно по интегрированіи въ выраженіи $nt + \varepsilon$ появляется членъ $-\frac{2k}{i}\sin(iv + \beta)$, а по уравненію (32) въ $\delta v'$ членъ $-\frac{2mk}{i}\sin(iv + \beta)$, такъ-что можно принять

$$\delta v' = -\sum \frac{2mk}{i} \cdot \sin(iv + \beta),$$

откуда варьяція

$$\frac{3m'u'^3}{h^2u^3}\,\delta v'.\sin(2v - 2v') = -\frac{3m'u'^3}{h^2u^3} \cdot \sin(2v - 2v').2m \sum \frac{k}{i} \sin(iv + \beta).$$

Но $\sum \frac{k}{i} \cdot \sin(iv + \beta) =$ выраженію $a\delta u$, въ которомъ вмѣсто косинусовъ взяты синусы и коэффиціенты раздѣлены на i. Возьмемъ-же второй членъ выраженія $a\delta u$, перемѣнимъ въ немъ cos на sin и составимъ произведеніе этого члена на

$$-2m.\frac{3m'\,u'^3}{h^2\,u^3} \cdot \sin(2v - 2v')$$

Мы получимъ такимъ образомъ:

$$-4m.\frac{3m'\,u'^3}{2h^2\,u^3} \cdot \sin(2v - 2v').Be\sin(v - 2mv + \varpi)$$

Въ виду незначительности множителя m, мы ограничимся только главнымъ членомъ въ разложеніи

$$\frac{3m'\,u'^3}{2h^2\,u^3} \cdot \sin(2v - 2v'),$$

данномъ на стр. 60, т. е. членомъ

$$\frac{3\overline{m}^2}{2a_1}\left[\left(1 - \frac{5}{2}e'^2\right) \cdot \sin(2v - 2mv)\right.$$

Перемножая, получимъ для искомой варьяціи:

$$-\frac{3\overline{m}^2}{2a_1}\left(1-\frac{5}{2}e'^2\right).\sin(2v-2mv).4m.Be\sin(v-2mv+\tilde\omega)=$$

$$-\frac{3\overline{m}^2}{a_1}\left(1-\frac{5}{2}e'^2\right).Bme\cos(cv-\tilde\omega)\ldots\ldots\ldots\ldots\ldots\ldots(42)$$

Варьяція члена $-\frac{3m'.u'^3}{2h^2u^4}\cdot\frac{du}{dv}.\sin(2v-2v')$ равняется

$$\frac{6m'u'^3}{h^2u^5}\cdot\frac{du}{dv}\cdot\delta u.\sin(2v-2v')-\frac{3m'.u'^3}{2h^2u^4}\cdot\frac{d\delta u}{dv}\cdot\sin(2v-2v')$$

$$+\frac{3m'u'^3.\delta v'}{h^2u^4}\cdot\frac{du}{dv}\cos(2v-2v')\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(43)$$

Но мы имѣемъ

$$\frac{6m'.u'^3}{h^2u^5}\cdot\frac{du}{dv}\cdot\delta u.\sin(2v-2v')=-\frac{3m'.u'^3}{h^2u^4}\cdot\frac{du}{dv}\cdot\sin(2v-2v').-\frac{4\delta u}{u}$$

или приблизительно:

$$-\frac{3m'.u'^3}{2h^2u^4}\cdot\frac{du}{dv}\cdot\sin(2v-2v').-4a\delta u.$$

Разсматривая выраженія (37 b) и (39), мы легко убѣждаемся, что произведеніе ихъ не даетъ ни одного члена разсматриваемаго порядка съ аргументомъ $(cv-\tilde\omega)$

Варьяція

$$-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{d\delta u}{dv}\cdot\sin(2v-2v')=-\left\{\frac{3m'u'^3}{2h^2u^4a}\cdot\sin(2v-2v')\right\}\frac{d(a\delta u)}{dv}.$$

Но

$$-\left\{\frac{3m'u'^3}{2h^2u^4a}\cdot\sin(2v-2v')\right\}=\frac{1}{2a}\cdot\frac{d}{dv}\left[-\frac{3m'}{h^2}\int\frac{u'^3dv}{u^4}\cdot\sin(2v-2v')\right]$$

$$=\frac{3\overline{m}^2}{2a_1}\left[-\left(1-\frac{5}{2}e'^2\right)\sin(2v-2mv)+2(1+m)e.\sin(v-2mv+\tilde\omega)\right.$$

$$-\frac{7e'}{2}\cdot\sin(2v-3mv+\tilde\omega')+\frac{1}{2}e'.\sin(2v-mv-\tilde\omega')+\frac{7(2+3m)}{2}ee'.$$

$$\left.\sin(v-3mv+\tilde\omega+\tilde\omega')-\frac{(2+m)}{2}\cdot ee'.\sin(v-mv+\tilde\omega-\tilde\omega')\right].$$

Умножая это на $\frac{d\,(a\delta u)}{dv}$, находимъ:

$$\frac{3\overline{m}^2}{2a_1}\left[\left(1-\frac{5}{2}e'^2\right).Be\sin(2v-2mv).\sin(v-2mv+\tilde{\omega}).(1-2m)\right.$$

$$-\frac{7e'^2}{2}\cdot eG.\sin(2v-3mv+\tilde{\omega}').\sin(v-3mv+\tilde{\omega}+\tilde{\omega}')(1-3m)$$

$$\left.+\frac{1}{2}e'^2.eF.\sin(2v-mv+\tilde{\omega}').\sin(v-mv+\tilde{\omega}-\tilde{\omega}')(1-m)\right].$$

Отсюда:

$$-\frac{3m'\,u'^3}{2h^2u^4}\cdot\frac{d(\delta u)}{dv}\cdot\sin(2v-2v')$$

$$=\frac{3\overline{m}^2}{4a_1}\left[B\left(1-\frac{5}{2}e'^2\right)+\frac{7}{2}\cdot Ge'^2-\frac{1}{2}F'.e'^2\right]e.\cos(cv-\tilde{\omega})\cdots(44)$$

Наконецъ 3-й членъ выраженія (43), въ который входитъ факторъ $\delta v'$, даетъ, какъ легко видѣть, только совершенно незначительные члены съ аргументомъ $(cv-\tilde{\omega})$.

Намъ остается разсмотрѣть варіяцію члена

$$-\frac{3m'}{h^2}\left(\frac{d^2u}{dv^2}+u\right)\int\frac{u'^3\,dv}{u^4}.\sin(2v-2v'),$$

который мы для краткости обозначимъ черезъ $-N\int Mdv$.

Мы имѣемъ

$$-\delta\left(N\int Mdv\right)=-N\int\left(\left(\frac{dM}{du}\right)\cdot\delta u+\left(\frac{dM}{dv'}\right)\cdot\delta v'\right)dv$$

$$-\delta N\int Mdv-N\int\left(\frac{dM}{du'}\right)\cdot\delta u'.dv$$

Если положимъ $\frac{d^2u}{dv^2}+u=\frac{1}{a}$, то первый членъ будетъ равенъ $\frac{12m'}{h^2a}\int\frac{u'^3dv}{u^4}.\frac{\delta u}{u}.\sin(2v-2v')$

Чтобы получить выраженіе этого члена, продифференцируемъ уравненіе (38):

$$-\frac{3m'}{h^2}\cdot\frac{u'^3\,dv}{u^4}\sin(2v-2v') = -3\overline{m}^2\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)$$

$$-2(1+m)e\sin(v-2mv+\tilde\omega)+\frac{7e'}{2}.\sin(2v-3mv+\tilde\omega')$$

$$-\frac{1}{2}e'.\sin(2v-mv-\tilde\omega')-\frac{7}{2}(2+3m)ee'.\sin(v-3mv+\tilde\omega+\tilde\omega')$$

$$+\frac{1}{2}(2+m)ee'.\sin(v-mv+\tilde\omega-\tilde\omega')\Big]$$

Замѣчая, что $-\frac{4}{a^2 u} = -\frac{4}{a}(1-e\cos(cv-\omega)-\ldots)$,

находимъ, по перемноженіи этихъ равенствъ:

$$\frac{12m'}{h^2a^2}\cdot\frac{u'^3\,dv}{u^4}\cdot\frac{1}{u}.\sin(2v-2v') = \frac{12\overline{m}^2}{a_1}dv\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)$$

$$-\Big(\frac{5}{2}+2m\Big)e.\sin(v-2mv+\tilde\omega)+\frac{7}{2}e'.\sin(2v-3mv+\tilde\omega')$$

$$-\frac{e'}{2}\sin(2v-mv-\tilde\omega')-\frac{7}{2}\Big(\frac{5}{2}+3m\Big)ee'.\sin(v-3mv+\tilde\omega+\tilde\omega')$$

$$+\frac{1}{2}\Big(\frac{5}{2}+m\Big)ee'.\sin(v-mv+\tilde\omega-\tilde\omega')\Big]$$

Помножимъ это равенство на $a\delta u$ и оставимъ во 2-й части произведенія только тѣ члены, изъ которыхъ образуются синусы аргумента $cv-\tilde\omega$:

$$\frac{12m'}{h^2a^2}\cdot\frac{u'^3\,dv}{u^4}\cdot\frac{a\delta u}{u}.\sin(2v-2v) = \frac{12\overline{m}^2}{a_1}dv\Big[\Big(1-\frac{5}{2}e'^2\Big).\sin(2v-2mv)$$

$$.Be.\cos(v-2mv+\tilde\omega)$$

$$+\frac{7}{2}e'.\sin(2v-3mv+\tilde\omega').\,Gee'.\cos(v-3mv+\tilde\omega+\tilde\omega')$$

$$-\frac{1}{2}e'.\sin(2v-mv-\tilde\omega').\,Fee'.\cos(v-mv+\tilde\omega-\tilde\omega')$$

$$= \frac{6\bar{m}^2}{a_1} dv \left[\left(1 - \frac{5}{2} e'^2\right) Be . \sin(cv - \tilde{\omega}) - \frac{7}{2} e'^2 Ge \sin(cv - \tilde{\omega}) - \frac{1}{2} e'^2 . Fe \sin(cv - \tilde{\omega})\right] \sin(cv - \tilde{\omega})$$

Интегрируя это выраженіе, находимъ

$$\frac{12m'}{h^2 a} \int \frac{u'^3 dv}{u^4} . \frac{\delta u}{u} . \sin(2v - 2v') = \frac{6\bar{m}^2}{a_1} \left\{ -B\left(1 - \frac{5}{2} e'^2\right) - \frac{7}{2} Ge'^2 + \frac{1}{2} Fe'^2 \right\} e \cos(cv - \tilde{\omega}) \ldots\ldots\ldots\ldots (45)$$

Слѣдующій членъ варьяціи т. е. $- N \int \frac{dM}{dv'} \cdot \delta v' dv$ равенъ $\frac{12m'}{h^2 a} \int \frac{u'^3 dv}{u^4} . \frac{1}{2} \delta v' . \cos(2v - 2v')$. Это выраженіе даетъ члены порядка $\frac{\delta v'}{a \delta u}$, т. е. m, въ сравненіи съ членами предъидущаго, поэтому въ выраженіи $- \frac{9}{4} \frac{m' u'^3}{h^2 u^4 . a} \cos(2v - 2v')$, (см. стр. 65), съ помощью котораго можетъ быть составленъ искомый членъ варьяціи, мы возьмемъ только первый, наибольшій, членъ:

$$- \frac{9\bar{m}^2}{4a_1}\left(1 - \frac{5}{2} e'^2\right) \cos(2v - 2mv)$$

Замѣчая, что вообще $\delta v' = - 2m \sum \frac{k}{i} \sin(iv + \beta)$, гдѣ $\Sigma k \sin(iv + \beta)$ представляетъ сумму членовъ выраженія подобнаго $a\delta u$, только съ перемѣною cos на sin, мы возьмемъ изъ этой суммы только членъ $- 2m Be . \sin(v - 2mv + \tilde{\omega})$, такъ-какъ аргументъ $v - 2mv + \tilde{\omega}$, комбинируясь съ $2v - 2mv$, даетъ $cv - \tilde{\omega}$. [1])

Итакъ, умножая $- \frac{9\bar{m}^2}{4a_1} \left(1 - \frac{5}{2} e'^2\right) . \cos(2v - 2mv)$ на $- \frac{8}{3} \delta v' . dv$ или на $\frac{16}{3} . mBe . \sin(v - 2mv + \tilde{\omega})\, dv$, получаемъ:

$$\frac{6\bar{m}^2}{a_1} Bm . \sin(cv - \tilde{\omega}) . dv, \text{ откуда}$$

$$\frac{12m'}{h^2 a} \int \frac{u'^3 dv}{u^4} \cdot \frac{1}{2} \delta v' . \cos(2v - 2v') = - \frac{6\bar{m}^2}{a_1} \cdot Bme \cos(cv - \tilde{\omega}) \left(1 - \frac{5}{2} e'^2\right) \ldots (46)$$

[1]) Полный видъ аргумента $v - 2mv + \tilde{\omega}$ есть $2v - 2mv - cv + \tilde{\omega}$.

Третій членъ разсматриваемой варьяціи равенъ

$$-\left(\frac{d^2\,\delta u}{dv^2}+\delta u\right)\int\frac{3m'u'^3}{h^2\,u^4}\,.\sin(2v-2v')\,dv$$

Составляя сначала $\frac{d^2\,\delta u}{dv^2}+\delta u$, находимъ:

$$\frac{d^2\,\delta u}{dv^2}+\delta u=\frac{1}{a}\Big[A(1-(2-2m)^2)\cos(2v-2mv)+Be(1-(1-2m)^2)$$
$$\cos(v-2mv+\tilde\omega)$$

$$+Ce'[1-(2-m)^2]\cos(2v-mv-\tilde\omega')+De'.(1-(2-3m)^2).$$
$$\cos(2v-3mv+\tilde\omega')$$

$$+Ee'.(1-m^2)\cos(mv-\tilde\omega')+Fee'.(1-(1-m)^2)\cos(v-mv+\tilde\omega-\tilde\omega')$$

$$+Gee'.(1-(1-3m)^2)\cos(v-3mv+\tilde\omega+\tilde\omega')+Hee'(1-(1+m)^2).$$
$$\cos(v+mv-\tilde\omega-\tilde\omega')$$

$$+Jee'.(1-(1-m)^2).\cos(v-mv-\tilde\omega+\tilde\omega')\Big]$$

Такъ какъ по приведеніи 4 послѣдніе члена этого выраженія оказываются заключающими множителя m, то при предстоящемъ умноженіи $\frac{d^2\,\delta u}{dv^2}+\delta u$ на $-\frac{3m'}{h^2}\int\frac{u'^3\,dv}{u^4}.\sin(2v-2v')$ мы ихъ опустимъ.

Произведеніе $-\left(\frac{d^2\,\delta u}{dv^2}+\delta u\right)\frac{3m'}{h^2}\int\frac{u'^3\,dv}{u^4}\sin(2v-2v')$ (см. ур. 38) даетъ:

$$\dot{3}\overline{m}^2\left[\frac{(1-\frac{5}{2}e'^2}{2-2m}.\cos(2v-2mv).Be(1-(1-2m)^2).\cos(v-2mv+\tilde\omega)\right]$$

$$=\frac{3\,\overline{m}^2}{2a_1}\left[\frac{4m\,Be\,(1-\frac{5}{2}e'^2)}{2}\right].\cos(cv-\tilde\omega')\,.\,.\,.\,.\,.\,.(47)$$

Что касается до послѣдняго члена, то-есть

$$-\left(\frac{d^2\,u}{dv^2}+u\right)\int\frac{9m'\,u'^2}{h^2\,u^4}.\delta u'\,dv.\sin(2v-2v'),$$

то онъ не даетъ ничего замѣтнаго. Въ самомъ дѣлѣ мы имѣемъ при-

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близительно $a'u'=1+e'\cos(v'-\tilde{\omega}')$, $a'\delta u' = -e'\delta v'.\sin(v'-\tilde{\omega}')$, а такъ-какъ $\delta v'$ есть величина порядка $\overline{m}a\delta u$, то $a'\delta u'$ — порядка $\overline{m}e'.\ a\delta u$, т. е. по крайней мѣрѣ 3-го, такъ-что разсматриваемое выраженіе можетъ дать только величины 6-го порядка.

Такимъ образомъ полная варьяція интеграла

$$-\frac{3m'}{h^2}\left(\frac{d^2u}{dv^2}+u\right)\int\frac{u'^3\,dv}{u^4}.\sin(2v-2v')$$

даетъ слѣдующіе члены съ аргументомъ $(cv-\tilde{\omega})$:

$$\frac{6\overline{m}^2}{a_1}\left\{-B\left(1-\frac{5}{2}e'^2\right)-\frac{1}{2}Bm\left(1-\frac{5}{2}e'^2\right)-\frac{7}{2}Ge'^2+\right.$$
$$\left.+\frac{1}{2}Fe'^2\right\}e\cos(cv-\tilde{\omega})\ldots\ldots\ldots\ldots\ (48)$$

25. Подставимъ теперь въ первое изъ уравненій (34) полученныя нами разложенія его членовъ и ихъ варіацій по уравненіямъ: (35), (36,) (37*b*), (38*a*), (38*b*), (40), (41), (42), (44), (45), (46), (47) и (48), причемъ сгруппируемъ вмѣстѣ члены съ одинаковыми аргументами.

Мы получимъ такимъ образомъ:

$$\frac{d^2u}{dv^2}+u-\frac{1}{a_1}+\frac{\overline{m}^2}{2a_1}\left(1+\frac{3}{2}e'^2\right)-\frac{3\overline{m}^2}{4a_1}\left\{2+3e'^2+3(H+J)e'^2\right.$$
$$+3B\left(1-\frac{5}{2}e'^2\right)+\frac{21}{2}\cdot Ge'^2-\frac{3}{2}\cdot Fe'^2+4Bm\left(1-\frac{5}{2}e'^2\right)$$
$$-B\left(1-\frac{5}{2}e'^2\right)-\frac{7}{2}Ge'^2+\frac{1}{2}Fe'^2+8B\left(1-\frac{5}{2}e'^2\right)$$
$$\left.+4Bm\left(1-\frac{5}{2}e'^2\right)+28Ge'^2-4Fe'^2\right\}e\cos(cv-\tilde{\omega})$$
$$+\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\left\{\left(1-\frac{5}{2}e'^2\right)+\frac{(1-\frac{5}{2}e'^2)}{1-m}\right\}\cos(2v-2mv)$$
$$+\frac{3\overline{m}^2}{a_1}\left\{-\frac{1}{4}(3+4m)\left(1-\frac{5}{2}e'^2\right)+\frac{1}{4}\left(1-\frac{5}{2}e'^2\right)\right.$$
$$\left.-\frac{2(1+m)}{1-2m}\left(1-\frac{5}{2}e'^2\right)\right\}e\cos(v-2mv+\tilde{\omega})$$

$$-\frac{3\overline{m}^2}{2a_1}\left\{\frac{1}{2}+\frac{1}{2-m}\right\}e'.\cos(2v-mv-\tilde{\omega}')$$

$$+\frac{3'}{|2}\frac{\overline{m}^2}{a_1}\left\{\frac{7}{2}+\frac{7}{2-3m}\right\}e'.\cos(2v-3mv+\tilde{\omega}'.)$$

$$+\frac{3}{2}\frac{\overline{m}^2}{a_1}\cdot e'.\cos(mv-\tilde{\omega}')+$$

$$+\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\left\{\frac{1}{4}(3+2m)-\frac{1}{4}+\frac{(2+m)}{1-m}\right\}ee'.\cos(v-mv+\tilde{\omega}-\tilde{\omega}')$$

$$-\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\left\{\frac{21}{4}(1+2m)-\frac{7}{4}+\frac{7(2+3m)}{1-3m}\right\}ee'.\cos(v-3mv+\tilde{\omega}+\tilde{\omega}')$$

$$-\frac{3}{2}\frac{\overline{m}^2}{a_1}.\left(\frac{3+2m}{2}\right)ee'.\cos(cv+mv-\tilde{\omega}-\tilde{\omega}')$$

$$-\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\left(\frac{3-2m}{2}\right)ee'\cdot\cos(cv-mv-\tilde{\omega}+\tilde{\omega}')=0\;\ldots\ldots(49)$$

Коэффиціенты при послѣднихъ 9 членахъ мы опредѣлимъ съ точностью лишь до величинъ m^2; поэтому, отбрасывая m вездѣ, гдѣ эта буква входитъ внутри скобокъ въ коэффиціентахъ, мы замѣнимъ 9 послѣднихъ періодическихъ членовъ уравненія (49) слѣдующими:

$$\frac{3\overline{m}^2}{a_1}\left(1-\frac{5}{2}e'^2\right)\cos(2v-2mv)-\frac{15}{2a_1}\cdot\left(1-\frac{5}{2}e'^2\right)\overline{m}^2\cdot e\cdot\cos v-2mv+\tilde{\omega})$$

$$-\frac{3}{2}\frac{m^2}{a_1}e'\cdot\cos(2v-mv-\tilde{\omega}')+\frac{21}{2}\cdot\frac{\overline{m}^2}{a_1}\cdot e'\cdot\cos(2v-3mv+\tilde{\omega}')$$

$$+\frac{3}{2.}\frac{\overline{m}^2}{a_1}\cdot e'\cdot\cos(mv-\tilde{\omega}')+\frac{15}{4}\cdot\frac{\overline{m}^2}{a_1}\cdot ee'\cdot\cos(v-mv+\tilde{\omega}-\tilde{\omega}')$$

$$-\frac{105}{4}\cdot\frac{\overline{m}^2}{a_1}\cdot ee'\cdot\cos(v-3mv+\tilde{\omega}+\tilde{\omega}')-\frac{9}{4}\cdot\frac{\overline{m}^2}{a_1}ee'\cdot\cos(cv+mv-\tilde{\omega}-\tilde{\omega}')$$

$$-\frac{9}{4}\cdot\frac{\overline{m}^2}{a_1}\cdot ee'\cdot\cos(cv-mv-\tilde{\omega}+\tilde{\omega}')\ldots\ldots\ldots\ldots\ldots\ldots(50)$$

Обозначивъ черезъ δu ту часть величины u, которая происходитъ отъ возмущеній, мы положили (стр. 62):

$$u=\frac{1}{a}\left(1+e\cos(cv-\tilde{\omega})\right)+\delta u,$$

и выразили u въ видѣ суммы періодическихъ членовъ съ неопредѣленными коэффиціентами.

Дифференцируя теперь это уравненіе два раза и обращая вниманіе на уравненіе (39), опредѣляющее δu, мы немедленно получимъ величины коэффиціентовъ $A, B, C \ldots J$, посредствомъ подстановки величинъ $\frac{d^2u}{dv^2}$ и u въ уравненіе (49 — 50) и сравненія коэффиціентовъ при одинаковыхъ косинусахъ.

Оставляя пока въ сторонѣ аргументъ $cv - \tilde{\omega}$, и замѣчая, что величина $\frac{d^2\delta u}{dv^2} + \delta u$ у насъ уже составлена на стр. (71), мы легко получаемъ:

$$A(1-(2-2m)^2) = -3\overline{m}^2 \cdot \frac{a}{a_1}\left(1-\frac{5}{2}e'^2\right)$$

$$B(1-(1-2m)^2) = \frac{15}{2} \cdot \overline{m}^2 \cdot \frac{a}{a_1}\left(1-\frac{5}{2}e'^2\right)$$

$$C(1-(2-m)^2) = \frac{3}{2} \cdot \overline{m}^2 \cdot \frac{a}{a_1}$$

$$D(1-(2-3\overline{m})^2) = -\frac{21}{2} \cdot \overline{m}^2 \cdot \frac{a}{a_1}$$

$$E(1-m^2) = -\frac{3}{2} \cdot \overline{m}^2 \cdot \frac{a}{a_1}$$

$$F(1-(1-m)^2) = -\frac{15}{4} \cdot \overline{m}^2 \cdot \frac{a}{a_1}; \quad G(1-(1-3m)^2) = \frac{105}{4}\overline{m}^2 \cdot \frac{a}{a_1}$$

$$H(1-(1+m)^2) = \frac{9}{4} \cdot \overline{m}^2 \cdot \frac{a}{a_1}; \quad J(1-(1-m)^2) = \frac{9}{4} \cdot \overline{m}^2 \cdot \frac{a}{a_1}$$

Что касается до отношенія $\frac{a}{a_1}$, то оно легко находится изъ разсмотрѣнія постоянныхъ членовъ въ уравненіи 49 послѣ подстановки величины $\frac{d^2u}{dv^2} + u$.

Мы получаемъ очевидно:

$$\frac{1}{a} = \frac{1}{a_1} - \frac{\overline{m}^2}{2a_1}\left(1+\frac{3}{2}e'^2\right),$$

откуда

$$a = a_1 + \frac{\overline{m}^2 a_1}{2}\left(1+\frac{3}{2}e'^2\right)$$

или, съ точностью до величинъ 4-го порядка,

$$a = a_1 + \frac{\overline{m}^2 a_1}{2}\left(1 + \frac{3}{2} e'^2\right) \ldots\ldots\ldots\ldots (51)$$

Такъ-какъ мы вычисляемъ коэффиціенты A, B, $C \ldots J$ съ точностью только до величинъ порядка m^2, то мы можемъ положить въ предъидущихъ уравненіяхъ $\frac{a}{a_1} = 1$, а также $m^2 = \overline{m}^2$ [1]). Рѣшая ихъ, мы получимъ:

$$A = m^2\left(1 - \frac{5}{2} e'^2\right);\ E = -\frac{3}{2} m^2$$

$$B = +\frac{15}{8} m\left(1 - \frac{5}{2} e'^2\right);\ F = -\frac{15}{8} m$$

$$C = -\frac{1}{2} m^2;\ G = +\frac{35}{8} m$$

$$D = +\frac{7}{2} m^2;\ H = -\frac{9}{8} m \text{ и } J = +\frac{9}{8} m \ldots\ldots\ldots\ldots (52)$$

26. Теперь мы можемъ опредѣлить и коэффиціентъ при $\cos(cv - \tilde{\omega})$.

Обозначая этотъ коэффиціентъ черезъ K, мы находимъ:

$$K = -\frac{3\overline{m}^2}{4a_1}\left\{2 + 3e'^2 + B\left(1 - \frac{5}{2} e'^2\right)(3 - 1 + 8 = 10)\right.$$

$$\left. + \frac{7}{2} G . e'^2 (3 - 1 + 8 = 10) - \frac{1}{2} F e'^2 (3 - 1 + 8 = 10)\right\} =$$

$$= -\frac{3\overline{m}^2}{4a_1}\left\{2 + 3e'^2 + \frac{75}{4} m\left(1 - \frac{5}{2} e'^2\right) + \frac{1225}{8} me'^2 + \frac{75}{8} me'^2\right.$$

$$= -\frac{3\overline{m}^2}{4a_1}\left\{2 + 3e'^2 + \frac{75}{4} m - \left(\frac{1225}{8} - \frac{750}{8} + \frac{75}{8} = \frac{275}{4} me'^2\right)\right\}$$

---

[1]) Ниже мы докажемъ, что $\overline{m}^2 = m^2\left(1 - \frac{1}{2} m^2 - \frac{3}{4} m^2 e'^2 + \ldots\right)$

Съ другой стороны, дифференцируя два раза выраженіе

$$u = \frac{1}{a}(1 + e\cos(cv - \tilde{\omega})) + \delta u,$$

мы получаемъ

$$\frac{d^2u}{dv^2} + u = \frac{1}{a}(1 - c^2)e.\cos(cv - \tilde{\omega}) + \cdots$$

Сравненіе коэффиціентовъ даетъ:

$$1 - c^2 = \frac{3m^2}{4}\left\{2 + 3e'^2 + \frac{75}{4}m + \frac{275}{4}me'^2\right\},$$

откуда

$$c = 1 - \frac{3}{4}m^2 - \frac{9}{8}m^2e'^2 - \frac{225}{32}m^3 - \frac{825}{32}m^3e'^2 + \cdots,$$

а такъ-какъ мы положили (см. стр. 50) $1 - c = \frac{d\tilde{\omega}}{dv} = \frac{d\tilde{\omega}}{ndt}$, то получается:

$$\frac{d\tilde{\omega}}{dt} = \frac{3}{4}m^2n + \frac{225}{32}m^2n + \left(\frac{9}{8}m^2 + \frac{825}{32}m^3\right)e'^2n,$$

откуда

$$\tilde{\omega} = \tilde{\omega}_1 + \left(\frac{3}{4}m^2n + \frac{225}{32}m^3n + \frac{9}{8}m^2n.E'^2 + \frac{825}{32}m^3n\,E'^2\ldots\right)t$$

$$+ \left(\frac{9}{8}m^2 + \frac{825}{32}m^3\right)\int(e'^2 - E'^2)\,ndt \cdots\cdots (53)$$

Такимъ образомъ мы нашли новый членъ $\frac{225}{32}m^3$ въ выраженіи средней скорости движенія перигея и новый членъ того-же порядка въ выраженіи вѣковаго уравненія перигея. Присоединяя сюда вѣковые члены, входящіе въ составъ выраженія для $\tilde{\omega}$, даннаго на стр. 25-й, мы находимъ коэффиціентъ при $\int(e'^2 - E'^2)\,ndt$ равнымъ:

$$\frac{9}{8}m^2 - \frac{9}{16}m^2e^2 - \frac{9}{4}m^2i^2 + \frac{825}{32}m^3$$

По подстановкѣ числовыхъ величинъ, получаемъ:

$$\left(\frac{9}{8}m^2 - \frac{9}{16}m^2 e^2 - \frac{9}{4}m^2 i^2 + \frac{825}{32}m^3\right)\int (e'^2 - E'^2)\, ndt = -$$

$$-(0.006295 - 0.000009$$

$$-0.000431 + 0.010790).\, 1260''.896\, i^2 = -7''.938 +$$

$$0''.012 + 0''.543 - 13''.605 = 20''.988.$$

Относительно уравненія (51) мы можемъ замѣтить, что оно представляетъ интегралъ дифференціальнаго выраженія для измѣненія большой полуоси, которое мы нашли на стр. 43-й.

Подставляя теперь величины коэффиціентовъ A, B, C... по ур. 52 въ ур. 39, мы находимъ:

$$u = \frac{1}{a}\Big\{1 + e\cos(cv - \tilde\omega) + m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv) +$$

$$+ \frac{15}{8}m\left(1 - \frac{5}{2}e'^2\right)e.\cos(v - 2mv + \tilde\omega) - \frac{1}{2}m^2 e'.\cos(2v - mv - \tilde\omega')$$

$$+ \frac{7}{2}m^2 e'.\cos(2v - 3mv + \tilde\omega') - \frac{3}{2}m^2 e'.\cos(mv - \tilde\omega') - \frac{15}{8}m.\, ee'.$$

$$\cos(v - mv + \tilde\omega - \tilde\omega') + \frac{35}{8}mee'.\cos(v - 3mv + \tilde\omega + \tilde\omega')$$

$$- \frac{9}{8}\cdot mee'.\cos(cv + mv - \tilde\omega - \tilde\omega') + \frac{9}{8}mee'.\cos(cv - mv - \tilde\omega + \tilde\omega')\cdots(54)$$

Перейдемъ теперь къ интегрированію 2-го изъ уравненій (34).

ГЛАВА VI.

Интегрированіе дифференціальнаго уравненія для средней долготы Луны. Обзоръ главнѣйшихъ періодическихъ неравенствъ параллакса и долготы.

27. Подставляя въ первый членъ 2-й части дифференціальнаго уравненія для t вмѣсто u величину $u + \delta u$ и разлагая $\frac{1}{(u+\delta u)^2}$ по биному, мы легко находимъ

$$dt = \frac{dv}{hu^2}\left(1 - \frac{2\delta u}{u} + \ldots\right)\left(1 + \frac{3}{2}\frac{m'}{h^2}\int\frac{u'^3 . dv}{u^4} . \sin(2v - 2v')\right)$$

$$= \frac{dv}{hu^2} - \frac{dv}{hu^2} \cdot \frac{2\delta u}{u} + \frac{3}{2} \cdot \frac{m'}{h^3 u^2} \cdot \int\frac{u'^3\, dv}{u^4} . \sin(2v - 2v') \ldots\ldots (55)$$

Остальными членами можно пренебречь. Кромѣ того, въ найденномъ выше выраженіи для δu (см. ур. 39 и 52) и въ выраженіи

$$\frac{3}{2} \cdot \frac{m'}{h^2 u^2}\int\frac{u'^3\, dv}{u^4} . \sin(2v - 2v') \ldots\ldots \text{см. ур. } (38)$$

мы отбросимъ при подстановкѣ этихъ величинъ въ (55) всѣ члены умноженные на ee'.

Первый членъ уравненія (55) даетъ, по подстановкѣ вмѣсто u его эллиптической величины:

$$\frac{dv}{hu^2} = \frac{a^{3/2}}{\sqrt{\mu}}\,(1 - 2e\cos(cv - \tilde{\omega}))$$

Замѣчая далѣе, что $h = \sqrt{\mu a_1}$ и что, въ виду малости множителя $2\delta u$, во второмъ членѣ дѣлитель u^3 можетъ быть замѣненъ черезъ a^{-3} мы находимъ:

$$-\frac{dv}{hu^2}\cdot\frac{2\delta u}{u} = -\frac{dv.a^2}{\sqrt{a_1}\sqrt{\mu}}\Big\{2m^2\Big(1 - \frac{5}{2}e'^2\Big)\cos(2v - 2mv) +$$

$$+\frac{15}{4}m\Big(1 - \frac{5}{2}e'^2\Big)\cos(v - 2mv + \tilde{\omega}) - m^2e'.\cos(2v - mv - \tilde{\omega}')$$

$$+7m^2e'.\cos(2v - 3mv + \tilde{\omega}') - 3m^2e'.\cos(mv - \tilde{\omega}')$$

Наконецъ уравненіе (38a) даетъ намъ:

$$\frac{3}{2}\cdot\frac{m'}{h^3u^2}\int\frac{u'^3\,dv}{u^4}.\sin(2v - 2v') = -\frac{3\overline{m}^2a^2}{4\sqrt{\mu a_1}}.\Big[\Big(1 - \frac{5}{2}e'^2\Big)\cos(2v - 2mv)$$

$$+\frac{7}{2}e'.\cos(2v - 3mv + \tilde{\omega}') - \frac{e'}{2}.\cos(2v - mv - \tilde{\omega}') - \ldots\Big]$$

Такимъ образомъ получается

$$dt = \frac{dv.a^2}{\sqrt{\mu a_1}}\Big\{1 - 2e\cos(cv - \tilde{\omega}) - \frac{11}{4}.m^2\Big(1 - \frac{5}{2}e'^2\Big).\cos(2v - 2mv)$$

$$-\frac{15}{4}me\Big(1 - \frac{5}{2}e'^2\Big)\cos(v - 2mv + \tilde{\omega}) + \frac{11}{8}m^2e'.\cos(2v - mv - \tilde{\omega}')$$

$$-\frac{77}{8}.m^2e'.\cos(2v - 3mv + \tilde{\omega}') + 3m^2e'.\cos(mv - \tilde{\omega}')$$

Интегрируя, умножая обѣ части равенства на n и замѣчая, что пост. часть производной $\frac{dt}{dv}$ должна быть равна $\frac{1}{n}$, и что слѣдовательно $\frac{a^2}{\sqrt{\mu a_1}} = \frac{1}{n}$ мы находимъ:

$$nt + \varepsilon = v - 2e\sin(cv - \tilde{\omega}) - \frac{11}{8}m^2\Big(1 - \frac{5}{2}e'^2\Big)\sin(2v - 2mv)$$

$$-\frac{15}{4}me\left(1-\frac{5}{2}e'^2\right)\sin(v-2mv+\varpi)+\frac{11}{16}m^2e'.\sin(2v-mv-\varpi')$$

$$-\frac{77}{16}m^2e'.\sin(2v-3mv+\varpi')+3me'.\sin(mv-\varpi')\ldots(56)$$

Таково выраженіе средней долготы Луны въ функціи истинной. Нетрудно получить отсюда и обратное выраженіе:—истинной долготы въ функціи средней. Для этого стоитъ только подставить во всѣ періодическія члены второй части уравненія (56) вмѣсто v, $nt+\varepsilon$. Дѣйствительно, примѣняя къ этому уравненію формулу Лагранжа и представляя его въ видѣ $v=nt+\varepsilon+\varphi(v)$, мы не получимъ въ квадратѣ функціи $\varphi(nt+\varepsilon)$ ни одного члена съ коэффиціентомъ того-же аналитическаго вида, какъ въ уравненіи 56; получатся только члены съ коэффиціентами me, e^2, m^2ee' и пр., которые мы условились отбрасывать.

28. Главное неравенство параллакса Луны выражается членомъ $e\cos(cv-\varpi)$, который соотвѣтствовалъ-бы простому эллиптическому неравенству, если-бы c было равно единицѣ; мы видѣли, что множитель c, весьма близкій къ 1, выражаетъ измѣненіе средней аномаліи Луны, происходящее отъ поступательнаго движенія перигея.

Второй періодическій членъ въ выраженіи параллакса равенъ $m^2\left(1-\frac{5}{2}e'^2\right)cn(2v-2mv)$. Если предположимъ, что при $v=0$ Луна была въ одномъ изъ сизигіевъ, то разность $v-mv$ представитъ приблизительно угловое разстояніе Луны отъ Солнца по долготѣ; а такъ-какъ $\cos 2(☾-☉)$ имѣетъ наибольшее значеніе, когда Луна въ сизигіяхъ, и наименьшее, когда она въ квадратурахъ, то мы заключаемъ, что — caeteris paribus — разстояніе Луны отъ Земли наименьшее въ сизигіяхъ и наибольшее въ квадратурахъ.

Если пренебречь въ выраженіи 3-го члена параллакса угломъ $2mv$, то можно соединить первый и третій члены въ выраженіи параллакса (или величины u, приблизительно ему пропорціальной) и выразить u формулою:

$$\frac{1}{a}\left[1+\left(e+\frac{15}{8}me\right)\cos(cv-\varpi)\right]\ldots\ldots\ldots(a)$$

Съ другой стороны, если предположимъ, что при $v = 0$, долгота Солнца равнялась $-90°$, т. е. что Луна была въ квадратурѣ, то, какъ легко видѣть, параллаксъ при данной величинѣ v выразится формулою

$$\frac{1}{a}\left\{1 + e\cos(cv - \bar{\omega}) + \frac{15}{8}\, me\cos(cv - \bar{\omega} + 180°)\right.$$

или

$$\frac{1}{a}\left(1 + \left(e - \frac{15}{8}\, me\right)\right)\cos(cv - \bar{\omega}) \ldots\ldots\ldots\ldots (b)$$

Формулы (a) и (b) показываютъ, что эксцентрицитетъ увеличивается на величину $\frac{15}{8}\, me$ когда ось лунной орбиты или линія апсидъ совпадаетъ съ линіей сизигіевъ и уменьшается на ту-же величину, когда линія апсидъ становится перпендикулярною къ линіи сизигіевъ. Само собою разумѣется, что эти заключенія могутъ быть приблизительно вѣрны только для короткихъ періодовъ времени, какъ напр. для времени одного обращенія Луны.

Соотвѣтственно разсмотрѣннымъ неравенствамъ параллакса, въ выраженіи долготы Луны находятся члены

$$2e\sin(cv - \bar{\omega}),\ \frac{11}{8}\, m^2\left(1 - \frac{5}{2}e'^2\right)\sin(2v - 2mv)$$

$$\text{и } \frac{15}{4}\, m\left(1 - \frac{5}{2}\, e'^2\right)\sin(v - 2mv + \bar{\omega}).$$

Второй изъ этихъ членовъ выражаетъ неравенство называемое варіаціею и, какъ видимъ, зависитъ отъ двойной разности истинныхъ долготъ Луны и Солнца.

Онъ равенъ 0, когда ☾ находится въ соединеніи; имѣетъ наибольшую положительную величину, когда разность долготъ $= 45°$, опять приближается къ 0 съ приближеніемъ Луны къ квадратурѣ и достигаетъ наибольшей отрицательной величины, когда разность долготъ ☾ — ☉ становится равной $135°$. Итакъ истинное мѣсто Луны впереди средняго отъ сизигія до квадратуры и позади его отъ квадратуры до сизигія.

Послѣдній членъ, зависящій отъ аргумента $v - 2mv + \bar{\omega}$ называется эвекціею. Этотъ членъ выражаетъ неравенство, которое послѣ эллиптическаго имѣетъ наибольшее вліяніе на мѣсто Луны въ ея орбитѣ; долгота ея отъ этого неравенства можетъ измѣняться на $\pm 1°15'$.

Въ выраженіи истинной долготы Луны есть еще членъ, зависящій отъ средней аномаліи Солнца. Выражаемое имъ неравенство называется годовымъ уравненіемъ. Когда Солнце приближается, въ своемъ относительномъ годовомъ движеніи вокругъ Земли, къ апогею, то истинное мѣсто Луны оказывается позади ея средняго мѣста, когда-же оно движется отъ апогея къ перигею, то истинное мѣсто Луны впереди средняго (конечно насколько это зависитъ отъ вліянія разсматриваемаго неравенства). Мы видѣли, что возмущающая сила Солнца обратно пропорціональна кубу разстоянія Земли отъ Солнца и потому она больше, когда Земля въ перигеліи, слѣдовательно въ этой части земной орбиты угловое движеніе Луны медленнѣе средняго, когда-же Земля движется отъ перигелія къ афелію, то скорѣе средняго. Наибольшая величина этого неравенства около $11'$. Измѣненіе въ параллаксѣ отъ годоваго уравненія почти нечувствительно.

ГЛАВА VII.

Интегрированіе дифференціальнаго уравненія для широты Луны.

29. При рѣшеніи 3-го изъ уравненія системы (34) мы положимъ для простоты эксцентрицитетъ лунной орбиты равнымъ 0 и при разложеніи членовъ этого уравненія удержимъ только члены съ аргументами: $gv - \theta$, $2v - 2mv - gv + \theta$ или $v - 2mv + \theta$ $gv + mv - \theta - \tilde{\omega}'$, $gv - mv - \theta + \tilde{\omega}'$, $2v - 2mv - gv + mv + \theta - \tilde{\omega}'$ или $v - mv + \theta - \tilde{\omega}'$ и $2v - 2mv - gv - mv + \theta + \tilde{\omega}'$ или $v - 3mv + \theta + \tilde{\omega}'$, такъ-какъ коэффиціенты этихъ членовъ при интегрированіи получаютъ малыхъ дѣлителей. Буква g имѣетъ здѣсь тоже значеніе, какъ c въ дифференціальномъ уравненіи для u.

Обозначая черезъ $(g-1)v$ обратное движеніе линіи узловъ, мы замѣняемъ въ уравненіи $s = \gamma \,.\, \sin(v - \theta)$ долготу восходящаго узла θ выраженіемъ $\theta_1 - (1-g)v$ и потомъ, отбрасывая значекъ при θ_1 разсматриваемъ θ уже какъ величину постоянную. Затѣмъ въ ходѣ вычисленія мы часто будемъ принимать въ коэффиціентахъ и въ аргументахъ $g=1$, такъ-какъ g отличается, какъ мы видѣли, отъ единицы только на величину порядка m^2. На томъ-же основаніи можно положить $a_1 = a$ и $\overline{m}^2 = m^2$.

Первый членъ въ 3-мъ ур. системы 34, который намъ нужно разложить, равенъ $\frac{3m'u'^3 . s}{2h^2 u^4}$.

Полагая $s = \gamma \,.\, \sin(gv - v)$ и $u = \frac{1}{a}$, находимъ

$$\frac{3m'u'^3}{2h^2 u^3} \cdot \frac{s}{u} = \frac{3m^2}{2a}\left(1 + \frac{3}{2}e'^2 + 3e' . \cos(mv - \tilde{\omega}') + \ldots\right) a\gamma \,.\, \sin(gv - \theta)$$

$$= \frac{3m^2\gamma}{2}\Big\{\left(1 + \frac{3}{2}e'^2\right)\sin(gv - \theta) + \frac{3}{2}e'.\sin(gv + mv - \theta - \varpi')$$

$$+ \frac{3}{2}e'.\sin(gv - mv - \theta + \varpi')\Big\}\dots\dots\dots\dots(59)$$

Разложеніе $\frac{3m'u'^3}{2h^2u^4}\cdot\cos(2v - 2v')$ даетъ намъ:

$$\frac{3m'u'^3}{3h^2u^3}\cdot\cos(2v - 2v').\frac{s}{u} = \frac{3m^2}{2}\Big\{\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv)$$

$$+\frac{7}{2}e'.\cos(2v - 3mv + \varpi') - \frac{1}{2}e'.\cos(2v - mv - \varpi')\Big\}\cdot\gamma.\sin(gv - \theta)$$

$$= \frac{3m^2}{4}\cdot\gamma\Big\{-\left(1 - \frac{5}{2}e'^2\right).\sin(v - 2mv + \theta) - \frac{7}{2}e'.\sin(v - 3mv + \theta + \varpi')$$

$$+\frac{1}{2}e'.\sin(v - mv + \theta - \varpi')\Big\}\dots\dots\dots\dots(60)$$

Далѣе находимъ:

$$-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{ds}{dv}\cdot\sin(2v - 2v') = -\frac{3m^2}{2a}\Big\{\left(1 - \frac{5}{2}e'^2\right)\sin(2v - 2mv)$$

$$+\frac{7}{2}e'.\sin(2v - 3mv + \varpi') - \frac{1}{2}e'.\sin(2v - mv - \varpi')\Big\}\cdot a\gamma.g\cos(gv - \theta)$$

$$= -\frac{3}{4}m^2g.\gamma.\Big\{\left(1 - \frac{5}{2}e'^2\right)\cdot\sin(v - 2mv + \theta) + \frac{7}{2}e'.\sin(v - 3mv + \theta + \varpi')$$

$$-\frac{1}{2}e'.\sin(v - mv + \theta - \varpi')\Big\}\dots\dots\dots\dots(61)$$

Послѣдній членъ въ ур. для s, т. е. $-\frac{3m'}{h^2}\left(\frac{d^2s}{dv^2} + s\right)\int\frac{u'^3\,dv}{u^4}\cdot\sin(2v - 2v')$ можно совершенно опустить, такъ-какъ $\frac{d^2s}{dv^2} + s = (1 - g^2)\gamma.\sin(gv - \theta)$, т. е. величина того-же порядка, какъ и интегралъ

$$-\frac{3m'}{h^2}\int\frac{u'^3\,hv}{u^4}\cdot\sin(2v - 2v'),$$

а именно порядка m^2, четвертыя-же степени m мы въ разсчетъ принимать не будемъ.

30. Положимъ теперь $s = \gamma . \sin . (gv - \theta) + \delta s =$

$\gamma \sin(gv-\theta) + M\gamma . \sin(v-2mv+\theta) + Ne'\gamma . \sin(gv+mv-\theta-\varpi')$

$+ Pe'\gamma . \sin(gv - mv - \theta + \varpi') + Qe'\gamma . \sin(v - mv + \theta - \varpi')$

$+ Re'\gamma . \sin(v - 3mv + \theta + \varpi')$ ....(62)

гдѣ δs та часть s, которая происходитъ отъ возмущающаго дѣйствія Солнца, и вычислимъ варьяціи разсмотрѣнныхъ нами членовъ, причемъ оставимъ только члены съ аргументомъ $(gv - \theta)$.

Варьяція члена $\frac{3m'u'^3 s}{2h^2 u^4}$ даетъ $\frac{3m' u'^3 \delta s}{2h^2 u^4} = \frac{3m' u'^3}{2h^2 u^3} \cdot a\delta s$.

Подставляя сюда вмѣсто $\frac{m' u'^3}{2h^2 u^3}$ разложеніе эти величины по ур. (35), а вмѣсто δs — предъидущее выраженіе по ур. (62), находимъ:

$$\frac{3m' u'^3}{2h^2 u^3} \cdot a\delta s = \frac{3m^2}{2}\left(1 + \frac{3}{2}e'^2 + 3e' . \cos(mv - \varpi')\right)\delta s =$$

$$= \frac{3m^2}{2} \cdot \delta s\left(1 + \frac{3}{2}e'^2\right) + \frac{9}{4}m^2 e'^2 \gamma . M \sin(v - 2mv + \theta) +$$

$$\frac{9}{2}m^2 e.' \cos(mv - \varpi) . \Big\{ Ne'\gamma . \sin(gv - mv - \theta - \varpi') + Pe'\gamma . \sin(gv - mv - \theta + \varpi') =$$

$$= \frac{3m^2}{2} \cdot \delta s\left(1 + \frac{3}{2}e'^3\right) + \frac{9}{4}m^2 e'^2 \gamma . M . \sin(v - 2mv + \theta) +$$

$$+ \frac{9}{4}m^2 e'^2 \gamma . (N + P) . \sin(gv - \theta) \text{ ....(63)}$$

Варьяція 4-го члена ур. для s даетъ:

$$\frac{3m' u'^3 \delta s}{2h^2 u^4} \cdot \cos(2v - 2v') = \frac{3m^2}{2a}\Big\{\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv) +$$

$$+ \frac{7}{2}e' . \cos(2v - 3mv + \varpi') - \frac{1}{2}e' . \cos(2v - mv - \omega')\Big\}\delta s =$$

$$= \frac{3m^2}{2}\gamma\Big\{\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv) . M \sin(2v - 2mv - gv + \theta)$$

$$+\frac{7}{2}\cdot e'.\cos(2v-3mv+\tilde\omega').Re'.\sin(2v-3mv-gv+\theta+\tilde\omega')$$

$$-\frac{1}{2}\cdot e'.\cos(2v-mv-\tilde\omega').Qe'.\sin(2v-mv-gv+\theta-\tilde\omega')\Big\}=$$

$$-\frac{3m^2\gamma}{4}\Big\{M\Big(1-\frac{5}{2}e'^2\Big)\sin(gv-\theta)+\frac{7}{2}e'^2.R\sin(gv-\theta)-$$
$$-\frac{1}{2}e'^2.Q\sin(gv-\theta)\Big\}\ldots(64)$$

Что касается до варьяціи члена $-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{ds}{dv}\cdot\sin(2v-2v')$, то нетрудно видѣть, что она получается изъ выраженія варьяціи $\frac{3m'u'^3\delta s}{2h^2u^4}\cdot\cos(2v-2v')$. Въ самомъ дѣлѣ такъ-какъ

$$\frac{d\delta s}{dv}=\Sigma M.k\cos(kv+\beta)=\Sigma M.k\sin(kv+\beta+90^\circ)$$

т. е. равняется тому-же выраженію δs, только съ перемѣною въ аргументахъ угловъ kv на $kv+90^\circ$ и съ прибавленіемъ множителей k, то обозначая измѣненное такимъ образомъ δs черезъ $(k\delta s)$ мы находимъ:

$$-\frac{3m'.u'^3.d(\delta s)}{2h^2u^4dv}\cdot\sin(2v-2v')=+\frac{3m'u'^3}{2h^2u^4}\cdot(k\delta s).\cos(2v+90^\circ-2v')$$

$$=\frac{3m^2\gamma}{2}\Big\{\Big(1-\frac{5}{2}e'^2\Big).\cos(2v+90^\circ-2v').M(2-2m-g).$$
$$\sin(2v+90^\circ-2mv-gv+\theta)$$

$$+\frac{7}{2}e'.\cos(2v+90^\circ-3mv+\tilde\omega').Re'(2-3m-g).$$
$$\sin(2v+90^\circ-3mv-gv+\theta+\tilde\omega').$$

$$-\frac{1}{2}e'.\cos(2v+90^\circ-mv-\tilde\omega').Qe'.(2-m-g).$$
$$\sin(2v+90^\circ-mv-gv+\theta-\tilde\omega')\Big\}$$

$$=-\frac{3m^2}{4}\cdot\gamma\Big\{\Big(1-\frac{5}{2}e'^2\Big)\Big(1-\frac{5}{2}e'^2\Big)M+\frac{7}{2}e'^2R-\frac{1}{2}e'^2Q\Big\}\cdot$$
$$\sin(gv-\theta)\ldots(65)$$

31. Подставляемъ теперь полученныя выраженія въ наше дифференціальное уравненіе и группируемъ вмѣстѣ члены съ одними и тѣми-же аргументами, полагая вездѣ $\overline{m}^2 = m^2$.

Такимъ образомъ находимъ:

$$\frac{d^2s}{dv^2} + s + \frac{3}{2}m^2\Big[1 + \frac{3}{2}e'^2 + \frac{3}{2}m^2e'^2(N+P) - M\left(1 - \frac{5}{2}e'^2\right)$$

$$+ \frac{1}{2}Q.e'^2 - \frac{7}{2}.Re'^2\Big]\gamma\sin(gv-\theta) - \frac{3}{2}m^2\gamma\left(1-\frac{5}{2}e'^2\right)\sin(v-2mv+\theta)$$

$$+ \frac{9}{4}m^2e'.\gamma.\sin(gv+mv-\theta-\tilde{\omega}') + \frac{9}{4}m^2e'.\gamma.\sin(gv-mv-\theta+\tilde{\omega}')$$

$$+ \frac{3}{4}m^2e'.\gamma\sin(v-mv+\theta-\tilde{\omega}') - \frac{21}{4}m^2e'.\gamma\sin(v-3mv+\theta+\tilde{\omega}')\Big] = 0 \cdots (66)$$

Изъ уравненія (62) находимъ

$$\frac{d^2s}{dv^2} + s = (1-g^2)\gamma.\sin(gv-\theta) + M[1-(1-2m)^2]\gamma\sin(v-2mv+\theta)$$

$$+ N[1-(1+m)^2]e'\gamma\sin(gv+mv-\theta-\tilde{\omega}') + P[1-(1-m)^2]e'.\gamma.$$

$$\sin(gv-mv-\theta+\tilde{\omega}') + Q[1-(1-m)^2]e'.\gamma.\sin(v-mv+\theta-\tilde{\omega}')$$

$$+ R[1-(1-3m)^2]e'\gamma.\sin(v-3mv+\theta+\tilde{\omega}').$$

Подставляя это выраженіе въ уравненіе (66) и приравнивая нулю коэффиціенты при синусахъ одинаковыхъ аргументовъ, получаемъ слѣдующія уравненія для опредѣленія M, N, P, Q, R и g:

$$(1-g^2) = -\frac{3}{2}m^2\Big[1 + \frac{3}{2}e'^2 + \frac{3}{2}m^2e'^2(N+P) - M\left(1-\frac{5}{2}e'^2\right)$$

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$$M[1-(1-2m)^2] = \frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)$$

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$$+\frac{7}{2}\cdot e'.\cos(2v-3mv+\varpi').Re'.\sin(2v-3mv-gv+\theta+\varpi')$$

$$-\frac{1}{2}\cdot e'.\cos(2v-mv-\varpi').Qe'.\sin(2v-mv-gv+\theta-\varpi')\Big\}=$$

$$-\frac{3m^2\gamma}{4}\Big\{M\left(1-\frac{5}{2}e'^2\right)\sin(gv-\theta)+\frac{7}{2}e'^2.R\sin(gv-\theta)-$$

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$$=-\frac{3m^2}{4}\cdot\gamma\left\{\left(1-\frac{5}{2}e'^2\right)\left(1-\frac{5}{2}e'^2\right)M+\frac{7}{2}e'^2R-\frac{1}{2}e'^2Q\right\}\cdot$$
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$$=\frac{3m^2\gamma}{2}\left\{\left(1+\frac{3}{2}e'^2\right)\sin(gv-\theta)+\frac{3}{2}e'.\sin(gv+mv-\theta-\tilde{\omega}')\right.$$

$$\left.+\frac{3}{2}e'.\sin(gv-mv-\theta+\tilde{\omega}')\right\}\dots\dots\dots\dots\dots\dots(59)$$

Разложеніе $\frac{3m'u'^3}{2h^2u^4}\cdot\cos(2v-2v')$ даетъ намъ:

$$\frac{3m'u'^3}{3h^2u^3}\cdot\cos(2v-2v').\frac{s}{u}=\frac{3m^2}{2}\left\{\left(1-\frac{5}{2}e'^2\right)\cos(2v-2mv)\right.$$

$$\left.+\frac{7}{2}e'.\cos(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\cos(2v-mv-\tilde{\omega}')\right\}\cdot\gamma.\sin(gv-\theta)$$

$$=\frac{3m^2}{4}\cdot\gamma\left\{-\left(1-\frac{5}{2}e'^2\right).\sin(v-2mv+\theta)-\frac{7}{2}e'.\sin(v-3mv+\theta+\tilde{\omega}')\right.$$

$$\left.+\frac{1}{2}e'.\sin(v-mv+\theta-\tilde{\omega}')\right\}\dots\dots\dots\dots\dots\dots(60)$$

Далѣе находимъ:

$$-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{ds}{dv}\cdot\sin(2v-2v')=-\frac{3m^2}{2a}\left\{\left(1-\frac{5}{2}e'^2\right)\sin(2v-2mv)\right.$$

$$\left.+\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')\right\}\cdot a\gamma.g\cos(gv-\theta)$$

$$=-\frac{3}{4}m^2g.\gamma.\left\{\left(1-\frac{5}{2}e'^2\right)\cdot\sin(v-2mv+\theta)+\frac{7}{2}e'.\sin(v-3mv+\theta+\tilde{\omega}')\right.$$

$$\left.-\frac{1}{2}e'.\sin(v-mv+\theta-\tilde{\omega}')\right\}\dots\dots\dots\dots\dots\dots(61)$$

Послѣдній членъ въ ур. для s, т. е. $-\frac{3m'}{h^2}\left(\frac{d^2s}{dv^2}+s\right)\int\frac{u'^3\,dv}{u^4}\cdot\sin(2v-2v')$ можно совершенно опустить, такъ-какъ $\frac{d^2s}{dv^2}+s=(1-g^2)\gamma.\sin(gv-\theta)$, т. е. величина того-же порядка, какъ и интегралъ

$$-\frac{3m'}{h^2}\int\frac{u'^3\,hv}{u^4}\cdot\sin(2v-2v'),$$

а именно порядка m^2, четвертыя-же степени m мы въ разсчетъ принимать не будемъ.

30. Положимъ теперь $s = \gamma . \sin . (gv - \theta) + \delta s =$

$\gamma \sin(gv - \theta) + M\gamma . \sin(v - 2mv + \theta) + Ne'\gamma . \sin(gv + mv - \theta - \tilde{\omega}')$

$+ Pe'\gamma . \sin(gv - mv - \theta + \tilde{\omega}') + Qe'\gamma . \sin(v - mv + \theta - \tilde{\omega}')$

$+ Re'\gamma . \sin(v - 3mv + \theta + \tilde{\omega}')$ .... (62)

гдѣ δs та часть s, которая происходитъ отъ возмущающаго дѣйствія Солнца, и вычислимъ варьяціи разсмотрѣнныхъ нами членовъ, причемъ оставимъ только члены съ аргументомъ $(gv - \theta)$.

Варьяція члена $\frac{3m'u'^3 s}{2h^2 u^4}$ даетъ $\frac{3m'u'^3 \delta s}{2h^2 u^4} = \frac{3m'u'^3}{2h^2 u^3} . a\delta s$.

Подставляя сюда вмѣсто $\frac{m'u'^3}{2h^2 u^3}$ разложеніе эти величины по ур. (35), а вмѣсто δs — предъидущее выраженіе по ур. (62), находимъ:

$$\frac{3m'u'^3}{2h^2u^3} . a\delta s = \frac{3m^2}{2}\left(1 + \frac{3}{2}e'^2 + 3e' . \cos(mv - \tilde{\omega}')\right)\delta s =$$

$$= \frac{3m^2}{2} \cdot \delta s\left(1 + \frac{3}{2}e'^2\right) + \frac{9}{4}m^2 e'^2 \gamma . M \sin(v - 2mv + \theta) +$$

$$\frac{9}{2}m^2 e.' \cos(mv - \tilde{\omega}) . \left\{ Ne'\gamma . \sin(gv - mv - \theta - \tilde{\omega}') + Pe'\gamma . \sin(gv - mv - \theta + \tilde{\omega}') = \right.$$

$$= \frac{3m^2}{2} \cdot \delta s\left(1 + \frac{3}{2}e'^3\right) + \frac{9}{4}m^2 e'^2 \gamma . M . \sin(v - 2mv + \theta) +$$

$$+ \frac{9}{4}m^2 e'^2 \gamma . (N + P) . \sin(gv - \theta) \text{ .... } (63)$$

Варьяція 4-го члена ур. для s даетъ:

$$\frac{3m'u'^3 \delta s}{2h^2u^4} \cdot \cos(2v - 2v') = \frac{3m^2}{2a}\left\{\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv) + \right.$$

$$\left. + \frac{7}{2}e' . \cos(2v - 3mv + \tilde{\omega}') - \frac{1}{2}e' . \cos(2v - mv - \omega')\right\}\delta s =$$

$$= \frac{3m^2}{2}\gamma\left\{\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv) . M \sin(2v - 2mv - gv + \theta)\right.$$

$$+\frac{7}{2}\cdot e'.\cos(2v-3mv+\tilde{\omega}').Re'.\sin(2v-3mv-gv+\theta+\tilde{\omega}')$$

$$-\frac{1}{2}\cdot e'.\cos(2v-mv-\tilde{\omega}').Qe'.\sin(2v-mv-gv+\theta-\tilde{\omega}')\Big\}=$$

$$-\frac{3m^2\gamma}{4}\Big\{M\Big(1-\frac{5}{2}e'^2\Big)\sin(gv-\theta)+\frac{7}{2}e'^2.\,R\sin(gv-\theta)-$$

$$-\frac{1}{2}e'^2.\,Q\sin(gv-\theta)\Big\}\ldots(64)$$

Что касается до варьяціи члена $-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{ds}{dv}\cdot\sin(2v-2v')$, то нетрудно видѣть, что она получается изъ выраженія варьяціи $\frac{3m'u'^3\delta s}{2h^2u^4}\cdot\cos(2v-2v')$. Въ самомъ дѣлѣ такъ-какъ

$$\frac{d\delta s}{dv}=\Sigma M.k\cos(kv+\beta)=\Sigma M.k\sin(kv+\beta+90^\circ)$$

т. е. равняется тому-же выраженію δs, только съ перемѣною въ аргументахъ угловъ kv на $kv+90^\circ$ и съ прибавленіемъ множителей k, то обозначая измѣненное такимъ образомъ δs черезъ $(k\delta s)$ мы находимъ:

$$-\frac{3m'.u'^3.d(\delta s)}{2h^2u^4dv}\cdot\sin(2v-2v')=+\frac{3m'u'^3}{2h^2u^4}\cdot(k\delta s).\cos(2v+90^\circ-2v')$$

$$=\frac{3m^2\gamma}{2}\Big\{\Big(1-\frac{5}{2}e'^2\Big).\cos(2v+90^\circ-2v').\,M(2-2m-g).$$
$$\sin(2v+90^\circ-2mv-gv+\theta)$$

$$+\frac{7}{2}e'.\cos(2v+90^\circ-3mv+\tilde{\omega}').\,Re'(2-3m-g).$$
$$\sin(2v+90^\circ-3mv-gv+\theta+\tilde{\omega}').$$

$$-\frac{1}{2}e'.\cos(2v+90^\circ-mv-\tilde{\omega}').Qe'.(2-m-g).$$
$$\sin(2v+90^\circ-mv-gv+\theta-\tilde{\omega}')\Big\}$$

$$=-\frac{3m^2}{4}\cdot\gamma\Big\{\Big(1-\frac{5}{2}e'^2\Big)\Big(1-\frac{5}{2}e'^2\Big)M+\frac{7}{2}e'^2R-\frac{1}{2}e'^2Q\Big\}\cdot$$
$$\sin(gv-\theta)\ldots(65)$$

31. Подставляемъ теперь полученныя выраженія въ наше дифференціальное уравненіе и группируемъ вмѣстѣ члены съ одними и тѣми-же аргументами, полагая вездѣ $\overline{m}^2 = m^2$.

Такимъ образомъ находимъ:

$$\frac{d^2s}{dv^2}+s+\frac{3}{2}m^2\Big[1+\frac{3}{2}e'^2+\frac{3}{2}m^2e'^2(N+P)-M\Big(1-\frac{5}{2}e'^2\Big)$$

$$+\frac{1}{2}Q.e'^2-\frac{7}{2}.Re'^2\Big]\gamma\sin(gv-\theta)-\frac{3}{2}m^2\gamma\Big(1-\frac{5}{2}e'^2\Big)\sin(v-2mv+\theta)$$

$$+\frac{9}{4}m^2e'.\gamma.\sin(gv+mv-\theta-\varpi')+\frac{9}{4}m^2e'.\gamma.\sin(gv-mv-\theta+\varpi')$$

$$+\frac{3}{4}m^2e'.\gamma\sin(v-mv+\theta-\varpi')-\frac{21}{4}m^2e'.\gamma\sin(v-3mv+\theta+\varpi')\Big]=0 \cdots (66)$$

Изъ уравненія (62) находимъ

$$\frac{d^2s}{dv^2}+s=(1-g^2)\gamma.\sin(gv-\theta)+M[1-(1-2m)^2]\gamma\sin(v-2mv+\theta)$$

$$+N[1-(1+m)^2]e'\gamma\sin(gv+mv-\theta-\varpi')+P[1-(1-m)^2]e'.\gamma.$$

$$\sin(gv-mv-\theta+\varpi')+Q[1-(1-m)^2]e'.\gamma.\sin(v-mv+\theta-\varpi')$$

$$+R[1-(1-3m)^2]e'\gamma.\sin(v-3mv+\theta+\varpi').$$

Подставляя это выраженіе въ уравненіе (66) и приравнивая нулю коэффиціенты при синусахъ одинаковыхъ аргументовъ, получаемъ слѣдующія уравненія для опредѣленія M, N, P, Q, R и g:

$$(1-g^2)=-\frac{3}{2}m^2\Big[1+\frac{3}{2}e'^2+\frac{3}{2}m^2e'^2(N+P)-M\Big(1-\frac{5}{2}e'^2\Big)$$

$$+\frac{1}{2}Qe'^2-\frac{7}{2}Re'^2\Big]$$

$$M[1-(1-2m)^2]=\frac{3}{2}m^2\Big(1-\frac{5}{2}e'^2\Big)$$

$$N[1-(1+m)^2]=-\frac{9}{4}m^2$$

$$P[1-(1-m)^2] = -\frac{9}{4}m^2$$

$$Q[1-(1-m)^2] = -\frac{3}{4}m^2$$

$$R[1-(1-3m)^2] = +\frac{21}{4}m^2$$

Отсюда, пренебрегая второю степенью m въ выраженіяхъ коэффиціентовъ M, N, P, Q и R находимъ

$$M = \frac{3}{8}m\left(1-\frac{5}{2}e'^2\right), N = +\frac{9}{8}m, P = -\frac{9}{8}m, Q = -\frac{3}{2}m, R = +\frac{7}{8}m$$

слѣдовательно

$$1-g^2 = -\frac{3}{2}m^2\left[1+\frac{3}{8}e'^2-\frac{3}{8}m(1-5e'^2)-\frac{3}{16}me'^2-\frac{49}{16}me'^2\right]$$

$$= -\frac{3}{2}m^2\left[1-\frac{3}{8}m+\frac{3}{2}e'^2+\left(\frac{30}{16}-\frac{3}{16}-\frac{49}{16}=-\frac{11}{8}\right)me'^2\right],$$

откуда

$$g^2 = 1+\frac{3}{2}m^2\left[1-\frac{3}{8}m+\frac{3}{2}e'^2-\frac{11}{8}me'^2\right]$$

$$g \doteq 1+\frac{3}{4}m^2-\frac{9}{32}m^3+\frac{9}{8}m^2e'^2-\frac{33}{32}m^3e'^2+\ldots \quad (67)$$

а такъ-какъ по положенію $(g-1)v$ выражаетъ обратное движеніе линіи узловъ, то мы имѣемъ

$$\frac{d\theta}{dt} = n.\frac{d\theta}{dv} = -\frac{3}{4}m^2n+\frac{9}{32}m^3n-\left(\frac{9}{8}m^2-\frac{33}{32}m^3\right)e'^2n$$

Такимъ образомъ найденное нами на стр. 33 выраженіе долготы узла должно быть дополнено тремя новыми членами:

$$\frac{9}{32}m^3nt, \frac{33}{32}m^3nE'^2.t \text{ и } \frac{33}{32}m^3\int(e'^2-E'^2)ndt.$$

Коэффиціентъ при $\int(e'^2-E'^2)\,ndt$ въ выраженіи θ будетъ слѣдовательно равенъ

$$-\left(\frac{9}{8}m^2+\frac{9}{4}m^2e^2-\frac{3}{8}m^2i^2-\frac{33}{32}m^3\right).$$

Подставляя числовыя величины, находимъ:

$$-\left(\frac{9}{8}m^2+\frac{9}{4}m^2e^2-\frac{3}{8}m^2i^2-\frac{33}{32}m^3\right)\int(e'^2-E'^2)\,ndt=7''937+$$
$$0''048-0''091-0''544=7''350.$$

По подстановкѣ найденныхъ выше величинъ коэффиціентовъ M, N, P... въ уравненіе 62 мы получаемъ слѣдующее выраженіе для интеграла 3-го дифференціальнаго уравненія системы (34):

$$s=\gamma.\ \sin(gv-\theta)+\frac{3}{8}m\left(1-\frac{5}{2}e'^2\right)\gamma.\ \sin(v-2mv+\theta)$$

$$+\frac{9}{8}me'.\gamma.\sin(gv+mv-\theta-\varpi')-\frac{9}{8}me'\gamma.\sin(gv-mv-\theta+\varpi')$$

$$-\frac{3}{8}m.e'\gamma.\sin(v-mv+\theta-\varpi')+\frac{7}{8}m.e'\gamma.\sin(v-3mv+\theta+\varpi')\cdots(68).$$

32. Первый членъ второй части этого уравненія показываетъ намъ, что движеніе Луны по широтѣ можетъ быть представлено какъ составное изъ двухъ. Луна движется въ плоскости, тангенсъ угла наклоненія которой относительно истинной эклиптики равенъ γ, плоскость-же лунной орбиты непрерывно перемѣщается такимъ образомъ, что пересѣченіе ея съ эклиптикой отступаетъ назадъ со скоростью, средняя величина которой относится къ скорости движенія Луны по долготѣ какъ $(g-1):1$. Величина g приблизительно равна 1.004022.

Второй членъ въ выраженіи тангенса широты Луны выражаетъ неравенство, называемое иногда эвекціею въ широтѣ.

Разсматривая движеніе Луны въ теченіе нѣкоторой части ея обращенія вокругъ Земли, можно пренебречь сравнительно незначительнымъ перемѣщеніемъ Солнца и линіи узловъ въ этотъ промежутокъ времени, т. е. принять въ аргументахъ $g=1$ и $m=0$. При такомъ предположеніи, первые два члена выраженія s дадутъ намъ:

$$s=\gamma\sin(v-\theta)+\frac{3m}{8}\gamma\sin(v-\theta+2\theta)=\gamma\left\{1+\frac{3}{8}m\cos 2\theta\right\}\sin(v-\theta)$$

$$+\frac{3}{8}\gamma m.\sin 2\theta.\cos(v-\theta)$$

Положимъ

$$K.\sin\lambda = \frac{3}{8} m\gamma.\sin 2\theta$$

$$K\cos\lambda = \gamma\left\{1 + \frac{3}{8} m\cos 2\theta\right\}$$

Отсюда приблизительно

$$\begin{cases} tg\lambda = \frac{3m}{8}\sin 2\theta \\ K = \gamma\left\{1 + \frac{3m}{8}\cos 2\theta\right\}, \end{cases}$$

и мы получаемъ

$$s = K.\sin(v - \theta + \lambda) = \gamma\left\{1 + \frac{3m}{8}\cos 2\theta\right\}.\sin\left(v - \theta + \frac{3m}{8}\sin 2\theta\right)$$

Такое-же выраженіе для s мы получили-бы, если-бы долгота узла была $\theta - \frac{3m}{8}\,sm\,2\theta$, а тангенсъ наклонности равнялся

$$\gamma\left(1 + \frac{3m}{8}\cos 2\theta\right).$$

Итакъ, если долгота узла θ была найдена въ предположеніи равномѣрнаго его отступленія, то изъ вычисленной такимъ образомъ долготы θ' мы должны вычесть $\frac{3}{8}\,m\sin 2\theta'$ и принимая разность $\theta' - \frac{3}{8}m\sin 2\theta'$ за истинную долготу узла, опредѣлить наклонность изъ уравненія $\operatorname{tg} i = \gamma\left(1 + \frac{3m}{8}\cos 2\theta\right)$.

Изъ этого выраженія заключаемъ между прочимъ, что наклонность бываетъ наибольшею, тогда θ равно 0 или 180°, т. е. когда линія узловъ совпадаетъ съ линіей сизигій (ибо мы предположили, что при $v = 0$ и долгота Солнца была равна 0) и наименьшею, когда $\gamma = 90°$ или 270°, т. е. когда линія узловъ въ квадратурахъ.

ГЛАВА VIII.

Опредѣленіе коэффиціента вѣковаго уравненія средняго движенія Луны съ точностью до величинъ порядка m^4.

33. Для упрощенія предстоящаго изслѣдованія, положимъ эксцентрицитетъ и наклонность лунной орбиты равными нулю. Интегрируя уравненія системы (34), мы принимали до сихъ поръ e' за величину постоянную, но это допущеніе возможно, если ограничиваться только первою степенью пертурбаціонной функціи. Переходя теперь къ болѣе подробному изслѣдованію вопроса, мы должны уже à priori разсматривать эксцентрицитетъ земной орбиты какъ функцію времени. Пусть $e' = E' - \alpha t$. Мы видѣли, что первое изъ уравненій (34) можетъ быть представлено ве формѣ $\frac{d^2u}{dv^2} + u - u_0 - \Sigma N \cos(kv + \alpha) = 0$, гдѣ коэффиціенты N суть функціи постоянныхъ величинъ и e'. Рѣшая второе уравненіе системы (34) мы опредѣляемъ t въ функціи долготы Луны, слѣдовательно въ строгомъ смыслѣ и коэффиціенты A должны быть разсматриваемы какъ функціи долготы Луны, и потому уравненіе для параллакса Луны, или для величины u, получаетъ видъ:

$$\frac{d^2u}{dv^2} + u - u_0 - \Sigma\left(B + C.\frac{de'}{dt} \cdot v\right) \cos(kv + \alpha) = 0$$

Интегрируя это уравненіе находимъ:

$$u = u_0 - \Sigma \frac{1}{k^2-1}\left(B + C.\frac{de'}{dt}\cdot v\right)\cdot \cos(kv+\alpha) + \frac{\Sigma 2C.\frac{de'}{dt}.k}{(k^2-1)^2}\cdot \sin(kv+\alpha)^{1})$$

Такимъ образомъ мы видимъ, что если по предъидущему принять $u = \frac{1}{a} + \delta u$, гдѣ $\frac{1}{a}$ непеpіодическая часть функціи u, то для того, чтобы наше дифференціальное уравненіе удовлетворялось, необходимо прибавить къ выраженію δu еще сумму членовъ съ тѣми-же аргументами, но подъ знаками синусовъ.

Возьмемъ теперь 1-е уравненіе системы (34) и подставимъ въ него вмѣсто u и $\frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}}$ соотвѣтственно величины $\frac{1}{a}+\delta u$ и $\frac{1}{a_1}$, а вмѣсто членовъ

$$\frac{m'u'^3}{2h^2u^3},\ \frac{3m'u'^3}{2h^2u^3}\cdot\cos(2v-2v'),\ -\frac{3m'u'^3}{2h^2u^4}\cdot\frac{du}{dv}\cdot\sin 2(v-v')$$

$$\text{и } -\frac{3m'}{h^2}\left(\frac{d^2u}{dv^2}+u\right)\int\frac{u'^3\,dv}{u^4}.\sin(2v-2v');$$

ихъ выраженія по ур. 35, 36, 37a, 37b, 38a, 38b, и ихъ варьяціи, отбрасывая вездѣ члены умноженные на e.

Мы получаемъ:

$$\frac{d^2u}{dv^2}+u = \frac{d^2\left(\frac{1}{a}\right)}{dv^2}+\frac{1}{a}+\frac{d^2(\delta u)}{dv^2}+\delta u;$$

$$-\frac{\mu}{h^2(1+s^2)^{\frac{3}{2}}} = -\frac{1}{a_1};$$

$$\frac{m'u'^3}{2h^2u^3}-\frac{3m'u'^3\delta u}{2h^2u^4}+\frac{3m'u'^3\delta u}{2h^2u^3} = \frac{\overline{m}^2}{2a_1}\left\{1+\frac{3}{2}e'^2+3e'.\cos(mv-\tilde{\omega}')\right\}$$

$$-\frac{3\overline{m}^2}{2a_1}\left[1+\frac{3}{2}e'^2+3e'\cos(mv-\tilde{\omega}')\right](a\delta u)+\frac{3\overline{m}^2}{2a_1}\cdot(a'\delta u');$$

$$\frac{3m'u'^3}{2h^2u^3}\cdot\cos(2v-2v')-\frac{9m'u'^3}{2h^2u^4}\cdot\delta u.\cos(2v-2v') =$$

---

1) Во всемъ послѣдующемъ изложеніи мы будемъ принимать $\frac{de'}{dt}$ за величину постоянную.

$$=+\frac{3}{2}\frac{\overline{m}^2}{a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)+\frac{7}{2}e'.\cos(2v-3mv+\tilde{\omega}')$$

$$-\frac{1}{2}e'.\cos(2v-mv-\tilde{\omega}')\Big]-\frac{9}{2}\cdot\frac{\overline{m}^2}{a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big).\cos(2v-2mv)$$

$$+\frac{7}{2}e'.\cos(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\cos(2v-mv-\tilde{\omega}')\Big)\Big](a\delta u);$$

$$-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{du}{dv}\cdot\sin(2v-2v')-\frac{3m'u'^3}{2h^2u^4}\cdot\frac{d\delta u}{dv}\cdot\sin(2v-2v')=$$

$$=-\frac{3}{2}\cdot\overline{m}^2.\frac{a}{a_1}\cdot\frac{d\left(\frac{1}{a}\right)}{dv}\cdot\sin(2v-2mv)-\frac{3\overline{m}^2}{2a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)$$

$$-\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')\Big].\frac{d(a\delta u)}{dv};$$

$$-\frac{3m'}{h^2}\Big(\frac{d^2u}{dv^2}+u\Big)\int\frac{u'^3dv}{u^4}\cdot\sin(2v-2v')-\frac{3m'}{h^2}\Big(\frac{a^2\,\delta u}{dv^2}+\delta u\Big)\int\frac{u'^3dv}{u^4}\sin(2v-2v')$$

$$+\frac{12m'}{h^2a}\int\frac{u'^3dv}{u^4}\cdot\frac{\delta u}{u}\cdot\sin(2v-2v')=-\frac{3\overline{m}^2.a}{a_1}\int\Big\{\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)+$$

$$+\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')\Big\}dv$$

$$-\frac{3\overline{m}^2}{a_1}\Big(\frac{d^2(a\delta u)}{dv^2}+a\delta u\Big)\int dv\Big[\Big(1-\frac{5}{2}e'^2\Big)\cdot\sin(2v-2mv)+$$

$$+\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')$$

$$-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega})\Big]+\frac{12\overline{m}^2}{a_1}\int dv\Big[\Big(1-\frac{5}{2}e'^2\Big).\sin(2v-2mv)$$

$$+\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')\Big]$$

Обозначая для краткости функцію $\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)$ $+\frac{7}{2}e'.\sin(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\sin(2v-mv-\tilde{\omega}')\Big]$ черезъ Ψ и соединяя полученныя выраженія, находимъ:

$$\frac{d^2\left(\frac{1}{a}\right)}{dv^2}+\frac{1}{a}+\frac{d^2(\delta u)}{dv^2}+\delta u-\frac{1}{a_1}+\frac{\overline{m}^2}{2a_1}\Big\{1+\frac{3}{2}e'^2+3e'.\cos(mv-\tilde{\omega}')\Big\}$$

$$-\frac{3\overline{m}^2}{2a_1}\Big[1+\frac{3}{2}e'^2+3e'\cos(mv-\tilde{\omega}')\Big](a\delta u)+\frac{3\overline{m}^2}{2a_1}(a'\delta u')$$

$$+\frac{3}{2}\frac{\overline{m}^2}{a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)+\frac{7}{2}e'.\cos(2v-3mv+\tilde{\omega}')$$

$$-\frac{1}{2}e'.\cos(2v-mv-\tilde{\omega}')\Big]-\frac{9}{2}\cdot\frac{\overline{m}^2}{a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big).\cos(2v-2mv)$$

$$+\frac{7}{2}e'.\cos(2v-3mv+\tilde{\omega}')-\frac{1}{2}e'.\cos(2v-mv-\tilde{\omega}')\Big](a\delta u)$$

$$-\frac{3}{2}.\overline{m}^2.\frac{a}{a_1}.\frac{d\left(\frac{1}{a}\right)}{dv}.\sin(2v-2mv)-\frac{3\overline{m}^2}{2a_1}.\Psi.\frac{d(a\delta u)}{dv}$$

$$-\frac{3\overline{m}^2}{a_1}\int\Psi.dv-\frac{3m^2}{a_1}\left(\frac{d^2(a\delta u)}{dv^2}+a\delta u\right)\int\Psi.dv+$$

$$+12\frac{\overline{m}^2}{a_1}\int(\Psi.dv.(a\delta u))=0\ldots(69)$$

Положимъ теперь

$$a\delta u=m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)+A_0\frac{e'\,de'}{ndt}.\sin(2v-2mv)$$

$$-\frac{3}{2}m^2e'.\cos(mv-\tilde{\omega}')+E_0\frac{de'}{ndt}.\sin(mv-\tilde{\omega}')$$

$$+\frac{7}{2}m^2e'.\cos(2v-3mv+\tilde{\omega}')+D_0\frac{de'}{ndt}.\sin(2v-3mv+\tilde{\omega}')$$

$$-\frac{1}{2}m^2e'.\cos(2v-mv-\tilde{\omega}')+C_0\frac{de'}{ndt}.\sin(2v-mv-\tilde{\omega}')\ldots(70)$$

и подставимъ это выраженіе въ уравненіе (69).

Коэффиціенты A_0, E_0, D_0 и C_0 немедленно опредѣлятся, если по подстановкѣ приравняемъ нулю коэффиціенты при синусахъ.

Разсматривая ур. (69), нетрудно убѣдиться, что синусы съ наибольшими коэффиціентами произойдутъ изъ членовъ $\frac{d^2\,\delta u}{dv^2}$, δu и $-\frac{3\overline{m}^2}{a_1}\int\Psi\,dv$.

Дифференцируя уравненіе (70) и отбрасывая при этомъ члены умноженные на m^3, мы находимъ:

$$\frac{d(a\delta u)}{dv} = -2m^2\left(1 - \frac{5}{2}e'^2\right)\sin(2v-2mv) - 5m^2 e'.\frac{de'}{dv}.\cos(2v-2mv)$$

$$+ 2A_0\frac{e'\,de'}{n.dt}\cos(2v-2mv) + \frac{3}{2}m^3e'\sin(mv-\omega')$$

$$-\frac{3}{2}m^2.\frac{de'}{dv}\cos(mv-\omega') + E_0\frac{de'}{ndt}\cdot m\cos(mv-\omega')$$

$$-7m^2e'.\sin(2v-3mv+\omega') + \frac{7}{2}m^2\cdot\frac{de'}{dv}.\cos(2v-3mv+\omega')$$

$$+ 2D_0.\frac{de'}{ndt}\cos(2v-3mv+\omega')$$

$$+ m^2e'.\sin(2v-mv-\omega') - \frac{1}{2}m^2.\frac{de'}{dv}.\cos(2v-mv-\omega')$$

$$+ 2C_0.\frac{de'}{ndt}\cos(2v-mv-\omega')$$

Дифференцируя еще разъ, получаемъ:

$$\frac{d^2(a\delta u)}{dv^2} = -4m^2\left(1-\frac{5}{2}e'^2\right)\cos(2v-2mv) + 10m^2e'.\frac{de'}{dv}\sin(2v-2mv)$$

$$+ 10m^2e'.\frac{de'}{dv}\cdot\sin(2v-2mv) - 4A_0.\frac{e'\,de'}{ndt}\sin(2v-2mv)$$

$$+\frac{3}{2}m^3\frac{de'}{dv}\cdot\sin(mv-\omega') + \frac{3}{2}m^3\cdot\frac{de'}{dv}\cdot\sin(mv-\omega')$$

$$- E_0\frac{de'}{ndt}\cdot m^2.\sin(mv-\omega) - 14m^2e'.\cos(2v-3mv+\omega')$$

$$-7m^2.\frac{de'}{dv}\cdot\sin(2v-3mv+\omega') - 7m^2.\frac{de'}{dv}\cdot\sin(2v-3mv+\omega')$$

$$-4D_0.\frac{de'}{ndt}\cdot\sin(2v-3mv+\omega')$$

$$+2m^2e'.\cos(2v-mv-\omega') + m^2.\frac{de'}{dv}\cdot\sin(2v-mv-\omega')$$

$$+m^2.\frac{de'}{dv}\cdot\sin(2v-mv-\omega') - 4C_0.\frac{de'}{ndt}\cdot\sin(2v-mv-\omega')$$

Что касается до интеграла $-\frac{3\overline{m}^2}{a_1}\int\Psi\,dv$, то онъ опредѣляется посредствомъ интегрированія по частямъ.

Мы имѣемъ

$$-\frac{3\overline{m}^2}{a_1}\int\Psi\,dv=\frac{3\overline{m}^2}{a_1}\int\Big[\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)+\frac{7}{2}e'.$$

$$\sin(2v-3mv+\tilde\omega')-\frac{1}{2}e'.\sin(2v-mv-\tilde\omega')\Big]dv=$$

$$=-\frac{3\overline{m}^2}{2a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big).\cos(2v-2mv)+$$

$$+\frac{7}{2}e'.\cos(2v-3mv+\tilde\omega')-\frac{1}{2}e'.\cos(2v-mv-\tilde\omega')\Big]$$

$$+\frac{3\overline{m}^2}{a_1}\Big[\frac{5}{2}\cdot\int\cos(2v-2mv).e'.\frac{de'}{dv}\cdot dv-\frac{7}{2}\int\cos(2v-3mv+\tilde\omega')\cdot\frac{de'}{dv}\cdot dv$$

$$+\frac{1}{2}\int\cos(2v-mv-\tilde\omega').\frac{de'}{dv}\cdot dv\Big].$$

Замѣняя $\frac{de'}{dv}$ черезъ $\frac{de'}{ndt}\cdot\frac{ndt}{dv}$ и разсматривая по прежнему $\frac{de'}{dt}$ какъ величину постоянную, представляемъ вторую часть нашего выраженія въ видѣ

$$+\frac{15\overline{m}^2}{2a_1}\cdot\frac{de'}{ndt}\int\cos(2v-2mv).\frac{e'ndt}{dv}\cdot dv$$

$$-\frac{21}{2}\cdot\frac{\overline{m}^2}{a_1}\cdot\frac{de'}{ndt}\int\cos(2v-3mv+\tilde\omega').\frac{ndt}{dv}\cdot dv$$

$$+\frac{3\overline{m}^2}{2a_1}\cdot\frac{de'}{ndt}\int\cos(2v-mv-\tilde\omega').\frac{ndt}{dv}\cdot dv$$

Для приближеннаго вычисленія этихъ интеграловъ можно положить $\frac{ndt}{dv}=1$ (см. ур. 56) и разсматривать въ первомъ изъ нихъ e' также какъ постоянную величину. Такимъ образомъ находимъ:

$$-\frac{3\overline{m}^2}{a_1}\int\Psi.\,dv=\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\Big[\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)+$$

$$+\frac{7}{2}e'.\cos(2v-3mv+\varpi')-\frac{1}{2}e'.\cos(2v-mv-\varpi')\Big]$$

$$+\frac{15}{4}\cdot\frac{\overline{m}^2}{a_1}\cdot e'\cdot\frac{de'}{ndt}\cdot\sin(2v-2mv)-\frac{21}{8}\cdot\frac{\overline{m}^2}{a_1}\cdot\frac{de'}{ndt}\sin(2v-3mv+\varpi')$$

$$+\frac{3}{8}\cdot\frac{\overline{m}^2}{a_1}\frac{de'}{ndt}\cdot\sin(2v-mv-\varpi')\ldots\ldots\ldots\ldots\ldots(71)$$

Отбирая въ выраженіяхъ функцій $\frac{a\delta u}{a}$ (уравненіе 70), $\frac{1}{a}\cdot\frac{d^2(a\delta u)}{dv^2}$ и $-\frac{3\overline{m}^2}{a_1}\int\Psi\,dv$ синусы аргументовъ $2v-2mv$, $mv-\varpi'$, $2v-3mv+\varpi'$ и $2v-mv-\varpi'$ и приравнивая нулю суммы коэффиціентовъ при одинаковыхъ синусахъ, получаемъ слѣдующія 4 уравненія для опредѣленія A_0, E_0, D_0 и C_0:

$$20m^2-3A_0+\frac{15}{4}m^2=0$$

$$3m^3+E_0=0$$

$$-14m^2-3D_0-\frac{21}{8}m^2=0$$

$$2m^2-3C_0+\frac{3}{8}m^2=0,\ \text{откуда}$$

$$A_0=\frac{95}{12}m^2, E_0=-3m^3, D_0=-\frac{133}{24}m^2, C_0=\frac{19}{24}m^2.$$

34. Перейдемъ теперь къ опредѣленію зависимости существующей между величинами a и a_1 и для этого вычислимъ непеaccording періодическую часть уравненія (69).

Отыщемъ прежде всего постоянные члены въ выраженіи

$$-\frac{3\overline{m}^2}{a_1}\int\Psi dv.$$

Въ интегралахъ, входящихъ во вторую часть этой функціи, мы приняли сначала $\frac{ndt}{dv}=1$, такъ-какъ намъ нужно было опредѣлить только первые члены этихъ интеграловъ. Возьмемъ теперь уравненіе (56) и, опустивъ въ немъ членъ зависящій отъ аргумента $mv-\varpi'$, подставимъ это выраженіе въ наши интегралы.

Мы имѣемъ

$$\frac{ndt}{dv} = 1 - \frac{11}{4} m^2 \left(1 - \frac{5}{2} e'^2\right) \cos(2v - 2mv)$$

$$- \frac{77}{8} m^2 e' . \cos(2v - 3mv + \tilde{\omega}') + \frac{11}{8} m^2 e' . \cos(2v - mv - \tilde{\omega}')$$

Первый интегралъ даетъ:

$$\frac{15}{2} \cdot \frac{\overline{m}^2}{a_1} \cdot \frac{de'}{ndt} \int \cos(2v - 2mv) . e' . \frac{ndt}{dv} \cdot dv =$$

$$- \frac{165}{8} \cdot \frac{m^4}{a_1} \int \cos^2(2v - 2mv) . e' . \left(1 - \frac{5}{2} e'^2\right) \frac{de'}{ndt} \cdot dv \cdots$$

$$= - \frac{165}{32} \cdot \frac{m^4}{a_1} \cdot e'^2 + \cdots$$

Точно также находимъ:

$$- \frac{21}{2} \cdot \frac{\overline{m}^2}{a_1} \cdot \frac{de'}{ndt} \int \cos(2v - 3mv + \tilde{\omega}') . \frac{ndt}{dv} \cdot dv =$$

$$+ \frac{21}{4} \cdot \frac{77}{16} \cdot \frac{m^4}{a_1} \cdot \int \frac{e' \, de'}{ndt} \cdot dv + \cdots = \frac{1617}{128} \cdot \frac{m^4}{a_1} \cdot e'^2 + \cdots$$

$$\frac{3}{2} \cdot \frac{\overline{m}^2}{a_1} \frac{de'}{ndt} \int \cos(2v - mv - \tilde{\omega}') . \frac{ndt}{dv} \cdot dv = \frac{3}{4} \cdot \frac{11}{16} \frac{m^4}{a_1} \int \frac{e' de'}{ndt} \cdot dv$$

$$= \frac{33}{128} \cdot \frac{m^4}{a_1} \cdot e'^2 + \cdots$$

Здѣсь мы приняли $\overline{m}^2 . m^2 = m^4$, что̀ не вполнѣ точно, но если ограничиваться только четвертою степенью величины m, то это допущеніе можетъ быть сдѣлано. Въ самомъ дѣлѣ, на стр. (75) мы нашли $a = a_1 + \frac{\overline{m}^2 a}{2} \left(1 + \frac{3}{2} e'^2\right)$, что-же касается до величины $\overline{m}$, то по положенію $\frac{m' a^3}{a'^3} = \overline{m}^2$.

Рѣшая уравненіе для t, мы положили $\frac{a^2}{\sqrt{a_1}} = \frac{1}{n}$, гдѣ n среднее движеніе Луны, слѣдовательно мы имѣемъ:

$$m = \frac{n'}{n} = \frac{\sqrt{m'}}{a'^{\frac{3}{2}}} \cdot \frac{a^2}{\sqrt{a_1}}, \quad m^2 = \frac{m'}{a'^3} \cdot \frac{a^4}{a_1}$$

Но $\frac{1}{a_1} = \frac{1}{a}\left(1 + \frac{1}{2}\overline{m}^2 + \frac{3}{4}\overline{m}^2 . e'^2\right)$, слѣд.

$$m^2 = \frac{m'.a^3}{a'^3}\left(1 + \frac{1}{2}\overline{m}^2 + \frac{3}{4}\overline{m}^2 e'^2\right) = \overline{m}^2\left(1 + \frac{1}{2}\overline{m}^2 + \frac{3}{4}\overline{m}^2 e'^2\right),$$

откуда приблизительно $\overline{m}^2 = m^2\left(1 - \frac{1}{2}m^2 - \frac{3}{4}m^2 e'^2\right) \cdots (72)$

Принимая слѣдовательно $\overline{m}^2 . m^2 = m^4$, мы ошибаемся только на величину порядка m^6.

Разсмотримъ теперь послѣдовательно всѣ члены ур. 69, начиная съ 7-го, т. е. съ $- \frac{3\overline{m}^2}{2a_1}\left\{1 + \frac{3}{2}e'^2 + 3e' . \cos(mv - \tilde{\omega}')\right\} a\delta u$

Этотъ членъ даетъ намъ:

$$+ \frac{3\overline{m}^2}{2a_1}\left\{1 + \frac{3}{2}e'^2 + 3e' . \cos(mv - \tilde{\omega}')\right\} \cdot \frac{3}{2}m^2 e' \cdot \cos(mv - \tilde{\omega}') =$$

$$= \frac{27}{8}\frac{m^4}{a_1} \cdot e'^2$$

Далѣе получаемъ

$$8)\ \frac{3\overline{m}^2}{2a_1}(a'\delta u') = - \frac{3\overline{m}^2}{2a_1} \cdot e' \cdot \sin(mv - \tilde{\omega}') \cdot m\delta v = - \frac{9}{4}\frac{m^4}{a_1} \cdot e'^2,$$

ибо приблиз. $m\delta v = \delta v' = + 3m^2 e' \sin(mv - \tilde{\omega}')$

$$10) - \frac{9}{2} \cdot \frac{\overline{m}^2}{a_1}\left[\left(1 - \frac{5}{2}e'^2\right)\cos(2v - 2mv) + \frac{7}{2}e' \cdot \cos(2v - 3mv + \tilde{\omega}')\right.$$

$$\left. - \frac{1}{2}e' \cdot \cos(2v - mv - \tilde{\omega}')\right] a\delta u = - \frac{9}{2} \cdot \frac{\overline{m}^2}{a_1}\left[\left(1 - \frac{5}{2}e'^2\right)^2 . m^2 .\right.$$

$$\cdot \cos^2(2v - 2mv) + \left(\frac{7}{2}e' \cdot\right)^2 . m^2 \cdot \cos^2(2v - 3mv + \tilde{\omega}')$$

$$\left. + \left(\frac{1}{2}e'\right)^2 . m^2 \cos^2(2v - mv - \tilde{\omega}')\right] = - \frac{9}{4} \cdot \frac{m^4}{a_1}(1 - 5e'^2)$$

$$- \frac{441}{16}\frac{m^4}{a_1} \cdot e'^2 - \frac{9}{16} \cdot \frac{m^4}{a_1}e'^2 = - \frac{9}{4} \cdot \frac{m^4}{a_1} - \frac{135}{8} \cdot \frac{m^4}{a_1} \cdot e'^2$$

$$12) -\frac{3\overline{m}^2}{2a_1}\Psi\cdot\frac{d(a\delta u)}{dv}=-\frac{3\overline{m}^2}{2a_1}\Big[-\Big(1-\frac{5}{2}e'^2\Big)^2 2m^2\cdot\sin^2(2v-2mv)$$

$$-\frac{49}{2}m^2e'^2\cdot\sin^2(2v-3mv+\tilde{\omega}')-\frac{1}{2}\cdot m^2.e'^2.\sin^2(2v-mv-\tilde{\omega}')\Big]$$

$$=\frac{3}{2}\cdot\frac{m^4}{a_1}(1-5e'^2)+\frac{147}{8}\cdot\frac{m^4}{a_1}e'^2+\frac{3}{8}\frac{m^4}{a_1}\cdot e'^2=\frac{3}{2}\cdot\frac{m^4}{a_1}+\frac{45}{4}\cdot\frac{m^4}{a_1}e'^2$$

13) Выше мы нашли: $-\frac{3\overline{m}^2}{a_1}\int\Psi\,dv=-\frac{165}{32}\frac{m^4}{a_1}\cdot e'^2$

$$+\frac{1617}{128}\frac{m^4}{a_1}\cdot e'^2+\frac{33}{128}\cdot\frac{m^4}{a_1}\cdot e'^2=\frac{495}{64}\cdot\frac{m^4}{a_1}\cdot e'^2$$

$$14) -\frac{3\overline{m}^2}{a_1}\Big(\frac{d^2(a\delta u)}{dv^2}+a\delta u\Big)\int\Psi\,dv=-\frac{3\overline{m}^2}{a_1}\Big\{-3m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)$$

$$-\frac{21}{2}m^2e'\cdot\cos(2v-3mv+\tilde{\omega}')+\frac{3}{2}m^2e'\cdot\cos(2v-mv-\tilde{\omega}')\Big\}.$$

$$\cdot\frac{1}{2}\Big[\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)+\frac{7}{2}e'\cdot\cos(2v-3mv+\tilde{\omega}')-$$

$$\frac{1}{2}e'\cdot\cos(2v-mv-\tilde{\omega}')\Big]$$

$$=-\frac{9}{4}\cdot\frac{m^4}{a_1}(1-5e'^2)-\frac{441}{16}\frac{m^4}{a_1}\cdot e'^2-\frac{9}{16}\cdot\frac{m^4}{a_1}\cdot e'^2=-\frac{9}{4}\frac{m^4}{a_1}-\frac{135}{8}\cdot\frac{m^4}{a_1}e'^2$$

$$15) \frac{12\overline{m}^2}{a_1}\int\Psi\cdot(a\delta u)\cdot dv=$$

$$=\frac{12\overline{m}^2}{a_1}\int\Big[m^2\Big(1-\frac{5}{2}e'^2\Big)\sin(2v-2mv)+\frac{7}{2}e'\cdot\sin(2v-3mv+\tilde{\omega}')$$

$$-\frac{1}{2}e'\cdot\sin(2v-mv-\tilde{\omega}')\Big]dv\cdot\Big\{\frac{95}{12}m^2\cdot\frac{e'\,de'}{ndt}\sin(2v-2mv)-$$

$$-\frac{133}{24}m^2\cdot\frac{de'}{ndt}\cdot\sin(2v-3mv+\tilde{\omega}')+\frac{19}{24}m^2\cdot\frac{de'}{ndt}\sin(2v-mv-\tilde{\omega}')\Big\}$$

$$=\frac{12\overline{m}^2}{a_1}\int dv\Big\{\frac{95}{24}m^2\cdot\frac{e'\,de'}{ndt}-\frac{931}{96}m^2\cdot\frac{e'de'}{ndt}-\frac{19}{96}m^2\cdot\frac{e'de'}{ndt}\Big\}$$

$$=-\frac{285}{4}\frac{m^4}{a_1}\int ndt\cdot\frac{e'de'}{ndt}=-\frac{285}{8}\cdot\frac{m^4}{a_1}e'^2$$

Подставляя полученные неперіодическіе члены въ уравненіе (69), и приравнивая нулю постоянную часть этого уравненія, находимъ:

$$\frac{1}{a}-\frac{1}{a_1}+\frac{1}{2}\cdot\frac{\overline{m}^2}{a_1}\left(1+\frac{3}{2}e'^2\right)+\frac{27}{8}\cdot\frac{m^4}{a_1}\cdot e'^2-\frac{9}{4}\cdot\frac{m^4}{a_1}\cdot e'^2$$

$$-\frac{9}{4}\cdot\frac{m^4}{a_1}-\frac{135}{8}\cdot\frac{m^4}{a_1}\cdot e'^2+\frac{3}{2}\cdot\frac{m^4}{a_1}+\frac{45}{4}\cdot\frac{m^4}{a_1}\cdot e'^2+\frac{495}{64}\cdot\frac{m^4}{a_1}e'^2$$

$$-\frac{9}{4}\cdot\frac{m^4}{a_1}-\frac{135}{8}\cdot\frac{m^4}{a_1}\cdot e'^2-\frac{285}{8}\cdot\frac{m^4}{a_1}e'^2=0$$

или

$$\frac{1}{a}-\frac{1}{a_1}\Big\{1-\frac{1}{2}\overline{m}^2-\frac{3}{4}\overline{m}^2\cdot e'^2+3m^4-\Big(\frac{27}{8}-\frac{9}{4}-\frac{135}{8}$$

$$+\frac{45}{4}+\frac{495}{64}-\frac{135}{8}-\frac{285}{8}=-\frac{3153}{64}\Big)m^4e'^2\Big\}=0 \quad \ldots\ldots (73a)$$

Подставляя сюда вмѣсто $\overline{m}^2$ выраженіе этой величины въ функціи m (по ур. 72), находимъ

$$a_1=a\left\{1-\frac{1}{2}m^2-\frac{3}{4}m^2\cdot e'^2+\frac{13}{4}m^4+\frac{3201}{64}m^4e'^2\right\},$$

откуда

$$a^2=a_1^2\left\{1+m^2-\frac{23}{4}m^4+\frac{3}{2}m^2e'^2-\frac{3129}{32}m^4e'^2\right\}\ldots\ldots (73b)$$

35. Теперь намъ остается только подставить найденныя величины $a\delta u$ и a^2 во второе дифференціальное уравненіе системы (34) и затѣмъ проинтегрировать его.

Мы имѣемъ

$$\frac{dt}{dv}=\frac{1}{hu^2}+\frac{3}{2}\frac{m'}{h^3u^2}\int\frac{u'^3}{u^4}\cdot dv\sin(2v-2v')$$
$$+\frac{27}{8}\cdot\frac{m'^2}{h^5u^2}\left[\int\frac{u'^3\,dv}{u^4}\cdot\sin(2v-2v')\right]^2$$

Подставляя въ первый членъ второй части вмѣсто u, $\frac{1}{a}+\delta u$ или $\frac{1}{a}(1+a\delta u)$, находимъ

$$\frac{1}{hu^2}=\frac{1}{h}\cdot a^2(1+a\delta u)^{-2}=\frac{a^2}{h}(1-2a\delta u+3a^2(\delta u)^2-\ldots)$$

$$= \frac{a^2}{\sqrt{a_1 \mu}} \left\{ 1 - 2\, a\delta u + \frac{3}{2} m^4 (1 - 5\, e'^2) + \frac{27}{8} m^4 e'^2 + \frac{147}{8} m^4 e'^2 + \right.$$
$$\left. + \frac{3}{8} m^4 e'^2 + \ldots \right\}$$

Функція

$$\frac{3}{2} \cdot \frac{m'}{h^3 u^2} \int \frac{u'^3}{u^4} dv \,.\, \sin(2v - 2v')$$

даетъ по предъидущему:

$$\frac{1}{2hu^2} \cdot \frac{3m'}{h^2} \int \frac{u'^3}{u^4} . dv \,.\, \sin(2v - 2v') = \frac{a^2}{2\sqrt{a_1 \mu}} \left\{ 1 - 2a\delta u + \frac{3}{2} m^4 (1 - 5e^2) + \ldots \right.$$

$$\left. + \ldots \right\} \cdot 3\overline{m}^2 \cdot \frac{a}{a_1} \int dv \left[\left(1 - \frac{5}{2} e'^2\right) \sin(2v - 2mv) + \frac{7}{2} e' . \sin 2v - 3mv - \tilde{\omega}') \right.$$
$$\left. - \frac{1}{2} e' . \sin(2v - mv - \tilde{\omega}') \right]$$

$$= \frac{a^2}{\sqrt{a_1 \mu}} \left[\frac{3}{2} \cdot \overline{m}^2 . \frac{a}{a_1} \int \Psi . dv - 3\overline{m}^2 . \frac{a}{a_1} \cdot a\delta u \int \Psi . dv \right]$$

Третій членъ

$$\frac{27}{8} \frac{m'^2}{h^5 u^2} \left[\int \frac{u'^3\, dv}{u^4} \sin(2v - 2v') \right]^2$$

равенъ

$$\frac{27}{8} \cdot \frac{m^4}{h^5 u^2} \frac{a^2 h^4}{a_1^2} \left[\int \Psi . dv \right]^2,$$

ибо

$$\int \frac{u'^3\, dv}{u^4} \sin(2v - 2v') = \frac{\overline{m}^2 . ah^2}{m' . a_1} \int \Psi\, dv,$$

и слѣдовательно

$$\frac{27}{8} \cdot \frac{m'^2}{h^5 u^2} \left[\int \frac{u'^3 . dv}{u^4} \sin(2v - 2v') \right]^2 =$$

$$\frac{27}{8} \cdot m^4 . \left(\frac{a}{a_1}\right)^2 . \frac{a^2}{\sqrt{a_1}} \left\{ 1 - 2\, a\delta u + \ldots \right\} \left[\int \Psi dv \right]^2$$

Остается найти варіацію втораго члена. Мы имѣемъ:

$$\frac{3}{2} \cdot \frac{m'}{h^3 u^2} \int \frac{u'^3\, dv}{u^4} \cdot \sin(2v - 2v') = \frac{3m'}{2h^2} \cdot \frac{a^2}{\sqrt{a_1}} \int \frac{u'^3\, dv}{u^4} \cdot \sin(2v - 2v'),$$

$$\delta \left[\frac{3m'}{2h^2} \frac{a^2}{\sqrt{a_1}} \int \frac{u'^3\, dv}{u^4} . \sin(2v - 2v') \right] = - \frac{6\, a^2}{\sqrt{a_1}} . \frac{m'}{h^2} \int \frac{u'^3\, \delta u}{u^5} \sin(2v - 2v')\, dv$$

$$= -\frac{6a^2}{\sqrt{a_1}}\cdot\frac{m'}{h^2}\int\frac{u'^3\,dv}{u^4}\,.\sin(2v-2v').\,a\delta u = -\frac{6a^2}{\sqrt{a_1}}\cdot\frac{m'\overline{m}^2.a.h^2}{h^2\,m'\,a_1}\int\Psi\,dv.a\delta u$$

$$= -\frac{6a^2}{\sqrt{a_1}}\overline{m}^2.\left(\frac{a}{a_1}\right)\int\Psi.\,dv\,.(a\delta u)$$

Итакъ мы получили:

$$\frac{dt}{dv} = \frac{a^2}{\sqrt{a_1}\,\mu}\Big(1 - 2\,a\delta u + \tfrac{3}{2}\,\overline{m}^4(1-5e'^2) + \tfrac{27}{8}\,m^4 e'^2$$

$$+ \frac{147}{8}m^4 e'^2 + \frac{3}{8}m^4 e'^2 + \frac{3}{2}\overline{m}^2.\,\frac{a}{a_1}\int\Psi\,dv$$

$$- 3\overline{m}^2\cdot\frac{a}{a_1}\cdot(a\delta u)\int\Psi\,dv + \frac{27}{8}m^4\cdot\left(\frac{a}{a_1}\right)^2\cdot\left[\int\Psi\cdot dv\right]^2$$

$$- 6\overline{m}^2\cdot\left(\frac{a}{a_1}\right)\int\Psi\cdot dv\cdot(a\delta u)\Big)$$

На страницахъ 98 и 100 мы нашли:

$$-\frac{3\overline{m}^2}{a_1}\int\Psi\,dv = +\frac{495}{64}\cdot\frac{m^4}{a_1}e'^2$$

и

$$12\frac{\overline{m}^2}{a_1}\int\Psi\,dv(a\delta u) = -\frac{285}{8}\cdot\frac{m^4}{a_1}e'^2$$

Отсюда

$$\frac{3}{2}\overline{m}^2.\,\frac{a}{a_1}\int\Psi\,dv = -\frac{495}{128}.\,m^4.\left(\frac{a}{a_1}\right)e'^2 \text{ (7-й членъ)}$$

$$-6\,\overline{m}^2.\left(\frac{a}{a_1}\right)\int\Psi\,dv\,(a\delta u) = +\frac{285}{16}m^4\left(\frac{a}{a_1}\right)e'^2 \text{ (10-й членъ)}$$

По ур. (70) и (71) имѣемъ

$$-3\overline{m}^2\cdot\frac{a}{a_1}(a\delta u)\int\Psi\,dv = \frac{a}{a_1}\cdot(a\delta u)\Big[\frac{3}{2}\cdot\frac{\overline{m}^2}{a_1}\cdot\left(1-\frac{5}{2}e'^2\right)\cos(2v-2mv)$$

$$+\frac{21}{4}.\overline{m}^2.e'.\cos(2v-3mv+\tilde{\omega}') - \frac{3}{4}.m^2.e'.\cos(2v-mv-\tilde{\omega}') +$$

$$+\frac{15}{4}.\overline{m}^2.\frac{e'de'}{ndt}.\sin(2v-2mv) - \frac{21}{8}.\overline{m}^2.\frac{de'}{ndt}.\sin(2v-3mv+\tilde{\omega}')$$

$$+\frac{3}{8}.\overline{m}^2.\frac{de'}{ndt}\sin(2v-mv-\tilde{\omega}')\Big]=\frac{a}{a_1}\Big[\frac{3}{2}.\overline{m}^2\Big(1-\frac{5}{2}e'^2\Big)^2.m^2\cos^2(2v-2mv)$$

$$+\frac{21}{4}.\frac{7}{2}\overline{m}^2.e'^2.m^2.\cos(2v-3mv+\tilde{\omega}')+\frac{3}{4}\cdot\frac{1}{2}\overline{m}^2.e'^2.m^2.\cos^2(2v-mv-\tilde{\omega})\Big]$$

$$=\Big(\frac{a}{a_1}\Big)\Big[\frac{3}{4}m^4(1-5e'^2)+\frac{147}{16}m^4e'^2+\frac{3}{16}m^4e'^2\Big]=$$

$$=\Big(\frac{a}{a_1}\Big)\Big[\frac{3}{4}m^4+\frac{45}{8}m^4e'^2\Big]$$

Наконецъ

$$\frac{27}{8}m^4\Big(\frac{a}{a_1}\Big)^2\Big[\int\Psi\,dv\Big]^2=\frac{3}{8}a^2.\Big[\frac{3\overline{m}^2}{a_1}\int\Psi\,dv\Big]^2$$

$$=\frac{27}{32}\Big(\frac{a^2}{a_1^2}\Big)m^4\Big[\frac{1}{2}(1-5e'^2)+\frac{49}{8}e'^2+\frac{1}{8}e'^2\Big]=$$

$$\frac{27}{64}\frac{a^2}{a_1^2}.\overline{m}^4+\frac{405}{128}\frac{a^2}{a_1^2}.m^4e'^2$$

Въ данной степени приближенія можно вездѣ положить $\frac{a}{a_1}=1$ и $\frac{a^2}{a_1^2}=1$. Присоединяя къ найденнымъ постояннымъ членамъ еще періодическіе члены происходящіе изъ

$$\frac{3}{2}\overline{m}^2.\frac{a}{a_1}\int\Psi\,dv \text{ (см. ур. 71)},$$

мы получаемъ:

$$\frac{dt}{dv}=\frac{a^2}{\sqrt{a_1\mu}}\Big\{1-2a\delta u+\frac{3}{2}m^4-\frac{15}{2}m^4e'^2+\frac{27}{8}m^4e'^2+\frac{147}{8}m^4e'^2$$

$$+\frac{3}{8}m^4e'^2-\frac{495}{128}m^4e'^2+\frac{3}{4}m^4+\frac{45}{8}m^4e'^2+\frac{27}{64}m^4+\frac{405}{128}m^4e'^2$$

$$+\frac{285}{16}m^4e'^2\Big\}+\frac{a^2}{\sqrt{a_1\mu}}\Big\{-\frac{3}{4}\overline{m}^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)$$

$$-\frac{21}{8}\overline{m}^2e'\cos(2v-3mv+\tilde{\omega}')+\frac{3}{8}\overline{m}^2e'.\cos(2v-3mv+\tilde{\omega}')$$

$$-\frac{15}{8}\overline{m}^2\cdot\frac{e'de'}{ndt}\cdot\sin(2v-2mv)+\frac{21}{16}\overline{m}^2\cdot\frac{de'}{ndt}\cdot\sin(2v-3mv+\tilde{\omega}')$$

$$-\frac{3}{16}\overline{m}^2\cdot\frac{de'}{ndt}\cdot\sin(2v-mv-\tilde{\omega}')\Big\}=\frac{a^2}{\sqrt{a_1\mu}}\Big\{1-2a\delta u+\frac{171}{64}m^4+$$

$$+\Big[\frac{-960+432+2352+48-495+720+405+2280}{128}=$$

$$=\frac{2391}{64}\Big]m^4e'^2\Big\}+\text{сумма періодическихъ членовъ.}$$

Подставляя сюда вмѣсто $2a\delta u$ выраженіе этой функціи по уравненію 70, въ которомъ коэффиціенты A_0, E_0, D_0 и C_0 должны быть замѣнены ихъ величинами найденными на стр. (97-й), и замѣняя $\overline{m}^2$ въ коэффиціентахъ періодическихъ членовъ черезъ m^2, мы находимъ:

$$\frac{dt}{dv}=\frac{a^2}{\sqrt{a_1\mu}}\Big\{1+\frac{171}{64}m^4+\frac{2391}{64}m^4e'^2-\frac{11}{4}m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2v-2mv)$$

$$-\frac{495}{24}m^2\cdot\frac{e'de'}{ndt}\cdot\sin(2v-2mv)+3m^2e'\cdot\cos(mv-\tilde{\omega})+6m^3\cdot\frac{de'}{ndt}\sin(mv-\tilde{\omega}')$$

$$-\frac{77}{8}m^2e'.\cos(2v-3mv+\tilde{\omega}')+\frac{595}{48}m^2\cdot\frac{de'}{ndt}\sin(2v-3mv+\tilde{\omega}')$$

$$+\frac{11}{8}m^2e'\cdot\cos(2v-mv-\tilde{\omega}')-\frac{85}{48}\cdot m^2\cdot\frac{de'}{ndt}\cdot\sin(2v-mv-\tilde{\omega}')$$

Замѣнимъ теперь a^2 величиною его по уравненію (73b) и обозначимъ для краткости суммы періодическихъ членовъ соотвѣтственно черезъ

$$\Sigma A\cos(jv+\beta)\ \text{и}\ \Sigma B.\sin(jv+\beta).$$

Мы имѣемъ

$$\frac{dt}{dv}=a_1^{\frac{3}{2}}\cdot\frac{1}{\sqrt{\mu}}\Big\{1+m^2-\frac{23}{4}m^4+\frac{3}{2}m^2e'^2-\frac{3129}{32}m^4e'^2+\frac{171}{64}m^4$$

$$+\frac{2391}{64}m^4e'^2+\Sigma A\cos(jv+\beta)+\Sigma B\sin(jv+\beta)\Big\}$$

$$=\frac{a_1^{\frac{3}{2}}}{\sqrt{\mu}}\Big\{1+m^2-\frac{197}{64}m^4+\frac{3}{2}m^2e'^2-\frac{3867}{64}m^4e'^2+$$

$$+\Sigma A\cos(jv+\beta)+\Sigma B\sin(jv+\beta)\Big\}.$$

Полагая теперь опять постоянную часть этого выраженія равною $\frac{1}{n}$, гдѣ n среднее движеніе Луны, и интегрируя, находимъ:

$$\int ndt = v - \frac{11}{8} m^2\left(1 - \frac{5}{2} e'^2\right)\sin(2v - 2mv) + \frac{295}{24} m^2 \cdot \frac{e'de'}{ndt} \cdot$$

$$\cos(2v - 2mv) + 3me'.\sin(mv - \tilde{\omega}') + 3 \cdot \frac{de'}{ndt} \cdot \cos(mv - \tilde{\omega}')$$

$$- \frac{77}{16} m^2 e' \sin(2v - 3mv + \tilde{\omega}') - \frac{413}{48} m^2 \cdot \frac{de'}{ndt} \cos(2v - 3mv + \tilde{\omega}')$$

$$+ \frac{11}{16} m^2 e'.\sin(2v - mv - \tilde{\omega}') + \frac{59}{48} m^2 \cdot \frac{de'}{ndt} \cdot \cos(2v - mv - \tilde{\omega}') \quad ..(74)$$

36. Мы положили

$$\frac{\sqrt{\mu}}{n} = a_1^{\frac{3}{2}}\left(1 + m^2 - \frac{197}{64} m^4 + \frac{3}{2} m^2 e'^2 - \frac{3867}{64} m^4 e'^2\right), \quad ...(75a)$$

гдѣ a_1 большая полуось лунной орбиты, какою-бы она была если-бы не было возмущеній.

Вслѣдствіе измѣненія e' измѣняется и n, т. е. среднее движеніе Луны. Дифференцируя предъидущее выраженіе по t, находимъ

$$-\frac{dn}{dt} \cdot \frac{1}{n^2} = 2a_1^{\frac{3}{2}} m . \frac{dm}{dt} + a_1^{\frac{3}{2}}\left(\frac{3}{2} m^2 - \frac{3867}{64} m^4\right) \cdot \frac{d(e'^2)}{dt} + \ldots$$

$$\text{Но } 2a_1^{\frac{3}{2}} m \frac{dm}{dt} = - 2a_1^{\frac{3}{2}} m^2 . \frac{dn}{ndt} = - 2 . \frac{1}{n^2} m^2 . \frac{dn}{dt},$$

слѣдовательно

$$\frac{dn}{dt} \cdot \frac{1}{n^2} = \frac{2m^2}{n^2} \cdot \frac{dn}{dt} - a_1^{\frac{3}{2}}\left(\frac{3}{2} m^2 - \frac{3867}{64} m^2\right) \frac{d(e'^2)}{dt}$$

или

$$\frac{1}{n} \cdot \frac{dn}{dt}(1 - 2m^2) = -\left(\frac{3}{2} m^2 - \frac{3867}{64} m^4\right) \frac{d(e'^2)}{dt}, \text{ откуда}$$

$$\frac{1}{n} \cdot \frac{dn}{dt} = \left(-\frac{3}{2} m^2 + \frac{3867}{64} m^4 - \frac{3}{2} m^4\right) \frac{d(e'^2)}{dt} = -\left(\frac{3}{2} m^2 - \frac{3771}{64} m^4\right) \frac{d(e'^2)}{dt}$$

Интегрируя, имѣемъ

$$n = n_0 - \left(\frac{3}{2}m^2 - \frac{3771}{64}m^4\right) \cdot (e'^2 - E'^2)\, n, \ldots . . (75b)$$

откуда

$$\int n dt = n_0 t + \varepsilon - \left(\frac{3}{2}m^2 - \frac{3771}{64}m^4\right) \int (e'^2 - E'^2)\, n dt \ldots \ldots (76)$$

Присоединяя члены зависящіе отъ e и i, найденные нами на стр. 37-й и подставляя (76) въ уравненіе 74 находимъ:

$$v = n_0 t + \varepsilon - \left(\frac{3}{2}m^2 - \frac{3771}{64}m^4 + \frac{27}{16}m^2 e^2 - \frac{27}{16}m^2 i^2\right) \int (e'^2 - E'^2)\, n dt$$

$$- 3me'.\sin(mv - \tilde{\omega}') - 3\frac{de'}{ndt}\cos(mv - \tilde{\omega}')$$

$$+ \frac{11}{8}m\left(1 - \frac{5}{2}e'^2\right)\sin(2v - 2mv) - \frac{295}{24}m^3 \cdot \frac{e'de'}{ndt}\cos(2v - 2mv)$$

$$+ \frac{77}{16}m^2 e'.\sin(2v - 3mv + \tilde{\omega}') + \frac{413}{48}m^2\frac{de'}{ndt}\cos(2v - 3mv + \tilde{\omega}')$$

$$- \frac{11}{16}m^2 e'.\sin(2v - mv - \tilde{\omega}') - \frac{59}{48}m^2 . \frac{de'}{ndt} \cdot \cos(2v - mv - \tilde{\omega}').$$

Подставляя численныя величины и замѣчая, что

$\int (e'^2 - E'^2)\, n dt = - 1260''.896\, i^2$, (гдѣ $i = 100$ юліанскимъ годамъ), имѣемъ:

$$\left(\frac{3}{2}m^2 - \frac{3771}{64}m^4 + \frac{27}{16}m^2 e^2 - \frac{27}{16}m^2 i^2\right) \int (e'^2 - E'^2)\, n dt =$$

$$- (0.008393 - 0.001845 + 0.000028 -$$

$$0.000323)\ 1260''.896\, i^2$$

$$= -(10''.583 - 2''.326 + 0''.036 - 0''.409) i^2 = - 7''.884\, i^2$$

Съ помощью формулъ (73) и (75a) нетрудно найти зависимость между величинами n и a.

Изъ уравненія

$$a^2 = a_1^2\left\{1 + m^2 - \frac{23}{4}m^4 + \frac{3}{2}m^2e'^2 - \frac{3129}{32}m^4e'^2\right\}$$

имѣемъ:

$$a_1^{\frac{3}{2}} = a^{\frac{3}{2}}\left\{1 - \frac{3}{4}m^2 + \frac{159}{32}m^4 - \frac{9}{8}m^2e'^2 + \frac{9387}{128}m^4e'^2 + \ldots\right\}$$

слѣдовательно по уравненію (75a)

$$\frac{\sqrt{\mu}}{n} = a^{\frac{3}{2}}\left\{1 + \frac{1}{4}m^2 + \frac{3}{8}m^2e'^2 - \left(\frac{159}{32} + \frac{3}{4} - \frac{197}{64} = \frac{169}{64}\right)m^4\right.$$

$$\left. + \left(\frac{9387}{128} - \frac{3867}{64} - \frac{9}{4} = \frac{1365}{128}\right)m^4e'^2\right\},$$

откуда

$$\mu = a^3n^2\left\{1 + \frac{1}{2}m^2 + \frac{3}{4}m^2e'^2 - \frac{167}{32}m^4 + \frac{1365}{64}m^4e'^2\right\} \quad \ldots\ldots (77)$$

Дифференцируя уравненіе (73b) и замѣчая, что

$$2m\frac{dm}{dt} = 3m^4\cdot e'\frac{de'}{dt} + \ldots\ldots,$$

находимъ

$$2a\cdot\frac{da}{dt} = a_1^2\left(3m^2 - \frac{3081}{16}m^4\right)\frac{e'de'}{dt},$$

откуда

$$2\cdot\frac{da}{adt} = \left(3m^2 - \frac{3081}{16}m^4\right)\left(1 - m^2 - \frac{3}{2}m^2e'^2 + \ldots\right)\frac{e'de'}{dt} =$$

$$= \left(3m^2 - \frac{3129}{16}m^4\right)\frac{e'de'}{dt}, \text{ или } \frac{da}{adt} = \left(\frac{3}{2}m^2 - \frac{3129}{32}m^4\right)\frac{e'de'}{dt}.$$

Какъ видимъ, измѣненіе большой полуоси оказывается еще меньше, чѣмъ то даетъ первое приближеніе.

Опредѣляя величину вѣковаго ускоренія, Лапласъ предполагалъ, что секторіальная скорость въ движеніи Луны не подвергается измѣненію, т. е. приписывалъ ускореніе средняго движенія Луны единственно уменьшенію средняго разстоянія Луны отъ Земли. Если допустить, что въ каждый моментъ между величинами r и v существуетъ соотношенія $r^2 \cdot \frac{dv}{dt} = \text{const.}$, то принимая для простоты разсужденія орбиту Луны за круговую и обозначая увеличеніе угловой скорости черезъ δn, легко находимъ, что между уменьшеніемъ $(-\delta a)$ средняго разстоянія и увеличеніемъ скорости имѣетъ мѣсто уравненіе $\delta n = -\frac{2\delta a}{a^3}$.

Лапласъ нашелъ[1]) $\delta a = \frac{3}{4} m^2 \delta(e'^2) \cdot a$; отсюда $\delta n = -\frac{3}{2}\frac{m^2}{a^2} \delta(e'^2)$, или полагая $a^2 n = 1$, имѣемъ $\delta n = -\frac{3}{2} m^2 n \cdot \delta(e'^2)$.

На самомъ дѣлѣ въ движеніи Луны ускореніе средняго движенія зависитъ не отъ одного уменьшенія средняго разстоянія, но также и отъ измѣненія тангенціальной слагающей возмущающей силы.

Изъ разсмотрѣнія нашихъ формулъ легко убѣдиться, что средняя величина секторіальной скорости Луны также подлежитъ вѣковому измѣненію, въ зависимости отъ измѣненія a и n, и очевидно, что величина этого измѣненія не можетъ быть объяснена одною эллиптическою гипотезою, т. е. предположеніемъ, что варьяція средней скорости происходитъ единственно отъ уменьшенія средняго разстоянія Луны отъ центра Земли.

Пулково, 12-го мая 1885 года.

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[1]) См. Hist. de l'Acad. 1786, p. 257.

ЗАМѢЧЕННЫЯ ПОГРѢШНОСТИ.

	Напечатано:	Должно быть:
Страница 13, строка 5 снизу	суммѣ	суммы
» 13, строка 4 снизу	элементы, земной орбиты	элементы земной орбиты
» 38, строка 15 сверху	на значеніи функіи λ	на значеніе функціи λ
» 39, строка 1 снизу	$-\ 1263.962\ i^2$	$-\ 1260''.896\ i^2$
» 40, строка 3 сверху	$-\ 10''.58227\ i^2$	$-\ 10''.58251\ i^2$
» 60, строка 1 снизу	$-\ 3\overline{m}^2$	$+\ 3\overline{m}^2$
» 61, строка 3 сверху	$-\ 3\overline{m}^2$	$+\ 3\overline{m}^2$
» 61, строка 5 сверху	$-\ 3\overline{m}^2$	$+\ 3\overline{m}^2$
» 91, строка 4 снизу	A	N

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THÉORIE
DES
MOUVEMENTS DIURNE,
ANNUEL ET SÉCULAIRE DE L'AXE DU MONDE;

PAR
F. FOLIE
MEMBRE DE L'ACADÉMIE ROYALE DE BELGIQUE.

TROISIÈME ET DERNIÈRE PARTIE.

Le livre III traite des variations séculaires.

J'y ai ajouté en appendice les formules qui expriment l'ensemble des variations en obliquité et en longitude, telles qu'elles résultent de ma théorie et de l'adoption des constantes de Struve et Peters pour la précession et la nutation, de Leverrier et Oppolzer pour la variation séculaire de l'écliptique.

On verra que mes formules relatives à l'obliquité concordent mieux qu'aucune des précédentes avec les observations. Il se manifeste encore, toutefois, lorsqu'on les applique aux observations les plus anciennes, des écarts qui restent à expliquer. Ils proviennent peut-être de ce que l'obliquité a été considérée comme constante dans l'intégration.

Dans une Addition au Livre I, j'ai fait voir que l'existence de la nutation diurne a pour conséquence indiscutable une irrégularité dans le mouvement de rotation de l'écorce solide du globe. Cette irrégularité, qui consiste en un balancement semi-diurne de la croûte autour de son axe de rotation, est une véritable nutation; et comme elle n'affecte que l'heure, on pourrait l'appeler nutation horaire. Son maximum peut s'élever à $0^s,06$, et se produit au bout de 6 heures, c'est-à-dire qu'une pendule dont la marche serait parfaite accuserait, comparativement au mouvement diurne du ciel, une avance ou un retard de $0^s,06$ après 6 heures. Cette quantité n'est plus négligeable aujourd'hui en astronomie. Dans les mêmes conditions, le déplacement linéaire d'un point de la croûte terrestre serait, sous la latitude de 45°, de 20 mètres environ plus grand ou plus petit que le chemin qu'il parcourrait dans le cas d'un mouvement de rotation uniforme de la croûte. Ce déplacement est peut-être assez sensible pour pouvoir être accusé par un flotteur qu'on maintiendrait bien immobile pendant quelques heures dans un liquide en repos, et qu'on abandonnerait ensuite à son inertie. Si, comme j'ai lieu de le penser, la résistance du liquide n'est pas suffisante pour vaincre cette inertie, on verra le flotteur se déplacer vers l'E. ou vers l'W., selon que le mouvement de l'écorce terrestre sera accéléré ou retardé.

J'ai installé à Cointe un appareil destiné à des observations de l'espèce; la condition essentielle d'un semblable appareil est une grande stabilité que je ne saurais obtenir à Bruxelles. D'autres expériences très intéressantes peuvent être faites dans le même ordre de recherches. A ce dernier également se rapporte la note que M. Ronkar a communiquée par mon intermédiaire à l'Académie dans sa dernière séance, et dont il a eu l'idée en recherchant le moyen le plus propre à mettre en évidence l'irrégularité, théoriquement démontrée, du mouvement de rotation de l'écorce terrestre.

J'ai cru utile de signaler aux physiciens une expérience du plus grand intérêt, qui n'est nullement dispendieuse, mais qui exige une installation d'une stabilité absolue, dans un milieu de température bien uniforme.

no. 12

CLASSE DES SCIENCES.

Note sur les oscillations d'un pendule produites par le déplacement de l'axe de suspension; par E. Ronkar.

Rapport de M. Folie.

« M. Ronkar a eu l'idée du présent travail en recherchant quel serait le procédé expérimental le plus propre à manifester les petites irrégularités que je viens de signaler dans le mouvement de l'écorce solide du globe.

Il s'est demandé si un pendule en repos, librement suspendu, ne pourrait pas prendre un mouvement oscillatoire dans le cas où son point de suspension éprouverait un mouvement de même nature.

Voici quelles sont les conclusions qu'il tire de l'analyse élégante à laquelle il a soumis la question proposée.

Lorsque l'axe d'un pendule au repos reçoit un certain nombre d'impulsions ondulatoires simples horizontales, le pendule peut conserver un certain mouvement oscillatoire, ou ne le peut pas, suivant les cas.

Lorsque la durée d'oscillation du pendule est la même que celle de l'axe, le pendule conserve un mouvement dont l'amplitude est proportionnelle au nombre d'impulsions reçues par l'axe.

En dehors de ce cas, le pendule peut ne conserver aucune trace d'oscillation, même si les périodes ci-dessus sont dans un rapport très simple, tandis qu'il peut prendre un mouvement sensible dans le cas contraire. Ce mouvement dépend de l'amplitude, du nombre et de la durée des impulsions ainsi que de la phase.

Ces résultats ne sont pas entièrement conformes à l'assertion de Rossi relativement aux pendules employés dans les observations sismiques, quand il dit que des pendules, qui reçoivent quelques impulsions conformes au rythme, sont naturellement fortement agités, et qu'au contraire, avec des impulsions qui se succèdent suivant un rythme différent, ils ne bougeront pas.

Quand on considère l'action d'une onde simple de longue durée, on peut assimiler, pendant son action, le mouvement du pendule à un mouvement oscillatoire, de la période propre au pendule, autour d'une certaine position moyenne qui est elle-même assujettie à un mouvement pendulaire dont la durée d'oscillation est celle de l'onde simple considérée.

L'amplitude de ces deux mouvements est inversement proportionnelle à l'intensité de la pesanteur pour le cas de longues périodes, et on conclut de là un procédé d'expérimentation pour la recherche d'irrégularités périodiques dans le mouvement de rotation diurne ; ces irrégularités, très faibles, peuvent être rendues plus sensibles en diminuant l'action de la pesanteur.

J'espère que nous arriverons prochainement, M. Ronkar et moi, à réaliser dans de bonnes conditions cette expérience, qui serait fondamentale pour l'astronomie.

Je propose à la Classe d'ordonner l'impression du travail de M. Ronkar au *Bulletin* et de voter des remerciements à l'auteur. » — Adopté.

Bruxelles. — Imprimerie de F. HAYEZ, rue de Louvain, 108.

Drawer Moon no. 13

POSTCRIPT, JUNE 1891.

The short note on "A possible cause for the Lunar libration other than an Ellipsoidal figure," to which these remarks are appended, having been put together rather hurriedly some time ago, with no opportunity for revision ere publication, I now add a few lines in the hope of making the case a little clearer. To begin with, it is neccessay to say that the discovery of the "Meridional shoal," as a cause for libration, (if it be the cause) arose more or less as a side issue, while looking for traces of what I call the great *Solar belt* or region about the moon's equator where for a long period the sun's heat must have retarded the junction of the advancing polar caps.

Like many others I had often noticed the position of the chain of Circulara maria, Imbrium, Serenetatis, Crisium and Smythii, lying on a great circle perpendicular to the plane of the moon's prime meridion, but until searching for the solar belt, it never occurred to me to suppose that a change in the lunar axis was either probable or possible.

On plotting out this series of circular Maria, as they would fall if we draw the N. Pole forward some 30°, they fulfilled so nearly the conditions I was in search of, that I accepted the series as evidence of the belt, and at once looked for the cause of the axis shifting.

In a more or less hazy way it seemed to me, that it must be due to the cause which set up and maintains the libration in longitude and latitude, but at the same time, I could not reconcile the Ellipsoidal figure of our moon with a period of former rapid rotation.

If, as seems to be generally agreed, our moon formerly rotated on her axis about as rapidly as our earth, and has been slowed down gradually by tidal friction, to once in 27 odd days, the ellipsoidal figure must have arisen during the latter stages, it could not have existed during the tidal era. My difficulty was in understanding how the Ellipsoidal figure arose, after the globe had so far solidified that there were no seas as now. And I was gradually led to belive that both the shift in the axis and present libration might be due, not to a 'meniscus', on our side, but to some local density of the surface on a spheroidal figure.

In looking over the question I will first notice the nature of Polar caps and their probable extension to the tropics, where, if glaciated, our moon should exhibit traces of the Solar belt.

Later on referring to the ellipsoidal figure, v. meridional shoal and the shifting of the axis as due to the latter.

POLAR CAPS.

On our Earth and Mars, the polar caps seem to be due to a gradual fall in the surface temperature, which cannot there be retarded by incident Solar heat, as about the equatorial regions.

It is well known that in Canada and Siberia, the soil below a certain depth is permanently frozen and that as we approach the tropic the snowline steadily rises. On our moon as Neison points out, radiation is independent of latitude while at the same time the effect of solar heat on the poler is nil, but at a maximum around the equator, (p. 37) we should therefore, it seems to me, expect to see polar caps on our moon, if the globe were constituted at all like ours. But polar caps being thus due to a local deficiency of solar heat, we may assume that they would not necessarily be developed on every globe.

It is doubtful if they would occur for instance on Neptune, and tolerably certain that they will not occur on our sun, when it cools down.

Again caps are not necessarily situated over the actual poles, Webb in his "Celestial Objects" says that Herschel found the caps on Mars not truly opposite one another. One would expect that they might have been diametrically opposite.

Madler and Secchi, he says " found the N. Zone concentric with the axis, but the South considerably eccentric," and it has been suggested by Beer and Madler that the poles of cold may not coinside with the poles of rotation.

On page 148 he tells us that " Secchi found the appearance at the poles irreconcilable with the idea of circular caps, and was forced to adopt the supposition of complicated and lobate forms. Sehiaparelli alludes to the possibility of a mass of floating ice."

Thus it has been taken for granted that the polar caps of Mars should not only be truly circular in form, but placed centrally over the axis of rotation, like the cloudcaps of Jupiter and Saturn.

If we look at our Earth we see the reason for the caps on Mars being eccentric and lobed.

Our S. polar cap is placed over the axis of rotation, and as we know does not drift about, having on it two Volcanoes, mounts Erebus and Terror, but as far as we can see the other cap is Greenland, and N. polar axis falls in a polar basin or open sed.

The character of the arctic and antarctic ice seems to bear this view out. In the south we have the immense flat topped bergs, often miles in extent,and as much as 2000 ft thick, evidently portions of the ice cap broken adrift, while in the North there is a preponderance of floe or fieldice which is formed on open water annually, a few flat topped bergs being occasionally seen near Franz Joseph land (Yeung),the angular bergs of the atlantic being mainly from Greenland (Greely).

If our N. polar basin is without many islands, it stands to reason that an ice cap could not be anchored there, the floe being broken up by tides, and carried off by winds and currents. Either islands or an extensive land surface seems to be neccessary even if eccentrically placed as Greenland to enable a permanent cap to form.

Thus the eccentric position and lobate form of the S. cap on Mars would seem to repeat and be explained by our Northern one. But our Earth and Mars exhibit, by the intervening space between the caps, what I call a belt of solar influence, and the extent of this depends on the secular temperature of the globes. The absence of all trace of such a belt in the present equatorial surfacing of our moon is therefore a distinct anomaly.

If the globe has been constituted at all like our Earth, there must, in the earlier stages of developement, have been an enormous vapor laden atmosphere, and as the secular temperature fell, the poles would constitute as in our case, two vast condensers, and at last refrigerators.

Tho formation and steady developement of caps would be in fact inevitable until they closed over the equator. A very general opinion seems to prevail that our moon was formerly in a heated or even semi molten condition and has slowly cooled down, but that the peculiarities of the present surfacing (poles even included) are results of Volcanic activity. That being so much smaller than our Earth, it has cooled down too rapidly to exhibit traces of an erosive era such as our Earth is now in, with river, valleys, &c., and that dessication more rapidly supervened on the heated stage.

Such a view it seems to me is quite untenable if we are to assume that the globe was constitued like ours and had its rapidity of rotation gradually slowed down by tides. If tides were possible to such an extent and for such a length of time, it is difficult to understand how an era of erosion can be denied, a collateral difficulty arises in the fact that judging by the surfacing, the tidal era must have *preceeded* the volcanic, if it occurred at all on the moon.

But it is remarkable that while our own Geological record shews clearly that our 'Erosive era' is coeval with the vast series of our sedimentary rocks,and that while yet so to speak in mid career, this era is characterised by the formation of *polar caps*. As far as we can see they are actually a neccessity of the conditions, and as secular temperature declines, and atmosphere becomes more attenuated, they will steadily extend, until they reach the tropics, where a prolonged struggle will ensue between the slow fall in temperature of the globe and the incident solar heat.

THE SOLAR BELT.

It is here—about equator—therefore that we might justly expect to find evidence on a glaciating globe of a very long struggle, between the slow refrigeration due to steady fall in temperature and the heating effect of the solar rays.

A ceaseless freezing and thawing, night and days for probably many thousands of years, the ending of which would most likely be comparatively sudden, the last remaining liquid areas retaining their general form and characters as seem near the close of the contest.

An irregular belt, in fact of restricted liquid areas, such as we see in the four circular maria, in Diagram C. Neccessarily this belt would have been on the then equator, and indeed in the breached ramparts on the east and west of Serenitatis and Imbrium; we seem to see the result of a restricted tidal action, and our next enquiry is, as to how such a change in the lunar axis can have taken place, as to carry this solar belt, to its present position, after the chain of maria had become glaciated.

SHIFT IN THE AXIS.

From the known nature of Lunar libration in both latitude and longitude, astronomers have come to the conclusion that the moon must be of an ellipsoidal figure, the longest axis directed towards our earth.

Proctor on page 132 of his "Moon" says, "If the Moon were a perfect sphere, the earth would have no grasp on her, so to speak, whereby to maintain the observed relation between the equator plane and the orbit plane."

The difficulty is to understand how and why this figure arose after the globe had so far solidified, as to appear after the tidal era was over, for it could hardly have existed or been neccessary *during* that era.

Apparently the moon has been looked on as a homogenous globe all over, and hence the neccessity of postulating the existence of a 'meniscus' on the sides facing us, to increase the mass there, which would account for libration.

But I cannot see why, judging by our Earth, we should assume that the lunar crust is of equal density all over. On our northern hemisphere where the continents so greatly preponderate, we see a case where the crust or outer layers must enormously outweigh that on the southern hemisphere, land being about 3 times as heavy as water. If our seas were all solidified and the moon's attraction eliminated, there can be little doubt that, with a slower rotation, our Earth would at last balance itself with the N. hemisphere directed towards the sun.

So that an increased density of the lunar *crust* over a large area would seem to obviate the neccessity of assuming an ellipsoidal figure, and the difficulties arising thence in re former tidal action.

In fact that increase of "Mass" due to greater density of the surface on our hemisphere, on a true spheroid, would produce the same results as a meniscus placed on a homogenous globe.

MERIDIONAL SHOAL.

At first sight it is not easy to see any thing which would uphold the view, neither in the Sinus Medii, or Mare Vaporum can we see any clue, but on taking a more extended view of the surface around the centre of the disc, we notice that three large Maria, Nubium, O. Procellarum and Imbrium cover the Eastern portion of the surface, while on the west again we have also 3 others, Nectaris, Tranquilitatis and Serenitatis.

These 6 Meria or seas, more or less symmetrically border the large meri-dional area from Walter to Cassini, on the S. portion of which we see the well known double series of vast walled plains. The natural inference on looking over this cental area bordered by well marked Maria is that it is not of the nature of a Sea, but rather of a land surface, in the main, and as we examine the detail more closely, bearing in mind the arguments advanced in the 'Theory of surfacing by glaciation' pp 12 to 24; the conviction becomes clearer that on the prime meridion, we have evidence of the existence of a vast shoal or rather submerged continent. The direction and position of the prevailing ridges, the direction and position of the clefts, viewed as cracks or fractures in a glaciated crust especially when viewed in relation to the surrounding maria (viewed as areas of subsidance) seem to me to afford cumulative evidence of such a shoal.

My view is that this shoal, extending some 1500 miles N. to S. by say about 400 East to west, is of greater specific gravity than the glaciated maria around, and answers the purpose of a meniscus as far as libration is concerned, the area colored brown in Diagram A. and outlined by a waved line in C. and D. giving roughly its position.

SHIFTING OF THE AXIS.

If we assume that the moon at one time rotated far more rapidly and has been slowed down by tidal friction, and that eventually libration set in, due to the Earth's attraction acting on a denser portion of the crust; we must remember that even now, she rotates on the axis once in 27 odd days, so that when libration first sets in, it would be a libration in *longitude*.

Not being a mathematician I am not in a position to prove this however, but as far as I can see, the first effect of libration would be to select the present prime meridion, the hemisphere on which the centre of greatest surface density lay and bring it gradually to the mean Geocentrie position.

Later on, when this prime meridion had become so to speak established by selection, the denser portion of the crust in that meridion N. or S. would be drawn forward, and the libration in latitude thereby establish a new Equator. It is noteworthy that if the above were correct, the old and new Equators would intersect each other in the line of the moon's orbit, and their greatest distance apart would be on the prime meridion. This would at once give us a valuable clue when searching for traces of the 'Solar belt,' which, if visible, must have been on the old equator, and by rotating the globe on the extreme east and west points, it should become visible. Now on rotating the globe as above, drawing the North pole forward 30° we find that the great circular Maria Imbrium, Serinitatis, Crisium and Sinythii ranges themselves in line on a great circle, perpendicular to the prime meridion forming a chain in such a manner, and at such a place, as to apparently preclude the possibility of its being accidental. This chain of maria therefore I take to be the remains of the great " Solar belt" above noted, the relics of the last area left liquid, ere the polar caps became united

Their enormous littoral ranges are the vast snow ramparts, raised during the final struggle between glacial and solar agencies, repeating exactly, but on a larger scale, the snow rings of the walled plains and craters.

Not being in a position myself to say whether such a large shift in the lunar axis as 30° was theoretically possible or not, I applied to Professer G. H. Darwin and he at once kindly replied, as follows "the shifting of the axis of rotation in the moon is undoubtedly a mechanical possibility. A large shift, such as you postulate, is far more likely to occur in the case of a body with slow rotation, as in the case of the moon, than in the Earth." It will be seen from the above that the feature put forward as causing the shift in the Lunar axis, (*i, e,* the greater relative density of a large portion of the surface) is also adduced as the cause of the present libration, and hence the necessity of assuming an Ellipsoidal figure obviated. If also the series of maria above alluded to (Diagram C.) are the remains of the " Solar belt" the inference seems justified that the polar caps have joined, and the Moon is glaciated from pole to pole.

July 22, 1891. S. E. P.

"Radhanath" Press; Dibrugarh.

no. 14

Ephemeride zur Beobachtung des Mondkraters Mösting A

mitgetheilt von Herrn Dr. J. Franz in Königsberg.

Ueber den Gebrauch der Ephemeride sei hier nur Folgendes bemerkt:

Um für irgend einen Beobachtungsort und für eine beliebige Beobachtungszeit die nöthigen Angaben aus der Ephemeride zu entnehmen, bediene man sich des Argumentes »Länge von Greenwich + Stundenwinkel des Mondes« ausgedrückt in Theilen des Tages und zwar so, dafs westliche Längen und westliche Stundenwinkel positiv, östliche Längen und östliche Stundenwinkel dagegen negativ genommen werden. Z. B. für Washington Jan. 12 sei der Mond beobachtet in dem östlichen Stundenwinkel $3^h\ 2^m\ 55^s$. Da nun Washington die Länge $5^h\ 8^m\ 12^s$ westlich von Greenwich hat, so wird das Argument = Jan. 12 + $5^h\ 8^m\ 12^s$ — $3^h\ 2^m\ 55^s$ = Jan. 12,087. Hiermit erhält man $a = 269^0\ 14',0$ und $d = -25^0\ 43',7$.

Für Meridianbeobachtungen (wo also im Argument: Stundenwinkel = 0 zu setzen ist), ist die Reduction vom Krater auf den Mondmittelpunkt:

$$\text{in AR.} = (\alpha_☾ - \alpha_k) + v_\alpha \,.\, \Delta\varphi$$
$$\text{in Decl.} = (\delta_☾ - \delta_k) + v_\delta \,.\, \Delta\varphi + v'_\delta \,.\, (\Delta\varphi)^2,$$

wenn man den Unterschied Polhöhe des Beobachtungsortes (φ) weniger Polhöhe von Greenwich ($\varphi_0 = +51^0,475$), also $\varphi - \varphi_0 = \Delta\varphi$ setzt.

$\Delta\varphi$ ist in Graden auszudrücken und darf nur eine mäfsige Anzahl Grade betragen, so dafs diese Formel etwa für die nördliche gemäfsigte Zone gilt. Z. B. für Washington Jan. 7 wird die Reduction $-9^s,91$ und $+65'',7$.

Für Beobachtungen aufserhalb des Meridians und für Meridianbeobachtungen aufserhalb der nördlichen gemäfsigten Zone entnehme man aus der nachstehenden Ephemeride a und d, ermittele ferner für die Beobachtungszeit die scheinbare (mit Parallaxe behaftete) AR. und Decl. des Mondmittelpunktes α und δ, sowie ebenso die scheinbare Horizontalparallaxe des Mondes p.

Mit Hülfe der letzteren ergiebt sich der anzuwendende scheinbare Mondhalbmesser h aus der Relation:

$$\log \sin h = \log \sin p + 9{,}43513 \,.$$

Hierauf berechne man:

$$\sin \pi \sin k = \cos d \sin (a - \alpha)$$
$$\cos \pi \sin k = \sin d \cos \delta - \cos d \sin \delta \cos (a - \alpha)$$
$$- \cos k = \sin d \sin \delta + \cos d \cos \delta \cos (a - \alpha)$$

$$\operatorname{tg} \sigma = \frac{\sin h \sin k}{1 - \sin h \cos k}$$

und hiermit die Reduction auf den Mondmittelpunkt:

$$\alpha_{☾} - \alpha_k = - \sigma \sin \pi \sec \delta$$
$$\delta_{☾} - \delta_k = - \sigma \cos \pi \,.$$

Näheres siehe Astron. Nachrichten Nr. 2917.

Librations-Ephemeride zur Beobachtung des Mondkraters Mösting A.

1892 Im Meridian von Greenwich	Lage des Mondäquators			Phys. Libr. in Länge	Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen				
							in AR.	Var. für 1° Breite	in Decl.	Var. für 1° Breite	Zweites Glied
	i	$\Delta-\mho$	Ω'	u	a	d	$\alpha_☾-\alpha_k$	v_α in $0^s,0000$	$\delta_☾-\delta_k$	v_δ in $0'',000$	v'_δ in $0'',0000$
	22°	2°	356°								
	′	′	′	′	° ′	° ′	s		″		
Jan. 7	35,6	52,5	52,5	−0,2	196 41,5	−11 26,0	− 9,82	+23	+73,5	+195	−19
8	35,0	52,2	52,8	−0,2	209 51,6	−16 7,5	−10,14	24	+50,5	217	16
9	34,6	52,2	52,9	−0,3	223 34,5	−20 8,4	−10,54	26	+29,8	238	14
10	34,1	52,4	52,8	−0,3	237 54,1	−23 13,6	−11,05	28	+13,4	247	12
11	33,6	52,6	52,5	−0,3	252 45,6	−25 9,0	−11,62	29	+ 2,6	251	11
12	33,3	53,0	51,9	−0,3	267 54,7	−25 44,5	−12,06	31	− 2,3	249	10
13	33,0	53,7	51,2	−0,3	283 2,0	−24 57,1	−12,20	30	− 2,2	242	10
14	32,8	54,4	50,5	−0,2	297 47,9	−22 51,1	−11,91	+29	+ 0,8	+232	−11
15	32,6	55,2	49,6	−0,1	312 0,3	−19 37,6	−11,19	26	+ 4,8	219	12
16	32,6	56,1	48,7	0,0	325 35,7	−15 31,1	−10,11	22	+ 8,6	204	13
17	32,7	56,8	48,0	+0,1	338 38,7	−10 47,7	− 8,77	16	+12,1	187	15
18	32,8	57,4	47,4	+0,2	351 19,6	− 5 3,0	− 7,24	14	+15,6	168	16
19	33,0	58,0	46,8	+0,3	3 50,2	− 0 31,7	− 5,58	12	+20,1	149	17
20	33,3	58,3	46,4	+0,5	16 23,4	+ 4 31,0	− 3,85	08	+26,8	129	18
Febr. 5	32,9	49,1	56,3	+0,7	218 8,6	−18 37,7	−11,79	+28	+25,8	+234	−14
6	32,4	49,0	56,4	+0,7	232 13,7	−22 6,4	−12,17	30	+10,6	244	12
7	32,0	49,2	56,2	+0,6	246 53,0	−24 30,7	−12,50	31	+ 0,9	247	10
8	31,6	49,6	55,8	+0,6	261 56,6	−25 38,5	−12,69	32	− 3,5	246	10
9	31,3	50,1	55,2	+0,7	277 6,1	−25 24,0	−12,62	31	− 3,5	241	10
10	31,0	50,8	54,5	+0,7	292 1,6	−23 48,4	−12,22	30	− 0,7	232	10
11	30,8	51,4	53,7	+0,8	306 27,6	−21 0,6	−11,48	27	+ 3,5	224	11
12	30,7	52,3	52,8	+0,9	320 17,3	−17 13,8	−10,45	+24	+ 7,9	+211	−12
13	30,7	53,1	52,0	+1,0	333 32,3	−12 43,7	− 9,19	20	+12,2	192	14
14	30,8	53,8	51,3	+1,1	346 20,8	− 7 46,0	− 7,74	17	+16,5	175	15
15	31,0	54,4	50,6	+1,2	358 54,0	− 2 36,0	− 6,13	13	+21,4	155	16
16	31,2	54,8	50,1	+1,3	11 24,9	+ 2 31,4	− 4,44	10	+27,9	134	18
17	31,4	55,0	49,9	+1,4	24 5,5	+ 7 21,8	− 2,68	06	+36,8	113	19
18	31,7	55,1	49,7	+1,5	37 6,3	+11 40,5	− 0,92	+02	+49,1	091	20
19	32,1	55,1	49,8	+1,5	50 33,5	+15 13,2	+ 0,69	−02	+65,7	069	21

1892 Im Meridian von Greenwich	Lage des Mondäquators			Phys. Libr. in Länge	Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen				
							in AR.	Var. für 1° Breite	in Decl.	Var. für 1° Breite	Zweites Glied
	i	$\Delta-☋$	$☊'$	u	a	d	$\alpha_{☾}-\alpha_k$	v_α in $0^s,0000$	$\delta_{☾}-\delta_k$	v_δ in 0″,000	v'_δ in 0″,0000
	22°	2°	356°								
	′	′	′	′	° ′	° ′	s		″		
März 6	30,0	45,9	59,7	+1,3	256 0,9	−25 19,5	−14,24	+36	− 6,6	+248	−09
7	29,6	46,4	59,3	+1,3	271 10,2	−25 37,8	−14,21	35	− 5,2	242	09
8	29,3	47,0	58,7	+1,3	286 12,9	−24 33,9	−13,67	33	− 0,9	234	10
9	29,1	47,6	57,9	+1,4	300 51,2	−22 13,6	−12,78	30	+ 4,4	223	11
10	28,9	48,4	57,1	+1,4	314 55,0	−18 48,9	−11,62	27	+ 9,5	210	12
11	28,8	49,2	56,3	+1,5	328 22,7	−14 34,7	−10,25	22	+ 14,2	196	13
12	28,9	50,0	55,5	+1,6	341 20,4	− 9 46,5	− 8,75	19	+ 18,9	179	15
13	28,9	50,6	54,7	+1,7	353 58,1	− 4 40,1	− 7,16	+15	+ 24,1	+160	−16
14	29,1	51,2	54,1	+1,8	6 28,6	+ 0 29,6	− 5,47	12	+ 30,5	137	17
15	29,4	51,6	53,6	+1,9	19 4,1	+ 5 28,2	− 3,72	08	+ 38,9	118	19
16	29,6	51,8	53,4	+2,0	31 55,9	+10 0,9	− 1,96	04	+ 50,0	096	20
17	29,9	51,9	53,3	+2,1	45 12,3	+13 53,4	− 0,26	+01	+ 64,6	074	20
18	30,2	51,6	53,6	+2,1	58 56,8	+16 51,6	+ 1,26	−03	+ 83,0	052	21
19	30,6	51,3	54,0	+2,2	73 6,0	+18 43,1	+ 2,39	−06	+105,0	033	22
20	30,9	50,7	54,7	+2,2	87 30,2	+19 19,4	+ 2,87	−07	+129,3	019	23
April 4	27,7	43,0	62,9	+1,5	280 19,9	−25 7,2	−15,23	+38	− 2,6	+239	−10
5	27,4	43,7	62,2	+1,5	295 9,6	−23 16,3	−14,46	35	+ 4,8	228	10
6	27,2	44,4	61,4	+1,6	309 27,5	−20 15,8	−13,27	31	+ 11,7	214	11
7	27,1	45,1	60,7	+1,6	323 9,0	−16 19,8	−11,82	27	+ 17,4	199	13
8	27,0	45,9	59,8	+1,7	336 17,3	−11 43,7	−10,23	22	+ 22,4	183	14
9	27,0	46,6	59,0	+1,7	349 1,4	− 6 43,2	− 8,56	18	+ 27,5	165	16
10	27,2	47,3	58,3	+1,8	1 33,4	− 1 33,3	− 6,85	15	+ 33,5	145	17
11	27,4	47,9	57,7	+1,9	14 5,4	+ 3 30,8	− 5,11	11	+ 41,3	124	18
12	27,6	48,2	57,3	+2,0	26 49,6	+ 8 15,2	− 3,38	+07	+ 51,6	+101	−19
13	27,9	48,4	57,1	+2,1	39 55,6	+12 25,1	− 1,70	+04	+ 64,9	079	20
14	28,2	48,3	57,2	+2,1	53 28,8	+15 46,4	− 0,17	00	+ 81,5	058	21
15	28,5	48,1	57,5	+2,2	67 29,0	+18 5,3	+ 1,07	−03	+101,4	038	21
16	28,8	47,6	58,0	+2,2	81 48,8	+19 12,4	+ 1,84	−05	+123,4	021	22
17	29,0	46,9	58,7	+2,3	96 15,3	+19 1,8	+ 1,97	−05	+145,6	010	23
18	29,2	46,2	59,5	+2,3	110 33,7	+17 33,0	+ 1,44	−04	+165,5	007	23
			357°								
Mai 4	25,3	41,1	5,1′	+1,3	317 48,9	−17 58,6	−13,83	+31	+ 19,1	+206	−13
5	25,2	41,8	4,3	+1,3	331 9,7	−13 36,9	−11,71	26	+ 25,4	189	14
6	25,2	42,5	3,4	+1,4	344 2,2	− 8 44,7	−10,01	22	+ 30,9	171	15
7	25,3	43,3	2,7	+1,4	356 37,6	− 3 37,4	− 8,26	18	+ 36,5	151	17
8	25,4	43,9	2,0	+1,5	9 8,2	+ 1 30,4	− 6,51	14	+ 43,4	130	18
9	25,6	44,4	1,5	+1,5	21 46,4	+ 6 24,1	− 4,78	10	+ 52,5	108	19
10	25,9	44,6	1,1	+1,6	34 42,8	+10 49,1	− 3,12	06	+ 64,6	085	20

1892 Im Meridian von Greenwich	Lage des Mondäquators			Phys. Libr. in Länge	Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen: in AR.	Var. für 1° Breite	in Decl.	Var. für 1° Breite	Zweites Glied
	i	$\Delta - \mho$	Ω'	u	a	d	$\alpha_☾ - \alpha_k$	υ_α in 0s,0000	$\delta_☾ - \delta_k$	υ_δ in 0'',000	υ'_δ in 0'',0000
	22°	2°	357°								
	′	′	′	′	° ′	° ′	s		″		
Mai 11	26,2	44,8	1,0	+1,7	48 5,1	+14 31,3	— 1,62	+04	+ 79,8	+063	—21
12	26,5	44,7	1,2	+1,7	61 55,4	+17 16,5	— 0,40	+01	+ 98,0	042	21
13	26,8	44,4	1,5	+1,8	76 8,7	+18 53,0	+ 0,39	—01	+118,4	025	22
14	27,1	43,9	2,0	+1,8	90 34,4	+19 13,3	+ 0,63	—02	+139,1	013	22
15	27,3	43,2	2,8	+1,8	104 57,7	+18 15,3	+ 0,29	—01	+157,6	006	23
16	27,5	42,3	3,7	+1,8	119 4,4	+16 3,5	— 0,49	+01	+171,6	009	23
17	27,5	41,3	4,7	+1,7	132 45,7	+12 47,5	— 1,49	04	+178,7	021	24
Juni 2	23,4	38,4	8,0	+0,6	338 58,9	—10 44,2	—11,04	+24	+ 32,4	+179	—15
3	23,4	39,1	7,2	+0,7	351 40,0	— 5 41,4	— 9,37	20	+ 38,5	158	16
4	23,5	39,8	6,5	+0,7	4 11,3	— 0 32,3	— 7,63	16	+ 45,0	137	18
5	23,7	40,4	5,9	+0,8	16 45,1	+ 4 28,4	— 5,89	13	+ 53,0	115	19
6	23,9	40,8	5,4	+0,8	29 33,0	+ 9 6,3	— 4,22	10	+ 63,6	098	20
7	21,2	41,0	5,2	+0,9	42 41,8	+13 7,0	— 2,69	06	+ 77,2	070	21
8	24,5	41,0	5,2	+0,9	56 21,0	+16 16,3	— 1,44	04	+ 94,0	049	21
9	24,8	40,8	5,3	+1,0	70 29,4	+18 21,4	— 0,62	01	+113,1	030	22
10	25,1	40,5	5,7	+1,0	84 52,2	+19 12,8	— 0,37	+01	+132,9	+016	—22
11	25,4	40,0	6,3	+1,0	99 18,5	+18 45,8	— 0,73	02	+150,9	009	23
12	25,6	39,3	7,1	+0,9	113 33,5	+17 2,5	— 1,56	04	+164,2	010	23
13	25,7	38,4	8,0	+0,9	127 25,8	+14 11,0	— 2,59	06	+170,9	021	24
14	25,8	37,4	9,1	+0,8	140 50,3	+10 23,0	— 3,57	09	+170,0	041	24
15	25,8	36,4	10,2	+0,8	153 49,6	+ 5 53,0	— 4,34	10	+161,6	070	24
16	25,7	35,3	11,2	+0,7	166 30,9	+ 0 55,8	— 4,91	12	+146,1	104	24
Juli 2	21,8	36,2	10,5	—0,2	11 45,0	+ 2 28,5	— 6,62	+14	+ 53,4	+123	—19
3	22,1	36,7	9,8	—0,2	24 26,3	+ 7 17,4	— 4,92	11	+ 62,6	100	20
4	22,3	37,1	9,5	—0,1	37 27,8	+11 34,5	— 3,31	06	+ 74,6	078	20
5	22,6	37,3	9,3	—0,1	50 55,7	+15 5,8	— 1,92	04	+ 89,6	054	21
6	22,9	37,3	9,2	0,0	64 51,0	+17 37,9	— 0,93	02	+107,5	037	22
7	23,2	37,1	9,5	0,0	79 8,3	+18 59,7	— 0,52	01	+127,0	021	22
8	23,5	36,6	10,0	0,0	93 35,1	+19 4,3	— 0,80	02	+145,5	012	23
9	23,7	36,0	10,7	0,0	107 56,8	+17 51,1	— 1,70	04	+159,8	010	23
10	23,9	35,2	11,5	—0,1	121 59,6	+15 25,8	— 2,95	+07	+167,1	+018	—24
11	24,1	34,3	12,5	—0,1	135 35,7	+11 58,9	— 4,22	10	+166,2	036	24
12	24,1	33,3	13,6	—0,2	148 44,8	+ 7 44,1	— 5,26	13	+157,1	064	24
13	24,1	32,2	14,8	—0,3	161 32,6	+ 2 56,2	— 6,01	14	+140,9	097	23
14	24,0	31,2	15,9	—0,4	174 9,0	— 2 10,0	— 6,48	15	+119,3	131	22
15	23,8	30,2	17,0	—0,5	186 46,0	— 7 19,9	— 6,82	16	+ 93,8	171	21
16	23,5	29,3	17,9	—0,6	199 37,1	—12 18,2	— 7,16	17	+ 66,2	214	18

1892 Im Meridian von Greenwich		Lage des Mondäquators			Phys. Libr. in Länge	Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen				
								in AR.	Var. für 1° Breite	in Decl.	Var. für 1° Breite	Zweites Glied
		i	$\Delta - ☋$	☊'	u	a	d	$\alpha_{☾} - \alpha_k$	υ_α in $0^s,0000$	$\delta_{☾} - \delta_k$	υ_δ in 0″,000	υ'_δ in 0″,0000
		22°	2°	357°								
		′	′	′	′	° ′	° ′	s		″		
Aug.	1	20,7	33,3	13,6	—1,0	45 31,2	+13 45,7	— 2,19	+05	+ 86,2	+063	—21
	2	21,0	33,4	13,5	—1,0	59 15,5	+16 42,9	— 0,92	02	+102,2	043	22
	3	21,3	33,3	13,5	—1,0	73 24,7	+18 34,0	— 0,14	00	+121,1	025	22
	4	21,6	33,0	13,8	—1,0	87 49,2	+19 10,1	— 0,04	00	+140,5	015	23
	5	21,9	32,5	14,4	—0,9	102 14,6	+18 28,1	— 0,69	02	+157,2	008	23
	6	22,1	31,8	15,2	—1,0	116 26,3	+16 31,0	— 1,95	05	+167,9	013	24
	7	22,3	31,0	16,1	—1,0	130 14,1	+13 27,5	— 3,47	09	+169,7	028	24
	8	22,4	30,1	17,1	—1,0	143 34,4	+ 9 30,6	— 4,93	+12	+161,7	+054	—24
	9	22,5	29,0	18,2	—1,1	156 30,4	+ 4 54,1	— 6,13	15	+144,9	088	24
	10	22,4	28,0	19,4	—1,2	169 10,8	— 0 6,4	— 7,04	17	+121,1	127	23
	11	22,3	27,0	20,5	—1,3	181 46,8	— 5 16,4	— 7,70	18	+ 93,1	165	21
	12	22,0	26,0	21,6	—1,4	194 31,3	—10 20,9	— 8,28	20	+ 63,7	200	19
	13	21,7	25,1	22,5	—1,5	207 36,7	—15 5,1	— 8,87	21	+ 35,5	227	17
	14	21,4	24,4	23,2	—1,7	221 13,1	—19 13,0	— 9,58	+24	+ 11,3	246	14
	30	19,4	29,3	17,9	—1,7	67 44,6	+17 56,2	+ 0,02	00	+118,8	+030	—21
	31	19,7	29,2	18,1	—1,6	82 3,8	+19 3,2	+ 0,64	—02	+135,0	006	22
Sept.	1	20,1	28,8	18,5	—1,6	96 30,0	+18 52,8	+ 0,56	—02	+153,4	001	23
	2	20,3	28,3	19,0	—1,6	110 48,2	+17 25,4	— 0,23	+01	+168,3	007	23
	3	20,6	27,6	19,8	—1,6	124 46,2	+14 47,6	— 1,55	04	+175,9	017	24
	4	20,7	26,7	20,8	—1,6	138 17,6	+11 10,8	— 3,10	08	+173,6	038	25
	5	20,8	25,8	21,8	—1,7	151 23,1	+ 6 48,8	— 4,61	11	+160,5	071	25
	6	20,9	24,7	23,0	—1,7	164 9,1	+ 1 56,5	— 5,93	+14	+137,4	+110	—24
	7	20,8	23,6	24,1	—1,8	176 46,1	— 3 11,3	— 7,08	17	+107,1	153	23
	8	20,6	22,6	25,2	—1,9	189 26,6	— 8 19,8	— 8,13	19	+ 73,1	193	20
	9	20,4	21,6	26,3	—2,0	202 23,3	—13 14,0	— 9,18	22	+ 39,8	225	18
	10	20,1	20,8	27,1	—2,1	215 47,4	—17 38,3	—10,31	26	+ 11,1	247	15
	11	19,7	20,2	27,9	—2,2	229 46,0	—21 17,0	—11,55	29	— 9,6	259	12
	12	19,3	19,7	28,4	—2,3	244 19,7	—23 54,5	—12,78	33	— 20,6	262	10
	13	18,8	19,4	28,7	—2,4	259 19,7	—25 18,3	—13,78	+36	— 21,9	259	09
	29	18,6	24,6	23,2	—1,8	105 8,5	+18 8,0	+ 0,80	—02	+164,7	+004	—23
	30	18,8	24,1	23,7	—1,8	119 14,4	+15 57,9	+ 0,07	00	+177,0	008	24
Oct.	1	19,1	23,3	24,5	—1,8	132 56,4	+12 43,7	— 1,03	+03	+181,8	023	24
	2	19,2	22,3	25,6	—1,8	146 11,6	+ 8 38,7	— 2,29	06	+176,8	049	25
	3	19,3	21,3	26,7	—1,9	159 4,2	+ 3 57,2	— 3,58	09	+160,8	086	24
	4	19,3	20,2	27,8	—1,9	171 43,6	— 1 5,7	— 4,83	12	+134,4	129	24
	5	19,2	19,1	29,0	—2,0	184 21,6	— 6 15,5	— 6,09	15	+ 99,9	175	20

1892 Im Meridian von Greenwich	Lage des Mondäquators			Phys. Libr. in Länge	Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen				
							in AR.	Var. für 1° Breite	in Decl.	Var. für 1° Breite	Zweites Glied
	i	$\Delta-\mho$	Ω'	u	a	d	$\alpha_☾ - \alpha_k$	v_α in $0^s,0000$	$\delta_☾ - \delta_k$	v_δ in 0″,000	v'_δ in 0″,0000
	22°	2°	357°								
	′	′	′	′	° ′	° ′	s		″		
Oct. 6	19,0	18,1	30,1	−2,1	197 10,9	−11 17,2	− 7,47	+18	+ 61,6	+217	−20
7	18,7	17,2	31,1	−2,2	210 23,7	−15 55,4	− 9,06	23	+ 25,1	247	17
8	18,4	16,4	31,9	−2,2	224 9,1	−19 54,5	−10,88	28	− 3,9	264	14
9	18,0	15,8	32,6	−2,3	238 30,4	−22 58,3	−12,80	33	− 21,3	270	11
10	17,6	15,4	33,1	−2,4	253 22,6	−24 52,9	−14,51	39	− 25,7	268	10
11	17,1	15,2	33,3	−2,5	268 31,3	−25 28,4	−15,58	40	− 19,4	259	09
12	16,7	15,2	33,3	−2,5	283 48,4	−24 40,7	−15,64	39	− 7,4	246	10
28	17,4	19,6	28,6	−1,5	127 33,9	+14 7,3	− 0,26	+01	+180,2	+014	−23
29	17,6	18,7	29,5	−1,5	140 58,5	+10 21,6	− 0,98	02	+181,7	033	24
30	17,7	17,7	30,5	−1,5	153 58,5	+ 5 53,7	− 1,75	04	+174,3	063	24
31	17,7	16,7	31,6	−1,6	166 41,2	+ 0 58,3	− 2,55	06	+157,1	101	24
Nov. 1	17,7	15,6	32,8	−1,6	179 17,8	− 4 9,9	− 3,44	08	+130,0	145	23
2	17,6	14,6	33,9	−1,7	192 0,6	− 9 16,2	− 4,53	11	+ 94,6	192	20
3	17,4	13,6	35,0	−1,7	205 2,5	−14 5,4	− 5,95	15	+ 55,1	232	17
4	17,1	12,7	36,0	−1,8	218 34,2	−18 22,2	− 7,85	+20	+ 17,5	+261	−15
5	16,8	11,9	36,8	−1,9	232 41,8	−21 50,3	−10,21	27	− 11,2	277	13
6	16,4	11,4	37,4	−1,9	247 23,5	−24 14,4	−12,75	34	− 25,6	281	11
7	15,9	11,0	37,8	−2,0	262 29,6	−25 22,6	−14,89	40	− 24,9	272	10
8	15,5	10,8	38,0	−2,0	277 40,4	−25 8,8	−16,14	42	− 12,9	259	10
9	15,1	10,9	37,9	−2,0	292 36,3	−23 34,8	−16,33	41	+ 4,2	243	11
10	14,7	11,1	37,6	−2,0	307 2,1	−20 49,4	−15,66	37	+ 21,3	223	11
27	16,2	13,0	35,6	−0,8	161 41,1	+ 2 58,7	− 1,95	+05	+162,0	+082	−23
28	16,3	12,0	36,7	−0,8	174 17,8	− 2 5,4	− 2,23	05	+143,3	121	23
29	16,2	10,9	37,9	−0,9	186 55,6	− 7 13,5	− 2,62	06	+116,5	163	22
30	16,1	9,9	39,0	−0,9	199 47,5	−12 10,8	− 3,30	08	+ 82,8	206	20
Dec. 1	15,8	8,9	40,1	−1,0	213 5,1	−16 42,1	− 4,47	11	+ 45,6	242	17
2	15,5	8,0	41,0	−1,0	226 57,0	−20 31,4	− 6,30	16	+ 10,3	268	14
3	15,2	7,3	41,8	−1,1	241 25,7	−23 23,1	− 8,79	23	− 15,7	282	12
4	14,8	6,8	42,4	−1,1	256 24,0	−25 3,3	−11,57	+31	− 26,9	+283	−10
5	14,4	6,5	42,7	−1,2	271 36,3	−25 22,8	−13,95	37	− 22,4	275	10
6	13,9	6,4	42,8	−1,2	286 41,8	−24 20,8	−15,34	40	− 6,7	260	11
7	13,5	6,5	42,7	−1,2	301 22,4	−22 2,8	−15,60	39	+ 13,1	240	12
8	13,2	6,8	42,3	−1,2	315 27,9	−18 40,7	−14,98	35	+ 32,0	218	14
9	12,8	7,3	41,9	−1,1	328 57,1	−14 29,6	−13,82	31	+ 47,7	193	15
10	12,6	7,9	41,2	−1,0	341 56,6	− 9 44,8	−12,33	27	+ 60,0	168	16

1892 Im Meridian von Greenwich	Lage des Mondäquators			Phys. Libr. in Länge	Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen				
							in AR.	Var. für 1° Breite	in Decl.	Var. für 1° Breite	Zweites Glied
	i	$\Delta-☋$	$☊'$	u	a	d	$\alpha_{☾}-\alpha_k$	v_α in $0^s,0000$	$\delta_{☾}-\delta_k$	v_δ in $0'',000$	v'_δ in $0'',0000$
	22°	2°	357°	′	° ′	° ′	s		″		
Dec. 26	14,8	7,3	41,9	+0,2	181 56,9	— 5 10,9	— 2,81	+07	+118,3	+146	—21
27	14,8	6,2	43,1	+0,1	194 40,7	—10 13,6	— 2,87	07	+ 91,3	184	20
28	14,6	5,2	44,2	+0,1	207 45,9	—14 56,6	— 3,21	08	+ 59,9	220	18
29	14,4	4,2	45,3	0,0	221 22,0	—19 4,1	— 4,07	10	+ 27,2	250	15
30	14,1	3,4	46,1	0,0	235 34,7	—22 20,4	— 5,62	14	— 2,1	270	13
31	13,8	2,7	46,9	0,0	250 21,2	—24 30,8	— 7,85	21	— 22,1	280	11
32	13,4	2,2	47,4	—0,1	265 29,7	—25 23,9	—10,37	28	— 28,2	281	10
1893											
Jan. 2	13,0	1,9	47,7	—0,1	280 40,7	—24 54,7	—12,51	+33	— 20,0	+272	—10
3	12,6	1,9	47,7	—0,1	295 33,9	—23 6,3	—13,76	35	— 2,3	256	11
4	12,3	2,1	47,5	—0,1	309 55,2	—20 8,4	—14,02	34	+ 19,0	235	13
5	11,9	2,4	47,2	—0,1	323 39,5	—16 15,2	—13,50	32	+ 39,0	209	15
6	11,6	2,9	46,6	0,0	336 50,6	—11 42,1	—12,47	29	+ 55,7	182	16
7	11,5	3,6	45,9	+0,1	349 37,2	— 6 44,8	—11,14	26	+ 69,0	155	18
8	11,4	4,3	45,1	+0,2	2 11,1	— 1 38,3	— 9,64	21	+ 79,7	128	19
9	11,4	5,1	44,3	+0,3	14 44,9	+ 3 23,0	— 8,06	17	+ 88,9	101	20

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no. 15

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OBSERVATIONS OF THE MOON MADE AT THE RADCLIFFE OBSERVATORY, OXFORD, DURING THE YEAR 1892; AND A COMPARISON OF THE RESULTS WITH THE TABULAR PLACES FROM HANSEN'S LUNAR TABLES.

ON THE CAUSE OF THE *PER SALTUM* CHANGE IN THE ERRORS OF HANSEN'S LUNAR TABLES AS USUALLY COMPARED WITH OBSERVATION, BEING AN APPENDIX TO A PAPER IN THE "MONTHLY NOTICES," VOL. LIV., NO. 1, WHICH EXHIBITS THESE ERRORS FROM 1847 TO 1892.

BY

E. J. STONE, M.A., F.R.S.,

Radcliffe Observer.

A

Observations of the Moon made at the Radcliffe Observatory, Oxford, during the Year 1892; and a Comparison of the Results with the Tabular Places from Hansen's Lunar Tables.

By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

The present paper contains the observations of the Right Ascensions and North Polar Distances of the Moon, made at the Radcliffe Observatory during the year 1892. These results are here compared with those deduced from Hansen's Lunar Tables on two suppositions—

(1) That the mean times found in the usual way, from the sidereal times at mean noon given in the *Nautical Almanac*, were not altered in scale, or affected with any different systematic errors of determination, by the adoption, in 1864, of a different ratio of the Julian year of 365¼ "mean solar days" to the mean tropical year.

(2) That the "mean solar times" which accurately correspond to given "local sidereal times" before and after 1864 are necessarily different from the adoption and use of a "Julian year" and "mean solar day" which are different fractional parts of the mean tropical year, and therefore are different intervals of absolute time.

If these differences are real, and are neglected by astronomers in referring the tabular positions of the Sun, Moon, &c., to the meridians for comparison with the local sidereal times at transit, the effects of the errors made will necessarily be thrown upon the theoretical expressions.

The differences of the errors thus made in practice, according to my views, before and after 1864, are given in Table III.

The periodical errors of Hansen's Tables, after correction, shown by the observations during the year 1892, are not larger than those which will be found to exist during the years 1847 to 1863.

The mean annual errors of Hansen's Tables show small but systematic periodical changes from 1847 to 1863, but since 1864 the errors in longitude have progressively increased until they amount in 1892 to $+19''\cdot7$. It will be seen that after the corrections, which I have indicated as necessary for a comparison of the changes of the errors of Hansen's Tables before and after 1864, have been applied, the *per saltum* change in 1864 is no longer shown. The mean annual error in longitude, 1847–63, is $-1''\cdot85$; the mean annual error in longitude, 1864–92, is $-1''\cdot56$; and the mean annual error in longitnde for the whole period, 1847–92, is $-1''\cdot67$.

For facilities for an accurate comparison between Hansen's Lunar Tables and Observations I am again indebted to the places published in the *Connaissance des Temps*.

TABLE I.

Radcliffe Observations of the Moon, 1892.

R.A.'s and N.P.D.'s of the Centre of the Moon, 1892; compared with Hansen's Tabular Places, **Uncorrected** *and* **Corrected** *for the change in the Unit of Mean Time introduced in the year 1864.*

Day, 1892.		Correction to be subtracted from M.T. for Change of Sidereal Time at Mean Noon since 1864.	Observer.	Limb observed in R.A.	Observed R.A.			R.A. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**	Limb observed in N.P.D.	Observed N.P.D.			N.P.D. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**
		s			h	m	s	s			s		°	′	″	″	″	″	″
Jan.	6	41·51	F.B.	I.	0	49	19·57	20·93	+1·36	−1·46	−0·10	S.	88	30	23·16	13·35	− 9·81	+10·98	+1·17
	7	41·52	W.	I.	1	41	51·40	52·80	+1·40	−1·48	−0.08	S.	82	3	10·94	1·25	− 9·69	+10·52	+0·83
	11	41·53	F.B.	I.	5	32	49·34	50·84	+1·50	−1·73	−0·23	N.	64	12	47·86	45·31	− 2·55	+ 2·93	+0·38
	13	41·54	R.	I. & II.	7	36	43·36	44·66	+1·30	−1·67	−0·37	N.	64	5	24·81	26·40	+ 1·59	− 2·64	−1·05
	15	41·55	R.	II.	9	29	56·92	58·07	+1·15	−1·47	−0·32	S.	69	53	54·79	60·20	+ 5·41	− 6·71	−1·30
Feb.	3	41·62	F.B.	I.	1	25	6·62	7·90	+1·28	−1·50	−0·22	S.	83	47	26·14	16·19	− 9·95	+10·95	+1·00
	8	41·64	R.	I.	6	13	52·97	54·34	+1·37	−1·71	−0·34	N.	63	17	14·81	13·80	− 1·01	+ 0·96	−0·05
	11	41·66	F.B.	I.	9	8	54·22	55·61	+1·39	−1·50	−0·11	N.	68	25	1·66	6·50	+ 4·84	− 6·13	−1·29
	16	41·68	F.B.	II.	13	2	50·93	52·13	+1·20	−1·23	−0·03	S.	93	35	54·13	60·35	+ 6·22	− 9·24	−3·02
Mar.	7	41·76	W.	I.	6	56	31·01	32·43	+1·42	−1·68	−0·26	N.	63	4	47·84	47·51	− 0·33	− 1·01	−1·34
	8	41·76	R.	I.	7	55	33·50	34·95	+1·45	−1·61	−0·16	N.	64	26	35·47	38·51	+ 3·04	− 3·50	−0·46
	9	41·76	F.B.	I.	8	51	27·19	28·64	+1·45	−1·52	−0·07	N.	67	10	11·16	14·47	+ 3·31	− 5·58	−2·27
	10	41·77	F.B.	I.	9	43	46·97	48·30	+1·33	−1·42	−0·09	N.	70	58	50·23	56·72	+ 6·49	− 7·18	−0·69
	11	41·77	R.	I.	10	32	46·55	47·55	+1·00	−1·34	−0·34	N.	75	35	29·77	37·80	+ 8·03	− 8·31	−0·28

Day, 1892.	Correction to be subtracted from M.T. for Change of Sidereal Time at Mean Noon since 1864.	Observer.	Limb observed in R.A.	Observed R.A.			R.A. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**	Limb observed in N.P.D.	Observed N.P.D.			N.P.D. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**
	s			h	m	s	s	s	s	s		°	′	″	″	″	″	″
Mar. 12	41·78	F.B.	I.	11	19	4·06	5·25	+1·19	−1·27	−0·08	N.	80	44	22·97	31·53	+ 8·56	− 9·03	−0·47
18	41·80	W.	II.	15	51	17·51	18·96	+1·45	−1·43	+0·02	S.	111	17	28·58	35·84	+ 7·26	− 6·22	+1·04
20	41·81	R.	II.	17	40	7·32	8·59	+1·27	−1·62	−0·35	S.	116	31	50·64	53·21	+ 2·57	− 2·31	+0·26
Apr. 4	41·87	W.	I.	7	37	9·89	11·23	+1·34	−1·66	−0·32	N.	63	39	36·14	37·93	+ 1·79	− 2·82	−1·03
5	41·87	R.	I.	8	34	38·08	39·41	+1·33	−1·56	−0·23	N.	66	1	36·43	41·47	+ 5·04	− 5·06	−0·02
6	1·88	F.B.	I.	9	28	7·35	8·51	+1·16	−1·45	−0·29	N.	69	33	48·60	54·04	+ 5·44	− 6·79	−1·35
7	41·88	W.	I.	10	17	52·28	53·41	+1·13	−1·36	−0·23	N.	73	58	2·86	10·28	+ 7·42	− 8·04	−0·62
8	41·88	R.	I.	11	4	35·92	36·94	+1·02	−1·29	−0·27	N.	78	58	20·90	29·30	+ 8·40	− 8·86	−0·46
9	41·89	F.B.	I.	11	49	13·47	14·59	+1·12	−1·24	−0·12	N.	84	21	2·89	11·44	+ 8·55	− 9·33	−0·78
11	41·90	W	I.	13	16	7·75	8·82	+1·07	−1·24	−0·17	N. & S.	95	26	52·39	61·40	+ 9·01	− 9·30	−0·29
17	41·92	R.	II.	18	18	21·22	22·29	+1·07	−1·64	−0·57	N.	117	15	16·68	18·89	+ 2·21	− 0·67	+1·54
May 6	42·00	R.	I.	11	34	47·56	48·47	+0·91	−1·26	−0·35	N.	82	30	17·18	26·49	+ 9·31	− 9·26	+0·05
9	42·01	W.	I.	13	45	29·37	30·34	+0·97	−1·26	−0·29	N.	99	2	6·64	14·39	+ 7·75	− 9·06	−1·31
10	42·01	R.	I.	14	30	49·07	50·12	+1·05	−1·31	−0·26	N.	104	10	16·87	24·88	+ 8·01	− 8·36	−0·35
12	2·02	F.B.	II.	16	9	19·57	20·95	+1·38	−1·48	−0·10	S.	112	39	11·08	19·17	+ 8·09	− 5·75	+2·34
14	42·03	R.	II.	18	0	17·61	18·79	+1·18	−1·63	−0·45	S.	117	4	14·43	16·59	+ 2·16	− 1·46	+0·70
18	42·04	R.	II.	21	52	47·03	48·64	+1·61	−1·56	+0·05	N.	108	26	56·56	48·12	− 8·44	+ 8·33	−0·11
June 3	42·11	R.	I.	12	2	45·14	45·99	+0·85	−1·25	−0·40	N.	86	3	53·54	61·66	+ 8·12	− 9·51	−1·39

Day, 1892.	Correction to be subtracted from M.T. for Change of Sidereal Time at Mean Noon since 1864.	Observer.	Limb observed in R.A.	Observed R.A.	R.A. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**	Limb observed in N.P.D.	Observed N.P.D.	N.P.D. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**
	s			h m s	s	s	s	s		° ′ ″	″	″	″	″
June 6	42·12	F.B.	I.	14 14 19·77	21·18	+1·41	−1·30	+0·11	N.	102 24 36·10	43·54	+ 7·44	− 8·66	−1·22
7	42·12	R.	I.	15 1 11·72	12·74	+1·02	−1·37	−0·35	N.	107 13 58·80	66·27	+ 7·47	− 7·70	−0·23
8	42·13	F.B.	I.	15 51 3·13	4·41	+1·28	−1·46	−0·18	N.	111 23 41·40	46·29	+ 4·89	− 6·36	−1·47
9	42·13	W.	I.	16 44 22·11	23·38	+1·27	−1·55	−0·28	N. & S.	114 37 47·59	53·37	+ 5·78	− 4·54	+1·24
12	42·14	R.	II.	19 39 47·51	49·05	+1·54	−1·68	−0·14	N.	116 16 11·10	10·33	− 0·77	+ 3·01	+2·24
13	42·15	F.B.	II.	20 38 46·93	48·84	+1·91	−1·64	+0·27	N.	113 43 33·39	31·13	− 2·26	+ 5·55	+3·29
14	42·15	W.	II.	21 35 48·52	50·35	+1·83	−1·58	+0·25	N.	109 46 49·18	44·13	− 5·05	+ 7·75	+2·70
17	42·16	F.B.	II.	0 15 19·88	21·42	+1·54	−1·47	+0·07	N.	92 13 69·38	56·91	−12·47	+11·33	−1·14
30	42·22	W.	I.	11 44 41·85	42·89	+1·04	−1·28	−0·24	N.	83 58 32·66	40·75	+ 8·09	− 9·58	−1·49
July 7	42·24	W.	I.	17 18 35·65	36·96	+1·31	−1·62	−0·31	S. & N.	116 4 8·78	10·73	+ 1·95	− 3·16	−1·21
Aug. 4	42·36	W.	I.	17 51 37·63	39·01	+1·38	−1·67	−0·29	S.	117 1 49·72	48·64	− 1·08	− 1·69	−2·77
5	42·36	R.	I.	18 51 33·36	34·73	+1·37	−1·72	−0·35	S.	117 14 44·81	40·70	− 4·11	+ 1·01	−3·10
12	42·39	W.	II.	1 27 18·37	20·04	+1·67	−1·52	+0·15	N.	82 33 51·12	39·20	−11·92	+11·29	−0·63
Sept. 5	42·48	W.	I.	22 21 20·77	22·28	+1·51	−1·61	−0·10	S.	105 19 40·60	30·99	− 9·61	+ 9·98	+0·37
7	42·49	W.	II.	0 11 36·69	38·26	+1·57	−1·55	+0·02	N.	92 7 45·52	33·55	−11·97	+12·09	+0·12
8	42·50	R.	II.	1 6 3·10	4·37	+1·27	−1·56	−0·29	N.	85 2 66·84	55·74	−11·10	+11·99	+0·89
Oct 1	42·59	W.	I.	20 55 27·01	28·38	+1·37	−1·65	−0·28	S.	112 48 38·19	32·44	− 5·75	+ 6·65	+0·90
3	42·60	R.	I.	22 48 38·41	39·60	+1·19	−1·58	−0·39	S.	102 26 32·38	21·44	−10·94	+10·79	−0·15

Day, 1892.	Correction to be subtracted from M.T. for Change of Sidereal Time at Mean Noon since 1864.	Observer.	Limb observed in R.A.	Observed R.A.	R.A. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**	Limb observed in N.P.D.	Observed N.P.D.	N.P.D. from Hansen's Tables for Uncorrected Mean Times.	Hansen *minus* Observed. **Uncorrected.**	Correction due to the Change in the Unit of Mean Time.	Hansen *minus* Observed. **Corrected.**
	s			h m s	s	s	s	s		° ′ ″	″	″	″	″
Oct. 5	42·60	R.	I.	0 38 28·39	29·66	+1·27	−1·58	−0·31	S.	88 35 46·82	35·19	−11·63	+12·44	+0·81
10	42·62	R.	II.	5 43 45·35	47·06	+1·71	−1·87	−0·16	N.	62 39 23·83	21·77	− 2·06	+ 2·24	+0·18
Nov. 1	42·71	R.	I.	0 9 13·70	14·72	+1·02	−1·54	−0·52	S.	92 28 11·16	1·67	− 9·49	+12·15	+2·66
3	42·72	R.	I.	2 0 40·21	41·40	+1·19	−1·67	−0·48	S.	78 16 27·66	16·23	−11·43	+11·65	+0·22
5	42·73	R.	II.	4 5 21·38	22·83	+1·45	−1·88	−0·43	N.	66 47 61·08	55·43	− 5·65	+ 7·33	+1·68
29	42·82	R.	I.	0 37 4·69	5·72	+1·03	−1·51	−0·48	S.	88 43 66·29	55·95	−10·34	+11·90	+1·56
30	42·83	R.	I.	1 30 50·17	51·28	+1·11	−1·58	−0·47	S.	81 49 12·48	2·33	−10·15	+11·76	+1·61
Dec. 3	42·84	W.	I. & II.	4 35 24·83	26·44	+1·61	−1·93	−0·32	N.	65 4 56·82	50·50	− 6·32	+ 5·88	−0·44
12	42·88	R.	II.	13 3 6·81	7·78	+0·97	−1·26	−0·29	S.	94 44 1·55	7·00	+ 5·45	− 9·79	−4·34
24	42·92	R.	I.	22 37 17·33	18·41	+1·08	−1·48	−0·40	S.	103 28 43·89	33·85	−10·04	+ 9·79	−0·25
26	42·93	W.	I.	0 17 58·41	59·49	+1·08	−1·46	−0·38	S.	91 1 33·43	23·36	−10·07	+11·47	+1·40
29	42·94	R.	I.	2 59 45·58	46·80	+1·22	−1·71	−0·49	S.	71 52 58·39	48·73	− 9·66	+ 9·52	−0·14
30	42·95	W.	I.	4 1 42·48	43·80	+1·32	−1·84	−0·52	S.	66 58 16·66	10·13	− 6·53	+ 7·22	+0·69
Mean of Errors, without regard to sign					...	1·290	...	0·263		...	...	6·696	...	1·117
Mean Errors for Year	...	...	...		...	+1·290	...	−0·233						

Observers: W., Mr. W. Wickham; R., Mr. W. H. Robinson; F.B., Mr. F. A. Bellamy.

Table II.

Radcliffe Observations of the Moon, 1892.

Errors of the Moon's Tabular Place in Longitude and Ecliptic Polar Distance, **Uncorrected** *and* **Corrected** *for the Change in the Unit of Mean Time introduced in the year 1864.*

Day, 1892.		Observer.	Limb observed in R.A.	Limb observed in N.P.D.	Errors of Longitude (Hansen *minus* Observed). Uncorrected.	Corrected.	Errors of E.N.P.D. (Hansen *minus* Observed). Uncorrected.	Corrected.
					"	"	"	"
Jan.	6	F.B.	I.	S.	+22·68	−1·84	−1·11	+0·50
	7	W.	I.	S.	+22·92	−1·41	−1·56	+0·35
	11	F.B.	I.	N.	+20·38	−3·22	−1·59	+0·23
	13	R.	I. & II.	N.	+17·60	−5·11	−1·28	−0·23
	15	R.	II.	S.	+17·16	−4·71	−0·01	+0·20
Feb.	3	F.B.	I.	S.	+21·43	−3·42	−2·16	−0·29
	8	R.	I.	N.	+18·35	−4·56	−1·46	+0·06
	11	F.B.	I.	N.	+20·04	−1·85	−1·06	−0·78
	16	F.B.	II.	S.	+19·01	−1·58	−1·16	−2·61
Mar.	7	W.	I.	N.	+18·90	−3·60	−2·16	−1·00
	8	R.	I.	N.	+19·92	−2·22	−0·82	−0·03
	9	F.B.	I.	N.	+20·27	−1·56	−2·28	−1·92
	10	F.B.	I.	N.	+20·02	−1·44	−0·13	−0·23
	11	R.	I.	N.	+16·55	−4·71	+2·08	+1·57
	12	F.B.	I.	N.	+19·61	−1·28	+0·95	+0·03
	18	W.	II.	S.	+21·35	+0·49	+2·82	+0·96
	20	R.	II.	S.	+17·15	−4·69	+1·97	+0·43
Apr.	4	W.	I.	N.	+18·09	−4·42	−1·22	−0·30
	5	R.	I.	N.	+18·99	−3·07	+0·35	+0·77
	6	F.B.	I.	N.	+17·27	−4·31	+0·02	0·00
	7	W.	I.	N.	+17·95	−3·33	+1·05	+0·62
	8	R.	I.	N.	+17·17	−3·86	+1·93	+1·11
	9	F.B.	I.	N.	+18·83	−1·96	+1·17	0·00
	11	W.	I.	N. & S.	+18·24	−2·47	+2·32	+0·69
	17	R.	II.	N.	+14·23	−7·66	+2·64	+1·31
May	6	R.	I.	N.	+16·16	−4·77	+3·18	+2·11
	9	W.	I.	N.	+16·21	−4·49	+2·11	+0·31
	10	R.	I.	N.	+17·01	−3·70	+2·78	+0·86
	12	F.B.	II.	S.	+20·31	−0·93	+4·41	+2·55
	14	R.	II.	S.	+15·79	−6·02	+2·16	+0·70
	18	R.	II.	N.	+24·56	+0·71	−0·16	+0·14

Day, 1892.		Observer.	Limb observed in R.A.	Limb observed in N.P.D.	Errors of Longitude (Hansen *minus* Observed). Uncorrected.	Corrected.	Errors of R.N.P.D. (Hansen *minus* Observed). Uncorrected.	Corrected.
					″	″	″	″
June	3	R.	I.	N.	+14·95	−6·06	+2·37	+1·11
	6	F.B.	I.	N.	+21·96	+1·11	+0·17	−1·69
	7	R.	I.	N.	+16·13	−4·88	+3·09	+1·18
	8	F.B.	I.	N.	+18·50	−2·77	+1·02	−0·91
	9	W.	I.	N. & S.	+17·95	−3·63	+3·51	+1·72
	12	R.	II.	N.	+20·63	−2·24	+2·74	+1·89
	13	F.B.	II.	N.	+26·06	+2·76	+4·50	+4·13
	14	W.	II.	N.	+26·18	+2·48	+3·58	+3·70
	17	F.B.	II.	N.	+26·24	+1·42	−2·28	−0·63
June	30	W.	I.	N.	+17·51	−3·89	+1·25	+0·06
July	7	W.	I.	S. & N.	+17·78	−4·26	+0·66	−0·90
Aug.	4	W.	I.	S.	+18·46	−3·92	−1·33	−2·72
	5	R.	I.	S.	+18·62	−4·38	−2·45	−3·51
	12	W.	II.	N.	+27·52	+2·31	−1·89	+0·24
Sept.	5	W.	I.	S.	+23·92	−1·49	−1·01	−0·18
	7	W.	II.	N.	+26·43	+0·23	−1·63	+0·23
	8	R.	II.	N.	+21·80	−4·35	−3·02	−0·83
Oct.	1	W.	I.	S.	+19·88	−3·99	−0·28	−0·21
	3	R.	I.	S.	+20·34	−5·24	−3·49	−2·31
	5	R.	I.	S.	+22·15	−4·60	−3·23	−1·08
	10	R.	II.	N.	+22·85	−2·14	−1·43	+0·12
Nov.	1	R.	I.	S.	+17·87	−8·24	−2·62	−0·66
	3	R.	I.	S.	+20·34	−6·69	−4·69	−2·23
	5	R.	II.	N.	+20·69	−6·14	−1·76	+0·53
Nov.	29	R.	I.	S.	+18·32	−7·25	−3·44	−1·40
	30	R.	I.	S.	+19·03	−7·07	−3·38	−1·07
Dec.	3	W.	I. & II.	N.	+22·60	−4·25	−3·13	−1·06
	12	R.	II.	S.	+15·49	−5·67	−0·52	−2·35
Dec.	24	R.	I.	S.	+18·43	−5·33	−3·41	−2·42
	26	W.	I.	S.	+18·92	−5·80	−2·82	−0·98
	29	R.	I.	S.	+19·41	−6·66	−4·36	−2·10
	30	W.	I.	S.	+19·17	−7·18	−2·83	−0·73
Mean of Errors, without regard to sign ...			...	...	19″·687	3″·775	2″·032	1″·076
Mean Errors for Year			...	...	+19″·687	−3″·410	...	...

Table III.

Mean Excess over Observation of the Moon's Tabular Place in Longitude for the years 1847 *to* 1892, *as computed from Hansen's Tables.*

Uncorrected *and* **Corrected** *for the change in the Unit of Mean Time introduced in the year* 1864.

Year.	Errors of Longitude (Hansen *minus* Observed). Uncorrected.	Corrected.	
	"	"	
1847	+ 0·51	+0·51	
1848	− 0·53	−0·53	
1849	− 1·08	−1·08	
1850	− 0·97	−0·97	
1851	− 1·93	−1·93	
1852	− 1·57	−1·57	
1853	− 2·18	−2·18	
1854	− 2·34	−2·34	Mean Annual Error in Longitude
1855	− 1·40	−1·40	from 1847 to 1863
1856	− 1·51	−1·51	= −1″·85.
1857	− 2·41	−2·41	
1858	− 2·61	−2·61	
1859	− 2·49	−2·49	
1860	− 3·62	−3·62	
1861	− 2·95	−2·95	
1862	− 2·83	−2·83	
1863	− 1·61	−1·61	
1864	+ 0·12	−0·81	
1865	+ 1·27	−0·22	
1866	+ 2·14	−0·22	
1867	+ 3·48	+0·36	
1868	+ 4·12	+0·28	
1869	+ 4·28	−0·35	
1870	+ 4·83	−0·66	
1871	+ 6·96	+0·44	
1872	+ 7·31	+0·10	
1873	+ 8·24	+0·20	
1874	+ 9·29	+0·56	
1875	+ 9·87	+0·36	
1876	+ 9·80	−0·50	Mean Annual Error
1877	+ 9·23	−1·90	in Longitude
1878	+ 8·22	−3·60	from 1864 to 1892
1879	+ 9·63	−3·12	(taken out with the
1880	+10·89	−2·77	corrected argument)
1881	+10·51	−4·06	= −1″·56.
1882	+12·68	−2·51	
1883	+14·71	−1·50	
1884	+14·65	−1·91	
1885	+15·20	−1·82	
1886	+15·34	−2·53	
1887	+15·70	−3·25	
1888	+17·68	−2·46	
1889	+17·37	−3·51	
1890	+18·02	−3·55	
1891	+19·30	−2·90	
1892	+19·69	−3·41	

1847 to 1879: Greenwich observations. 1880 to 1882: Mean of Greenwich and Radcliffe. 1883 to 1892: Radcliffe observations.

Radcliffe Observatory, Oxford:
1893 *November* 9.

A 4

On the Cause of the per saltum *Change in the Errors of Hansen's Lunar Tables as usually compared with Observation, being an Appendix to a paper in the "Monthly Notices," vol. liv. No.* 1, *which exhibits these errors from* 1847 *to* 1892. By E. J. Stone, M.A., F.R.S., Radcliffe Observer.

I have recently received several inquiries on points connected with the explanation which I have given of the *per saltum* change, which certainly took place about 1864, between the tabular places given by Hansen's Lunar Tables and the corresponding observations of the Moon, when compared in the usual manner. These inquiries show that I have not yet succeeded in making my views clearly understood. I have thought it, therefore, desirable to draw up the following statement of my views, which I hope may render them at least intelligible to mathematical astronomers.

But before proceeding with these explanations, it is perhaps desirable to first call attention to the fact that if P sin $(\alpha t+\beta)$ denotes a long inequality, neither the omission of a real inequality of this form from the tabular expression of the Moon's longitude nor the erroneous introduction of such an inequality can introduce *per saltum* changes in the errors of the Tables. It will render the determinations of the values of the "epoch correction" and "mean motion" from a discussion of residuals erroneous; and it will lead, therefore, at some time or other, to progressive changes in the tabular errors for many successive years, but not to such changes in the errors of the Tables as those shown on page 10 of the *Monthly Notices*, vol. liv. No. 1.

The errors discussed in the present paper will be found to arise from the use of two independent systems of time-measures, whilst the ratio of the units is only approximately known.

In the comparisons between the theoretical and observational results of astronomy for the determination of the numerical values of the data involved, it is necessary to fix the positions of the meridian of the observer in terms of the variable, t, which is employed to fix the geocentric positions of the centres of gravity. And it becomes necessary for my purpose to discuss the accuracy with which this is effected by the methods adopted in practical astronomical work.

The "mean tropical year" referred to in my papers is that of practical astronomy; an interval of time in terms of which motions can be, and are, measured; and not any fictitious or imaginary interval of time which could not be employed for such purposes.

At the equinoxes the Sun's centre passes through the plane of the Equator, and its north polar distance is 90°.

The times by the observatory clocks of these equinoctial passages can either be directly observed at different stations on the Earth's surface, or deduced from the observations usually made. The recorded times will be subject, as usual, to errors of observation and reduction; but these errors can be confined within small limits, and will not be accumulative with the time. The intervals between successive transits of the Sun through the same equinox are the "tropical years" with which we are concerned.

The lengths of the "tropical years" can, therefore, be determined directly from observation expressed in terms of the mean sidereal day, hours, minutes, and seconds, provided the errors of the observatory clocks on the local sidereal times can be correctly determined.

The lengths of the tropical years can be corrected for secular and periodic variations from the results of mathematical investigations, in which the mean tropical year itself, or some definite part of that interval of time, is taken as the unit; and the length of the mean tropical year at the selected epoch will thus become determinable in terms of mean sidereal seconds, subject to the condition already mentioned. The interval of time which can thus be deduced from observation expressed in mean sidereal seconds is the mean tropical year with which we are concerned in practical astronomical work.

The ratio of the mean tropical year to a mean sidereal day will be absolutely independent of any particular fraction of a mean tropical year, which we may, as a matter of convenience, adopt for the "unit day"; and its numerical value can be approximated to without limit provided only the local sidereal times can be accurately deduced from observation.

The following is a general statement of the course followed to render the local sidereal times determinable from observation. It will be found that the invariability of the ratio of the mean tropical year to the mean sidereal day is a fundamental condition of the success of the methods adopted for the determination of the local sidereal times; and that no approximative value of this ratio is used in any part of the work, except in terms so small that any errors which may arise from its use will be confined within very small limits; and that such errors, if sensible, are finally eliminated from the determinations of the local sidereal times, and prevented from accumulation.

The tabular right ascensions of the clock stars are brought up from epoch in terms of the mean tropical year and some definite sub-multiple,

$$\frac{N}{2\pi} \text{ (of a mean tropical year)},$$

where N is the circular measure of some assigned angle, as convertible units of time. The fractional part of the mean tropical year thus employed as a unit in stellar astronomy is treated as the "mean solar day." These are mere statements of facts.

The local sidereal times are deduced by the aid of clocks, whose rates are steady over intervals of a sidereal day, from comparisons between the recorded clock times at the transits of the different clock-stars over the meridian and their tabular right ascensions; and, if any sensible errors exist in these local sidereal times, they are determined, and when sensible duly allowed for in practice, from direct comparisons between the local sidereal times at the transits of the Sun's centre over the meridian and the corresponding right ascensions of the Sun's centre as deduced from the observed north polar distances of the Sun and the obliquity of the ecliptic.

The result of such a process must be to entirely prevent the accumulation of error on the local sidereal times, as found from observation, due either to errors in the adopted tabular right ascensions of the clock stars at epoch; the use of slightly erroneous values of the precession constant, and of proper motions of the stars; and of any small errors due to imperfect allowances for the changes of the right ascensions of the clock-stars between the times for which they are directly computed and the times of meridian passage. The local sidereal times can thus be correctly found from observation, and they will strictly correspond to definite "mean solar times" on the scale of time-measurement adopted in stellar astronomy.

We are placed in a position to approximate without limit, by the process described, to the ratio of the "mean tropical year at epoch" to the "mean sidereal day at epoch."

If we denote the exact ratio of the "mean tropical year" to the "mean sidereal day" by

$$\frac{2\pi}{N_0}+1,$$

where N_0 is the circular measure of an angle to whose value we are continually approximating; and, if Ω denotes the mean motion in right ascension of an observer's meridian in the "unit day," equal to

$$\frac{N}{2\pi}\,.\,\text{(mean tropical year)},$$

which is adopted in stellar astronomy, we shall have

$$\Omega=2\pi\frac{N}{N_0}+N.$$

If we knew and adopted the accurate value of N_0 to fix the proportionate part of the mean tropical year which is used as the "unit day" in stellar astronomy, we should then have

$$\Omega_0=2\pi+N_0,$$

and the "unit day" would, in this case, be the interval of time which astronomers have wished to adopt as their fundamental "unit day," an interval of time which, for distinction between the different "unit days" which are adopted in practice, I shall call "a physical mean solar day."

The "physical mean solar day" is the interval between consecutive transits of the meridian over the Sun's centre when cleared from secular and periodic variations, which in this case are as large proportionately to the interval of time measured as in the corresponding case of the tropical years already mentioned.

But if we adopt a definite angle N to fix the "unit day," the expression for the right ascension of the observer's meridian, A, as a function of the time, t, on the scale under consideration, assumes the form

$$2\pi\iota + A = A_0 + \left(2\pi\frac{N}{N_0} + N\right) t + \sigma t^2 + \text{nutation in R.A.},$$

where ι is an integer, A_0 and N_0 angles whose values have to be found from a discussion of these equations when formed with corresponding values of A and t.

The *exact values* of the angles

$$A_0 \text{ and } \Omega = 2\pi\frac{N}{N_0} + N$$

are unknown with the system of time measures, t, which is used in practical stellar astronomy, and which is fundamentally based upon the Sun's motion and the mean tropical year.

But in finding the local sidereal times from observation, if A_r denotes the right ascension of the observer's meridian at the time $t = r$, and $r + \delta r$ the required time when the right ascension of the meridian is A, we shall have

$$A_r = A_0 + \left(2\pi\frac{N}{N_0} + N\right) r + \sigma r^2 + \text{nut.} - 2\pi\iota$$

$$A = A_r + \left(2\pi\frac{N}{N_0} + N\right) \delta r + \sigma\ \ (2r\delta r + \overline{\delta r}|^2) + \frac{\delta\ .\ \text{nut.}}{\delta t}\ .\ \delta r.$$

The small terms

$$\sigma\,(2r\delta r + \overline{\delta r}|^2) + \frac{\delta\ .\ \text{nut.}}{\delta t}\ .\ \delta r$$

can be applied, if it is thought necessary, with an approximate value of δr, but they are so small that they can, in practice, be generally neglected; and we have

$$\delta r = \frac{A - A_r}{2\pi\frac{N}{N_0} + N}.$$

If we adopt for N the nearest approximative value to N_0 available at the time we shall have

$$\frac{N}{N_0} = 1 + x,$$

where x will be generally some small fraction, but may be zero; and if we adopt for A_0, as an approximative value, l, we shall have

$$A_r = l + Nr + \sigma r^2 + \text{nutation} - 2\pi\,(\iota - r) + \delta l + 2\pi x r\,;$$

and, therefore,

$$t = r + \frac{A - (l + Nr + \sigma r^2 + \text{nutation} - 2\pi(\iota - r)) - \delta l - 2\pi x r}{2\pi + N + 2\pi\,.\,x}\,;$$

or if

$$A'_r = l + Nr + \sigma r^2 + \text{nutation} - 2\pi\,(\iota - r)$$

$$t = r + \frac{A - A'_r - \delta l - 2\pi x r}{2\pi + N + 2\pi x}.$$

But if

$$t = r + \delta r$$

is the time which corresponds to the transit of a star, the centre of the Sun, or of the Moon, &c., over the meridian of the observer; A, the right ascension of the meridian at the time of transit, and, therefore, the right ascension of the object observed at the time

$$t = r + \delta r\,;$$

$\phi\,(t)$ the function which gives the tabular right ascension for the time; and $\delta\phi$ the true error of the tables in right ascension; we shall have

$$A = \phi\,(r) + \phi'\,.\,\delta r + \phi''\,.\,\left.\frac{\overline{\delta r}}{1\,.\,2}\right|^2 + \&c. + \delta\,.\,\phi$$

where the expansion has to be carried to the required degree of approximation. We can, of course, compute the values of $\phi\,(t)$ for each hour

$$\phi\left(r + \frac{1}{24}\right),\ \phi\left(r + \frac{2}{24}\right)\&c.$$

and then determine

$$\phi\left(r + \frac{h}{24} + \delta t\right)$$

by expansion. We have, therefore,

$$A = \phi\,(r) + \phi'\,.\,\frac{A - A'_r - \delta l - 2\pi x r}{2\pi + N + 2\pi x} + \&c. + \delta\,.\,\phi.$$

And if, in finding the tabular right ascensions at the meridian transits for comparison with the true right ascensions, we neglect the effects of the small terms δl, $2\pi x$, we necessarily include in the differences between the observed right ascensions and the "tabular right ascensions" the effects of the neglected terms δl and $2\pi x$ amongst those due to the errors of the Tables, $\delta\phi$; and similar remarks, of course, apply to the formation of the residual errors in N.P.D.

The importance of the errors introduced from neglecting the terms δl and $2\pi x$ will depend upon the magnitudes of the factors $\phi'(r)$, &c. In the case of the stars they will be practically insensible. But if, as is usual, we neglect the effect of these small terms in finding the tabular right ascensions a_1, a_2, a_3, . . . at the transits of the different clock-stars over the meridian of an observer on a given "sidereal day," we shall have the following exact equations of condition.

$$(1) \ldots \quad A_r + \left(2\pi\frac{N}{N_0} + N\right)\delta r_1 = \phi_1(r) + \phi_1' \,.\, \delta r_1 + \&c. + \delta \,.\, \phi_1 = a_1 + \delta a_1$$

$$(2) \ldots \quad A_r + \left(2\pi\frac{N}{N_0} + N\right)\delta r_2 = \phi_2(r) + \phi_2' \,.\, \delta r_2 + \&c. + \delta \,.\, \phi_2 = a_2 + \delta a_2$$

where δa_1, δa_2, δa_3 will denote the total corrections which the tabular right ascensions of the clock-stars require, including any which may be due to the neglect of the small terms δl and $2\pi x$. But if c_1, c_2, c_3 are the times in seconds indicated by the observatory clock at the meridian transits of the clock-stars; if we denote by c_0 the *unknown clock* time which accurately corresponds to the time $t = r$, on the scale of time under consideration; and if the "unit day" which equals the $\frac{N}{2\pi}$th part of a mean tropical year is put equal to 86400 seconds, we shall have

$$\delta r_1 = \frac{\rho\,(c_1 - c_0)}{86400},\quad \delta r_2 = \frac{\rho\,(c_1 - c_0)}{86400},\ \&c.,$$

where ρ will be some number which will remain constant so long as the clock has any steady rate, and the same "unit day" is adopted and put equal to 86400 seconds. We shall have, therefore,

$$(3) \ldots\ldots \quad A_r + \frac{2\pi}{86400}\left(\frac{N}{N_0} + \frac{N}{2\pi}\right) \,.\, \rho\,(c_1 - c_0) = a_1 + \delta a_1$$

$$(4) \ldots\ldots \quad A_r + \frac{2\pi}{86400}\left(\frac{N}{N_0} + \frac{N}{2\pi}\right) \,.\, \rho\,(c_2 - c_0) = a_2 + \delta a_2$$

$$\&c. = \&c.$$

The angle N_0 which appears in these equations is a constant, although its exact value is unknown, and N is the value assigned to the angle which is actually used in practice to fix the length of the "unit day."

If, therefore,

$$\rho\left(\frac{N}{N_0} + \frac{N}{2\pi}\right) = R,$$

R will be a constant, so long as ρ is a constant or the rate of the clock used is steady, and the same "unit day" is adopted. We shall have, therefore,

$$(5) \quad . \quad . \quad . \quad \frac{A_r}{2\pi} . 86400 + R(c_1 - c_0) = \frac{\alpha_1}{2\pi} . 86400^s + \frac{\delta\alpha_1}{2\pi} . 86400^s$$

$$\&c. = \&c.$$

where both sides of the equations are expressed in terms of the "mean solar seconds" of the "unit day," and from these equations values of the constants

$$\frac{A_r}{2\pi} . 86400^s - Rc_0)$$

and of R could be found. But as we require the clock errors on local sidereal time we proceed otherwise. The equivalent of the "unit day," or 86400 mean solar seconds, on the scale under consideration, in sidereal seconds is

$$86400^s \left(\frac{N}{N_0} + \frac{N}{2\pi}\right)$$

and therefore replacing the 86400 mean solar seconds by its equivalent in the equations (3); (4); &c., we have

$$(6) \quad . \quad . \quad . \quad A_r + \frac{2\pi}{86400^s} . \rho(c_1 - c_0) = \alpha_1 + \delta\alpha_1$$

$$(7) \quad . \quad . \quad A_r + \frac{2\pi}{86400^s} . \rho(c_2 - c_0) = \alpha_2 + \delta\alpha_2$$

$$\text{or } (8) \quad . \quad . \quad \frac{A_r}{2\pi} . 86400^s - \rho c_0 + \rho c_1 = \frac{\alpha_1}{2\pi} . 86400 + \frac{\delta\alpha_1}{2\pi} . 86400^s$$

$$\&c. = \&c.$$

where both sides of the equation are now expressed in terms of mean sidereal seconds.

The "rates" R and ρ and the corrections

$$\frac{A_r}{2\pi} . 86400^s - Rc_0$$

$$\frac{A_r}{2\pi} . 86400^s - \rho c_0$$

are different in the two cases, and the values which would be found for them would necessarily differ when deduced from observation. The adjustments for such changes of scale are necessarily included in the determination of clock errors and rates when accurately found from observation.

Denoting

$$\frac{A^r}{2\pi} . 86400^s \text{ by } A_r^s$$

$$\frac{\alpha_1}{2\pi} . 86400^s \text{ by } \alpha_1^s, \&c.,$$

we have

(9) . . . . $A_r^s - \rho \,.\, c_0 + \rho \,.\, c_1 = \alpha_1^s + \delta\alpha_1^s$

(10) . . . . $A_r^s - \rho \,.\, c_0 + \rho \,.\, c_2 = \alpha_2^s + \delta\alpha_2^s$

&c. = &c.,

where both sides of the equations are now expressed in mean sidereal seconds.

When the equations are thus expressed in sidereal seconds we can reduce them to $0^h\ 0^m\ 0^s$ sidereal for comparison with each other and with the results found from the observations made on a consecutive "day," and thus facilitate the determination of the rate constant ρ.

If we put $\tau^s + \delta\tau^s$ for the unknown difference of time

$$A_r^s - \rho c_0,$$

which appears in all these equations, we shall have

(11) . . . . $\tau^s + \rho c_1 + \delta\tau^s = \alpha_1^s + \delta\alpha_1^s$

(12) . . . . $\tau^s + \rho c_2 + \delta\tau^s = \alpha_2^s + \delta\alpha_2^s$,

&c. = &c.

And, if we provisionally neglect the small errors $\delta\alpha_1^s$, $\delta\alpha_2^s$, and find definite values of τ^s and ρ, from the equations

(13) . . . . $\tau^s + \rho c_1 = \alpha_1^s$

(14) . . . . $\tau^s + \rho c_2 = \alpha_2^s$, &c.,

the observatory clock when corrected by the " errors τ^s and ρ will give the "local sidereal times" as directly deduced from observation.

It will not, however, be possible to deduce the right ascension, A, of the observer's meridian accurately from the local sidereal times as thus found from the observed clock-time c by the formula

(15) . . . . $A = \left(\frac{\tau^s + \rho c}{86400}\right) 2\pi$

unless the effects of the errors $\delta\alpha_1^s$, $\delta\alpha_2^s$, which have been neglected in finding τ^s and ρ, are insensible.

But if $c_\odot$ is the recorded time of the Sun's transit by the clock on the "sidereal day" under consideration, and we put the Sun's right ascension, α,

$$= \frac{\tau^s + \delta\tau^s + \rho c_\odot}{86400} \,.\, 2\pi,$$

the correction $\delta\tau^s$, which the local "sidereal time" requires when found as described, can be deduced from this equation if the Sun's right ascension can be otherwise found from observation.

But if the north polar distances of the Sun's centre are observed, the right ascensions can be found in terms of the observed north polar distances, and of the obliquity of the ecliptic. When the observations are made near the equinoxes, any small errors in the adopted value of the obliquity will not sensibly affect the resulting values of the right ascensions of the Sun thus deduced, and a mean value of $\delta\tau^s$ can thus be found. And by extending the discussion to observations of the Sun made over a year, the mean correction, $\delta\tau^s$, required by the local sidereal times found from the tabular right ascensions of the clock stars, and any correction required by the adopted value of the obliquity of the ecliptic, can be deduced from a general discussion of the equations of condition,

$$(16) \quad \ldots \ldots \quad \frac{\alpha}{2\pi} \cdot 86400 = \tau^s + \delta\tau^s + \rho c_{\odot},$$

where α is the R.A. of the Sun's centre, as deduced from the observed N.P.D., and corrected obliquity. This is the method which is practically followed by astronomers. If the corrections $\delta\tau^s$ thus found were sufficiently large to render such a step necessary, they would have to be applied to the "local sidereal times" at once. But in the present state of astronomy the mean errors, $\delta\tau$, are so small that it is found sufficient to apply them every few years to the right ascensions of the clock-stars found from the local sidereal times

$$\tau^s + \rho c_1, \ \tau^s + \rho c_2, \ \&c.$$

In this way more accurate tabular right ascensions of the clock-stars are continually deduced and adopted for future use; and any accumulation of the effects of errors of the tabular right ascensions of the stars on the local sidereal times is thus rendered impossible.

The time, t, which, on this scale, will correspond to a given local sidereal time A^s will therefore be

$$(17) \quad \ldots \quad t = r + \frac{\rho(c - c_0)}{86400 \,.\, \text{unit seconds}}$$

$$= r + \frac{\rho(c - c_0)}{86400\left(\frac{N}{N_0} + \frac{N}{2\pi}\right) \text{sidereal seconds}}$$

$$= r + \frac{(\rho c + \tau + \delta\tau) - (\rho c_0 + \tau + \delta\tau)}{86400\left(\frac{N}{N_0} + \frac{N}{2\pi}\right)}$$

$$= r + \frac{A^s - A_r^s}{86400^s\left(\frac{N}{N_0} + \frac{N}{2\pi}\right)}$$

where the seconds are mean sidereal seconds.

The use of an erroneous value, $A_r^{s\prime}$ for A_r^s, will necessarily throw upon the determinations of the time, t, which correspond to a given local sidereal time A^s, any errors made. In fact, if we employ

$$A^{s\prime}_r = \left[\frac{l + Nr + \sigma r^2 - 2\pi(\iota - r)}{2\pi}\right] . 86400^s$$

instead of

$$A^s_r = A^{s\prime}_r + \frac{\left(\delta l + 2\pi r \dfrac{N - N_0}{N_0}\right)}{2\pi} . 86400^s = A^{s\prime}_r + \delta l^s + 86400^s . xr$$

we cannot escape throwing the errors made by neglecting

$$\delta l \text{ and } x = \frac{N - N_0}{N_0}$$

upon the determinations of the time, and therefore upon the epoch corrections and mean motions which are deduced from residual errors thus erroneously formed.

The exact errors of the times, t, when thus determined from the "local sidereal times" by neglecting δl and x, cannot be assigned so long as δl and x are unknown, but the differences between the errors made when using different constants, l' and N', instead of l and N, in the expression of the Sun's longitude can be computed with very great accuracy if the differences between the angles N, N', and N_0 are small. And the times, t, on the required scale can always be found from accurate observed geocentric positions of the Sun subject only to any errors which may arise from imperfect allowances for the small secular and periodic terms in the theoretical expressions of the Sun's co-ordinates.

The values of $A^{s\prime}_r$ are the "sidereal times at mean noon" for the standard meridian given in the Nautical Almanacs. If the "mean noons" are defined as the instants when the local sidereal time on a given sidereal day is equal to $A^{s\prime}_r$, the results are arithmetically correct. But if the usual assumption is made that $t = r$ (the value for which $A^{s\prime}_r$ is computed) when the scale of time-measurement is that which is generally adopted in practical astronomical work, the assumption will not be true unless l and N happen to have assigned to them certain definite values, l_0 and N_0; and this can only be true for one pair of such values at a given epoch. In other cases an unknown time, τ, must be introduced to transfer the "epoch" to a "mean noon" as defined.

The conclusions at which I have arrived respecting the errors made in referring the tabular places to the observer's meridian and in finding the mean solar times, t, which correspond to a given local sidereal time, A^s, follow as necessary consequences of the fact that the system of time-measures, t, in stellar astronomy is based fundamentally upon the motion of the Sun by assigning definite angles, l and N, to the epoch "constant" and "mean motion" in the expression of the Sun's longitude as a function of t; and that the assigned values are not generally such that $A_0=l$; $N_0=N$. But I shall prove that not only is this the system of time-measurement adopted in stellar astronomy, but that it does not differ from the system which is adopted in gravitational astronomy unless the adopted value of the precession constant is in error, and that the two systems then only differ in the ratio

$$\frac{2\pi-P-\delta P}{2\pi-P}=1-\frac{\delta P}{2\pi-P},$$

where P is the adopted value, and $P+\delta P$ the exact value of the precessional constant referred to the mean tropical year as the unit of time.

The corrections for such differences, if sensible, can be duly allowed for at any time, as more accurate values of $P+\delta P$ become available, but these differences are altogether unimportant as compared with the differences due to the adoption of such different angles, N and N′ for N_0, as those in use before and after 1864.

It may, perhaps, avoid misconception if I point out that I am perfectly aware of the fact that it is possible to adopt other systems of time-measurement than that which I have here considered. It is possible to base our time-measures fundamentally upon the motion of the observer's meridian in right ascension by assigning definite values to the constants A_0 and Ω in the equation

$$2\pi\iota+A=A_0+\Omega t+\sigma t^2+\text{nutation.}$$

If we assign values

$$0,\ l,\ l' \text{ to } A_0$$

$$2\pi,\ 2\pi+N,\ 2\pi+N' \text{ to } \Omega$$

we shall have three systems of *convertible times*, t_1, t_2, t_3, where the "times" which correspond to a given right ascension, A, of the observer's meridian will be determinable from the equations

$$2\pi\iota+A=2\pi\,.\,t_1+\sigma_1 t_1^2+\text{nutation}$$

$$2\pi\iota+A=l+(2\pi+N)t_2+\sigma_2 t_2^2+\text{nutation}$$

$$2\pi\iota+A=l'+(2\pi+N')t_3+\sigma_3 t_3^2+\text{nutation.}$$

If either of these systems was adopted and used to bring up the tabular right ascensions of the clock stars from epoch; and if the precessional motions and proper motions were expressed in terms of the same time-scale; there would be no difficulty in determining the "local sidereal times" from observation by following a method similar in principle to that at present adopted. Taking the second system as an illustration, we should have if

$$A'_r = l + Nr + \sigma_2 r^2 + \text{nutation} - 2\pi(\iota - r)$$

$$t_2 = r + \delta r = r + \frac{A - A'_r}{2\pi + N},$$

and the equations for the determination of times with the observatory clock would be

$$\delta r_1 = \frac{\rho'(c_1 - c_0')}{86400^s} \qquad \delta r_2 = \frac{\rho'(c_2 - c_0')}{86400^s},$$

where if the clock was the same as that before under consideration ρ' would not be the same constant, because the "unit day" of 86400^s is different, nor would c_0' be the same as c_0, because different instants of absolute time are denoted by the same number, r, on the two scales; but the equivalent of 86400 mean solar seconds in this case would be

$$86400\left(1 + \frac{N}{2\pi}\right) \text{ sidereal secs.};$$

and we should thus have from the equation

$$A'_r + (2\pi + N)\frac{\rho'(c_1 - c_0')}{86400 \text{ mean solar secs.}} = \phi_1(r) + \phi' \,.\, \delta r_1 = a_1 + \delta a_1$$

$$A'_r + \frac{2\pi\rho'(c_1 - c_0')}{86400 \text{ mean sidereal secs.}} = a_1 + \delta a_1$$

or

$$A'^s_r - \rho' c_0' + \rho' c_1 = a_1^s + \delta a_1^s$$
$$\&c. = \&c.$$

The necessary adjustments between this system of time-measures and that which we have previously discussed would thus be thrown upon the clock-corrections

$$A'^s_r - c_0'\rho' \text{ and } \rho',$$

and if this method was systematically carried out, the local sidereal times could be correctly found from observation. This method would possess the advantage that the equivalent in sidereal seconds of a "unit day" would be accurately known; and if the "unit days" of practical astronomy did contain

$$86400\left(1 + \frac{N}{2\pi}\right) \text{ sidereal seconds},$$

the times found from the formula

$$t_2 = r + \frac{\rho'(c_1 - c_0')}{86400 \text{ mean solar secs.}}$$

$$= r + \frac{\rho' c_1 + \tau' + \delta\tau' - (\rho' c_0' + \tau' + \delta\tau')}{86400\left(1 + \frac{N}{2\pi}\right)}$$

$$= r + \frac{A^s - A_r'^s}{86400\left(1 + \frac{N}{2\pi}\right)}$$

would be thus correctly found.

But unless N happens to be identically equal to N_0, and l equal to a definite but unknown angle l_0, it will not be possible for $A_r'^s$ to be equal to A_r^s. It is easily seen that the intervals of time between instants marked by observational facts are correctly measured on all these systems of time-measures t, t_1, t_2, t_3, when the proper allowances are made for the different intervals of time which are in these several cases called a "day," and put equal to 86400 seconds.

For if we wish to express the interval between the instants when

on the ι_0 sidereal day from epoch, L.S.T. $= A_0^s$

and

on the ι sidereal day from epoch, L.S.T. $= A^s$

we have

$$(t - t_0)\ 86400 \text{ mean secs.} = (t - t_0)\ 86400\left(\frac{N}{N_0} + \frac{N}{2\pi}\right) \text{sidereal secs.}$$

and this is identically equal to

$$86400\ (\iota - \iota_0) + A^s - A_0^s.$$

And in the corresponding case of the system of time-measures, t_2, we should have

$$(t_2 - t_0')\ 86400 \text{ mean secs.} = (t_2 - t_0')\ 86400\left(1 + \frac{N}{2\pi}\right) \text{sidereal secs.}$$

$$= 86400\ (\iota - \iota_0) + A^s - A_0^s.$$

No errors, therefore, are made in measuring time by these different systems of time-measures, provided we do not forget that the "unit days" in the different systems denote different intervals of absolute times, and contain, therefore, different numbers of mean sidereal seconds.

It will, I hope, be thus clearly understood that I do not deny that the systems of time-measurement t_1, t_2, t_3, and others might be adopted. I simply deny, as a matter of fact, that they are exclusively adopted in practical work.

If

$$\text{a "unit day"} = \frac{N}{2\pi} \text{ (of a mean tropical year)}$$

is used, we necessarily throw away for each count, one, of the "unit day," the true sidereal equivalent,

$$86400^s \left(\frac{N}{N_0} + \frac{N}{2\pi}\right),$$

by the process employed for the determination of the errors of the sidereal clocks. If we used in practice

$$\text{a "unit day"} = 86400 \left(1 + \frac{N}{2\pi}\right) \text{ sidereal secs.}$$

for the determination of the errors and rates of our clocks, we should necessarily throw away

$$86400 \left(1 + \frac{N}{2\pi}\right) \text{ sidereal secs.}$$

as the equivalents for each count of a "unit day."

The local sidereal times of a meridian after r "unit days" on each scale are therefore necessarily different in the two cases by

$$86400 \,.\, \frac{N - N_0}{N_0} \times r \text{ (sidereal secs.)}$$

and it is an allowance for the difference of this correction computed with the actual values of N and N′ adopted for N_0 before and after 1864 which accounts for the *per saltum* change in the errors of Hansen's Lunar Tables, as compared with observation when the differences

$$86400 \frac{N - N_0}{N_0} \,.\, r$$

and

$$86400 \frac{N_1 - N_0}{N_0} \,.\, r$$

are neglected.

But even if it were true that the times as found from observation were on the scale of time-measurement, t_2, it would not be generally true that these are the times required for the accurate determination of the constants contained in the differential equations and their integrals which represent the positions of the planets; and they would not, therefore, be the times required in practical astronomical work.

The general principle involved in these questions of time-determination is that definite values have to be assigned to some epoch correction, $n\tau$, and to some mean motion, n, in one of the theoretical expressions which represents a motion, which can be compared with the results of observations to render the time, t, observationally determinate.

The selection of a particular motion for this purpose is merely a question of practical convenience. The adoption of the measures t_1, t_2, or t_3 would simplify the direct determinations of the time from observation, but would greatly complicate the mathematical investigations of gravitational astronomy; and this is the reason why they have not been exclusively adopted.

The differential equations of motion are formed on the assumed truth of Newton's law of universal gravitation, "that every particle of matter in the universe attracts every other particle with a force which is proportional to the mass of the attracting particle directly and to the square of the distance between the particles inversely."

The law thus generally stated is, however, of no practical value, as a basis of mathematical investigation, until we have defined how the masses are to be measured, and the relationship between the force of gravitation, the mass, and the distance practically established.

If we assume that the law of gravitation is true when the masses are directly measured by their accelerating effects, the law becomes available as a basis of calculation, and it is upon this assumption that the differential equations of motion used by astronomers are formed; but it will be found that when the masses are thus measured the measures themselves are not independent of the units of time and of length which are adopted, but change their numerical values with changes of these units; and if, to avoid this inconvenience, we agree not to measure "masses" by their accelerating effects, but to take the "mass" of the Sun as "the unit of mass," and denote the masses of the planets *Mercury*, *Venus*, *Earth*, &c., by the fractions

$$\nu_1, \nu_2, \nu_3, \&c.$$

which express the ratio of their "masses" to that of the Sun, so that the masses

$$1, \nu_1, \nu_2, \nu_3, \&c.$$

are independent of the adopted units of length and time, it will be found necessary, before we can form the differential equations of motion of the planets, to introduce a factor, f, such that

$$f \cdot 1, f \cdot \nu_1, f \cdot \nu_2, f \cdot \nu_3, \&c.$$

will denote the accelerating effects of the respective masses. And, in this case, the numerical values which will have to be assigned to the factor f will depend upon the particular units of time and of length which we may adopt.

The factor f, when thus introduced, never appears in our equations unless multiplied by one of the masses

$$1, \nu_1, \nu_2, \nu_3, \&c.$$

and, conversely, a "mass"

$$1, \nu_1, \nu_2, \nu_3, \&c.$$

never appears in these equations unless multiplied by the factor f or as a ratio to other masses.

And it will be imposible, therefore, to correctly determine the numerical values of the fractions

$$\nu_1, \nu_2, \nu_3, \&c.$$

from the accelerating effects produced by the masses

$$f \cdot 1, f \cdot \nu_1, f \cdot \nu_2, f \cdot \nu_3, \&c.$$

unless the factor f has the same numerical value assigned to it in all our mathematical expressions, or unless any changes in the values adopted for f are accurately allowed for in evaluating the accelerating effects produced by the different masses.

The differential equations of motion for the different planets, the Moon, and for the rotation of the Earth about its axis are not isolated systems of three equations, but coexisting simultaneous equations.

And since the same symbols appear in many of these equations to represent the same physical quantities, it is absolutely necessary, in order that the results found from the integration of one set of equations may be transferable to the others, that the same units of length and of time should be adopted in the formation of all the equations of motion; or if different units are employed in special investigations, that all the physical quantities should be transferred to common units before they are used for the formation of any set of differential equations. And this will require that the ratios of the different units employed should be accurately known.

There is absolutely nothing to restrict our selection of units of time and of length in the mathematical investigations of astronomy, so long as no numerical value is assigned to f, and they are conducted in a purely symbolical form, and without the introduction *of any conditions which can only be satisfied by the adoption of particular units of time and length.* But to speak of the variable t in such cases as denoting "mean solar time" is nomenclature devoid of any physical significance. And if we adopt for the unit of time either the "mean sidereal day" or the "physical mean solar day," and for unit of length "a statute mile," "ninety-two millions of statute miles," or "the equatorial radius of the Earth"—units in terms of which the exact numerical value of the factor f is unknown—we shall have to overcome the mathematical difficulties which will arise from the appearance of an unknown factor, f, to all the measures of mass in our expressions.

But if $f_0 \times 1$ denotes the accelerating effects of the Sun's mass at the unit of distance in the unit of time, when such definite units as those described are adopted; and $f \times 1$, the value when units which bear to the former the ratios $\frac{1+p}{1}$, $\frac{1+q}{1}$ respectively are adopted, it can be directly proved by a simple change of the variables in the differential equations of motion that

$$f = \frac{f_0(1+q)^2}{(1+p)^3} \text{ identically.}$$

If, therefore, we adopt for f_0 an approximate value, f', it matters not in the slightest degree how f' may have been found, or why we have adopted it; we must, in order to correctly form the differential equations of motion in terms of definite units of length and of time, such as those above mentioned, substitute for f_0 its value

$$\frac{f' \cdot (1+p')^3}{(1+q')^2},$$

where $1 + p'$ and $1 + q'$ will denote the ratios of units of length and of time, for which the accelerating effects of the Sun's mass is equal to f', to the definite units of length and of time in terms of which we propose to express all the linear and time-quantities found from observation, and to determine the numerical values of the constants which appear in our integrals.

If we put $p' = 0$ and $q' = 0$ and form the differential equations of motion with the definite value f', then, if our mathematical work is accurate, and Newton's law of gravitation is true, the resulting integrals of the differential equations of motion will express the coordinates required in terms of such units of length and of time that for these units $f' \times 1$ does equal the Sun's accelerating effects at the unit of distance in the unit of time; but not in terms of the definite units of length and time in terms of which we proposed to express our linear and time quantities.

If we insist, therefore, upon the adoption of such definite units as those of the equatorial radius of the Earth and the "mean sidereal day" or the "physical mean solar day," we shall have, when using a definite value, f' for f_0, to retain in all our equations of motion, and throughout all our mathematical work, the expression

$$\frac{f'(1+p')^3}{(1+q')^2}$$

for f_0, and when we have succeeded in establishing exact relations between

$$\frac{f'(1+p')^3}{(1+q')^2}$$

and some linear quantity and some interval of time determinable from observation in terms of the selected units of length and of time, we shall be able to determine the numerical values of p' and q', and thus deduce the required value f_0.

But the numerical values of the numbers p' and q' found in this way from a discussion of observations would be liable not only to fallible errors of observation but to serious systematic errors if any long inequalities

$$P \,.\, \sin(\alpha t + \beta)$$

with sensible coefficients, P, were neglected in the theoretical expressions compared with the corresponding results from observation. For in such cases

$$P \,.\, \sin(\alpha t + \beta) = P \,.\, \sin\beta + \alpha t \,.\, P \cos\beta - \frac{\alpha^2 t^2}{1.2} \,.\, P \sin\beta \quad . \quad . \quad . \quad \&c.$$

and the effects of the terms

$$P \,.\, \sin\beta \,;\; P\alpha \,.\, \cos\beta, \&c.,$$

would be thrown upon the determinations of the epoch corrections and mean motions, and, therefore, upon the determinations of p' and q', and of

$$f_0 = \frac{f'(1+p')^2}{(1+q')^2},$$

and the approximate values of f_0 thus found would be liable to systematic errors which would not even be constant but subject to determinate laws when found for different epochs, and from observations extending over different intervals of time. It is unnecessary to point out that the mathematical difficulties presented by the problems of physical astronomy would be enormously increased from the causes indicated if such definite units of time and of length were adopted. In such cases we should be practically replacing in our mathematical work the fundamental constant, f_0, by a quantity which was a complicated function of the time.

It is possible, however, to avoid these mathematical difficulties with respect to variations in the adopted values of f by the selection of suitable units of length and of time; but if such units are selected to render f definite, then for accuracy all the linear quantities and time-quantities, which enter into our equations must be determined in terms of the same units as those for which f expresses the accelerating effects of the Sun's mass at the unit of distance in the unit of time; and if the unit of time is not equal

$$86400 \text{ sidereal seconds } \left(1 + \frac{N}{2\pi}\right),$$

then the use of the times t_2, found from the equation

$$2\pi\iota + A = l + (2\pi + N)\, t_2 + \sigma t_2^{\,2} + \text{nutation}$$

would certainly lead to error.

In fact, in these and other similar cases, the simplifications introduced in one stage of the work by the selection of particular units are to some extent counterbalanced by additional complications introduced in the other parts of the work.

We shall next consider how the factor f, which is used in gravitational astronomy, is connected in practice with measures of time and of length, which fix the units in terms of which it will measure the accelerating effects of the Sun's mass at the unit of length in the unit of time, if Newton's law of gravitation is true. The conditions which are made use of in practice to fix the value of f are from the positions of the observing stations almost necessarily directly connected with the Sun's geocentric motion.

If the constants introduced in the integration of the differential equations which result from the neglect of the disturbing masses in the differential equations of the Sun's geocentric motion are denoted by a, e, ω, τ, Ω, and γ; and we denote by $a+\delta a$, $e+\delta e$, &c., the functions which when substituted for a, e, &c., in the expressions

$$\mathrm{R} \,.\, \cos \mathrm{L} \,.\, \cos \lambda = \mathrm{R}\,[\cos v \,.\, \cos \Omega - \sin v \,.\, \sin \Omega \,.\, \cos \gamma]$$

$$\mathrm{R} \,.\, \sin \mathrm{L} \,.\, \cos \lambda = \mathrm{R}\,[\cos v \,.\, \sin \Omega + \sin v \,.\, \cos \Omega \,.\, \cos \gamma]$$

$$\mathrm{R} \,.\, \sin \lambda = \mathrm{R} \sin v \,.\, \sin \gamma,$$

where

$$\mathrm{R} = a\,(1 - e \cos \phi)\,; \quad \tan \frac{v-\omega}{2} = \sqrt{\frac{1+e}{1-e}} \,.\, \tan \phi$$

$$\phi - e \sin \phi = \sqrt{\frac{f'\,(1+p')^3}{(1+q')^2}} \,.\, (t+\tau)$$

satisfy the differential equations of the Sun's motion, and the three subsidiary equations usually introduced to render the six functions

$$a+\delta a,\ e+\delta e,\ \text{\&c., determinate,}$$

it is found that the geocentric longitude, L, and the distance, R, will be expressed under the forms

$$2\pi\iota' + \mathrm{L} = \sqrt{\frac{f'\,(1+p')^3}{a^3\,(1+q')^2}}\,(1+\nu_3) \,.\, \tau(1+\delta\tau) + \sqrt{\frac{f'(1+p')^3}{a^3(1+q')^2}}\,(1+\nu_3) \,.\, (1+\delta \mathrm{P}) \,.\, t$$

$$+ (2e_1 - \qquad) \,.\, \sin \zeta + \text{\&c.}$$

$$\mathrm{R} = a\,(1+\delta\theta)\left[1 + \frac{e_1^{\,2}}{2} - (e_1 - \qquad) \,.\, \cos \zeta + \text{\&c.},\right.$$

where

$$e_1 = e\,(1+\delta\phi)$$

is a constant, and ι' is an integer.

The symbols $\delta\tau$, $\delta \mathrm{P}$, $\delta\theta$, and $\delta\phi$ denote collections of small terms independent of the time which all contain, one or more, of the disturbing masses as a factor. The geocentric longitude, L, is here supposed to be measured from a fixed initial time.

If p' and q' have the numerical values assigned to them which renders the equation

$$f_0 = \frac{f'(1+p')^3}{(1+q')^2}$$

exact with the adopted value, f', when the unit of length is the mean equatorial radius of the Earth, and the unit of time an interval which contains

$$86400\left(1+\frac{N}{2\pi}\right) \text{ sidereal seconds,}$$

and if D denotes the mean distance of the Sun from the Earth, expressed in terms of the adopted unit of length, and n_s, l_s, the exact values, on the scale of time-measurement adopted, of the mean motion and epoch correction, we shall have

$$\sqrt{\frac{f'(1+p')^3}{(1+q')^2} \cdot \frac{1+\nu_3}{a^3}} \cdot (1+\delta P) = n_s \qquad n_s \cdot \frac{\tau\,(1+\delta\tau)}{1+\delta P} = l_s$$

$$D = a\,(1+\delta\theta)\left[1+\frac{e_1^2}{2}\right].$$

But if n is the Sun's mean motion in the interval of time which contains

$$(1+q')\,86400\left(1+\frac{N}{2\pi}\right) \text{ sidereal seconds,}$$

or in the interval for which, when adopted as the unit of time, f' is the exact value of the Sun's accelerating effects at the distance of the equatorial radius of the Earth, we shall have

$$n = n_s\,(1+q'),$$

and if we adopt the interval of time,

$$(1+q')\,86400\left(1+\frac{N}{2\pi}\right) \text{ sidereal seconds}$$

as the unit of time, instead of

$$86400\left(1+\frac{N}{2\pi}\right) \text{ sidereal seconds,}$$

we directly eliminate the factor $(1 + q')$ from our equations. In this case we have

$$\sqrt{\frac{f'(1+p')^3}{a^3} \cdot (1+\nu_3)} \cdot (1+\delta P) = n;$$

but when this method is adopted, directly a value is assigned to n in the expression for the Sun's longitude, the unit of time is absolutely fixed; it will in this case be the

$$\frac{n}{2\pi} \text{ part of an interval of time, } \sigma,$$

where σ is generally called by astronomers the mean sidereal year.

Again, since

$$\sqrt{\frac{f'(1+p')^3}{a^3(1+q')^2}} \cdot (1+\nu_3) \cdot \tau(1+\delta\tau) = l_s = \frac{n\tau(1+\delta\tau)}{1+q'} = n\tau',$$

where τ' is an interval of time measured on the new time-scale, we can change the "epoch" or instant from which t is measured by putting

$$\tau' = \frac{l}{n},$$

where l is some assigned angle.

The times t then become determinate from the equation

$$2\pi\iota' + \mathrm{L} = l + nt + (2e' \quad) \cdot \sin\zeta + \&c.$$

But we can further simplify the expression for the factor f, which expresses the accelerating effects of the Sun's mass at the unit of distance in the unit of time by adopting a suitable unit of length.

If we denote by D′ the number which expresses the mean distance of the Sun in terms of a unit which equals

$(1+p')$ times the equatorial radius of the Earth,

we shall have

$$\mathrm{D}'(1+p') = \mathrm{D} = a(1+\delta\theta)\left[1+\frac{e_1^2}{2}\right],$$

and

$$\sqrt{\frac{f'}{\mathrm{D}'^3}} \cdot (1+\nu_3)(1+\delta\theta)^3\left(1+\frac{e_1^2}{2}\right)^3(1+\delta\mathrm{P}) = n.$$

If, therefore, we take the unit such that

$$\mathrm{D}' = (1+\nu_3)^{\frac{1}{3}}(1+\delta\theta)\left(1+\frac{e_1^2}{2}\right)(1+\delta\mathrm{P})^{\frac{1}{3}},$$

we have

$$\sqrt{f'} = n,$$

and, conversely, if we adopt n, the coefficient of t, in the expression of the Sun's geocentric longitude to fix the value of $\sqrt{f'}$, and assign a definite value to

$$l = n\tau\frac{(1+\delta\tau)}{1+\delta\mathrm{P}},$$

the mathematical results obtained from the integration of the differential equations of motion when complete, or sensibly complete, will give the true coordinates of the Sun, Moon, &c., when the linear and time-quantities which they contain are all expressed in terms of the units of time and of length thus fixed. But the times t must in such cases be necessarily found subject to the condition

$$2\pi\iota' + \mathrm{L} = l + nt + \&c.,$$

where n is the angle adopted to fix the value of $\sqrt{f^1}$; or if the longitude L′ is measured from the equinox, we shall have

$$2\pi\iota' + L' = l + (n + p + \delta p)\,t + \&c.,$$

where p is the adopted value, and $p + \delta p$ the true value, of the precessional motion on the scale of time-measurement under consideration. Comparing this with the expression adopted in stellar astronomy,

$$2\pi\iota' + L' = l + (n + p)\,t',$$

it follows that the time-scale adopted in stellar astronomy is not identically the same as that required unless $\delta p = 0$; but the differences are insignificant as compared with the differences between the times t, t_1, t_2, t_3, &c., and the necessary corrections can be easily and directly applied at any time if the error δp is found to be sensible.

But instead of fixing the unit of length in the way described, the following method is in practice more convenient, as it is generally employed in the deduction of the geocentric coordinates from the heliocentric coordinates.

Let the number p' be so selected that with the unit of length

$$(1 + p') \times \text{equatorial radius of the Earth.}$$

$$R' = \frac{a}{1 + p'}(1 + \delta\theta)\,.\left[1 + \frac{e_1^2}{2} - (e' - \qquad)\cos\zeta + \&c.\right]$$

becomes

$$R' = 1 + \frac{e_1^2}{2} - (e' - \qquad)\cos\zeta + \&c.$$

We then have

$$a = \frac{1 + p'}{1 + \delta\theta};$$

and the equation

$$\sqrt{\frac{f'(1 + p')^3}{a^3}(1 + \nu_3)(1 + \delta P)^2} = n$$

gives

$$f' = \frac{n^2\,.\,a^3}{(1 + p')^3\,(1 + \nu_3)\,(1 + \delta P)^2} = \frac{n^2}{(1 + \nu_3)(1 + \delta P)^2(1 + \delta\theta)^3}$$

$$= n^2\frac{\mathrm{a}^3}{1 + \nu_3} \quad \text{or} \quad n^2\mathrm{a}_1^3,$$

where

$$\mathrm{a} = \mathrm{a}_1(1 + \nu_3)^{\frac{1}{3}} = (1 + \delta P)^{-\frac{2}{3}}(1 + \delta\theta)^{-1},$$

where a and a_1 will be linear quantities whose exact ratios to the adopted unit can only be found from the mathematical investigations conducted in terms of the unit of length thus fixed and with times, t, subject to the condition

$$2\pi\iota' + L = l + nt + \&c.,$$

or

$$2\pi\iota' + L' = l + (n + p + \delta p)t + \&c.,$$

when n has assigned to it the same numerical value as that employed in fixing the value of $\sqrt{f}$.

The equatorial radius of the Earth must in such cases be expressed in terms of the adopted unit of length; but this presents no difficulty, for its numerical expression is not required when the equatorial horizontal parallaxes are introduced.

Any errors made in fixing the positions of the meridians in terms of the variable, t, due to the neglect of sensible corrections

$$\delta l \text{ and } x = \frac{N - N_0}{N_0}$$

will appear, from reflex action, as apparent corrections (but they will be mixed up with the effects of any neglected long inequalities), to the adopted epoch constant, l, and mean motion, n, in the expression of the Sun's longitude. We shall thus be able to ultimately determine the corrections δl and x; but the effects of these errors will be much larger in the case of the Moon than in the case of the Sun during the progress of the approximations; and the errors thus introduced are not due to errors of gravitational theory, but to errors in the determination of the time.

This completes the proofs which are required to establish the necessity of the corrections which I have indicated in fixing the relative positions of the centre of gravity of the Sun and of the positions of the meridians in terms of the measure of time, t, which is adopted in practical astronomical work.

It has been shown that the system of time-measures adopted in gravitational astronomy is based on the geocentric motion of the centre of gravity of the Sun; that, subject to the possible small correction for error δp, the same system is adopted in stellar astronomy; that the local sidereal times are correctly found with this system of time-measures; but that errors are made in fixing the position of the meridian in terms of t or in finding the time on the required scale from the local sidereal times; and that these errors enter directly in the work in referring the tabular positions to the meridian for a comparison with the local sidereal times; and the "Tables" which I have given (*Monthly Notices*, vol. liv. No. 1) show that the differences thus made, before and after 1864, account for the *per saltum* change of the errors of the Tables which undoubtedly took place about 1864.

Note.

To avoid unnecessary complication of the formulæ, the secular terms have not been written down in the expressions for L and R on page 60; but they are included under the &c., and they must be applied to deduce the numerical values of the constants for the epoch from observations made at different times.

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Ephemeride zur Beobachtung des Mondkraters Mösting A

mitgetheilt von Herrn Prof. Dr. J. Franz in Königsberg.

Die Ephemeride ist, abweichend von den Vorjahren, nach der im Abschnitt 5 der Abhandlung »Darlegung der Ephemeridenrechnung von Mösting A« (Astr. Nachr. Nr. 3241) angegebenen Methode berechnet.

Zur Anwendung derselben auf Meridianbeobachtungen des Kraters interpolire man $\alpha_☾ - \alpha_k$, $\delta_☾ - \delta_k$ und $\log \sin p_k$ unter strenger Berücksichtigung der zweiten Differenzen mit dem Argument »Länge des Beobachtungsortes von Greenwich« so, dafs die Länge westlich positiv, östlich negativ genommen wird. Dann befreie man die beobachtete Decl. des Kraters von der Höhenparallaxe, indem man diese in der bekannten Weise mit dem Argument der wahren Kraterdeclination (nicht Monddeclination), unter Benutzung der Horizontaläquatorialparallaxe p_k des Kraters berechnet, welche aus der Horizontaläquatorialparallaxe des Mondes, wie sie der Nautical Almanac angiebt, abgeleitet ist. Bringt man alsdann $\alpha_☾ - \alpha_k$ und $\delta_☾ - \delta_k$ an die Beobachtung an, so hat man die AR. und Decl. des Mondes, wie sie vom Mittelpunkt der Erde aus beobachtet wäre, für die Beobachtungszeit, d. h. für die Zeit der Culmination des Kraters (nicht des Mondes). Diese Methode gilt für jeden Beobachtungsort auf der Erde, nicht nur (wie die Methode der Vorjahre) für die nördliche gemäfsigte Zone. Ueber die Reduction der Meridianbeobachtungen findet man Näheres Astr. Nachr. Nr. 3262.

Für Beobachtungen aufserhalb des Meridians interpolire man mit dem Argument »westliche Länge von Greenwich + westlicher Stundenwinkel des Mondes« a und d, ermittele ferner für die Beobachtungszeit die scheinbare (mit Parallaxe behaftete) AR. und Decl. des Mondmittelpunktes α und δ, sowie die scheinbare Horizontalparallaxe des Mondes p. Mit Hülfe der letzteren ergiebt sich der anzuwendende scheinbare Mondhalbmesser h aus der Relation:

$$\log \sin h = \log \sin p + 9{,}43513.$$

Hierauf berechne man:

$$\sin\pi \sin k = \cos d \sin(a-\alpha)$$
$$\cos\pi \sin k = \sin d \cos\delta - \cos d \sin\delta \cos(a-\alpha)$$
$$-\cos k = \sin d \sin\delta + \cos d \cos\delta \cos(a-\alpha)$$
$$\operatorname{tg}\sigma = \frac{\sin h \sin k}{1-\sin h \cos k},$$

man erhält dann die scheinbare, mit Parallaxendifferenz behaftete, Reduction auf den Mondmittelpunkt:

$$\alpha_{☾} - \alpha_k = -\sigma \sin\pi \sec\delta$$
$$\delta_{☾} - \delta_k = -\sigma \cos\pi .$$

Lage des Mondäquators.

1895	☊′	Δ — ☋	i
	° ′	° ′	° ′
Jan. 1,0	+0 17,82	—0 16,44	21 56,20
21,0	+0 22,32	—0 20,59	21 56,35
Febr. 10,0	+0 26,81	—0 24,74	21 56,53
März 2,0	+0 31,30	—0 28,87	21 56,75
22,0	+0 35,77	—0 33,00	21 56,99
April 11,0	+0 40,22	—0 37,11	21 57,27
Mai 1,0	+0 44,66	—0 41,20	21 57,59
21,0	+0 49,08	—0 45,28	21 57,93
Juni 10,0	+0 53,48	—0 49,33	21 58,31
30,0	+0 57,86	—0 53,37	21 58,72
Juli 20,0	+1 2,21	—0 57,39	21 59,16
Aug. 9,0	+1 6,54	—1 1,38	21 59,63
29,0	+1 10,84	—1 5,35	22 0,14
Sept. 18,0	+1 15,11	—1 9,28	22 0,67
Oct. 8,0	+1 19,36	—1 13,20	22 1,24
28,0	+1 23,57	—1 17,07	22 1,84
Nov. 17,0	+1 27,74	—1 20,92	22 2,47
Dec. 7,0	+1 31,88	—1 24,74	22 3,13
27,0	+1 35,98	—1 28,52	22 3,82

Librations-Ephemeride zur Beobachtung des Mondkraters Mösting A.

1895 Im Meridian von Greenwich	Physische Libration in selenocentr.		Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridian-beobachtungen		
					in AR.	in Decl.	Parallaxe
	Länge	Breite	a	d	$\alpha_☾ - \alpha_k$	$\delta_☾ - \delta_k$	$\log \sin p_k$
Jan. 3	+0,1′	−1,2′	186° 0,7′	− 5° 45,1′	+ 0,60s	+ 35,9″	8,20816
4	0,2	1,2	198 44,9	−10 41,6	+ 2,03	+ 20,2	21388
5	0,2	1,2	211 51,2	−15 17,1	+ 3,07	+ 1,7	22068
6	0,3	1,2	225 29,0	−19 16,1	+ 3,50	− 18,8	22812
7	0,3	1,2	239 43,0	−22 23,3	+ 3,01	− 39,2	23572
8	0,3	1,2	254 30,1	−24 24,4	+ 1,37	− 54,5	24278
9	0,3	1,2	269 38,1	−25 8,8	− 1,31	− 57,7	24851
10	0,2	1,2	284 47,7	−24 31,8	− 4,38	− 43,8	25219
11	+0,1	−1,2	299 39,1	−22 36,9	− 7,02	− 13,6	8,25343
12	+0,1	1,2	313 58,6	−19 34,4	− 8,80	+ 26,8	25208
13	0,0	1,2	327 42,2	−15 38,3	− 9,75	+ 69,5	24847
14	0,0	1,2	340 53,6	−11 4,2	−10,16	+108,5	24314
15	0,0	1,2	353 42,1	− 6 7,6	−10,29	+140,3	23675
16	−0,1	1,2	6 19,7	− 1 3,4	−10,32	+162,9	22997
17	−0,1	1,2	18 59,0	+ 3 53,9	−10,45	+177,4	22333
18	0,0	1,2	31 51,3	+ 8 29,8	−10,67	+183,0	21721
Febr. 2	+1,2	−1,1	220 1,5	−17 45,2	+ 3,65	− 13,2	8,21852
3	1,3	1,1	233 59,8	−21 15,0	+ 4,09	− 31,2	22539
4	1,3	1,1	248 33,0	−23 44,2	+ 3,64	− 47,8	23274
5	1,2	1,1	263 33,0	−25 0,6	+ 2,14	− 58,2	24001
6	1,2	1,1	278 43,4	−24 56,8	− 0,28	− 56,5	24650
7	1,1	1,1	293 43,9	−23 32,8	− 2,99	− 38,6	25143
8	1,1	1,1	308 17,8	−20 56,5	− 5,38	− 5,5	25416
9	+1,0	−1,1	322 16,7	−17 20,1	− 7,13	+ 37,3	8,25429
10	1,0	1,1	335 41,9	−12 59,3	− 8,30	+ 82,5	25179
11	0,9	1,1	348 38,8	− 8 9,7	− 9,09	+123,5	24699
12	0,9	1,1	1 20,4	− 3 6,2	− 9,71	+156,1	24058
13	0,8	1,1	13 58,4	+ 1 56,1	−10,31	+178,1	23323
14	0,8	1,1	26 45,2	+ 6 42,8	−10,96	+189,2	22568
15	0,8	1,1	39 50,6	+10 59,1	−11,63	+190,2	21852
16	0,9	1,1	53 20,9	+14 30,8	−12,21	+182,5	21215

1895 Im Meridian von Greenwich	Physische Libration in selenocentr. Länge	Physische Libration in selenocentr. Breite	Selenocentrische AR. und Decl. von Mösting A. a	Selenocentrische AR. und Decl. von Mösting A. d	Reduction für Meridianbeobachtungen in AR. $\alpha_{☾} - \alpha_k$	Reduction für Meridianbeobachtungen in Decl. $\delta_{☾} - \delta_k$	Reduction für Meridianbeobachtungen Parallaxe log sin p_k
	′	′	° ′	° ′	s	″	
März 3	+2,0	−1,0	242 44,9	−22 52,7	+ 3,37	− 44,9	8,22383
4	1,9	1,0	257 34,7	−24 39,5	+ 2,80	− 56,0	22994
5	1,9	1,0	272 41,8	−25 8,4	+ 1,44	− 59,5	23620
6	1,8	1,0	287 47,2	−24 16,5	− 0,45	− 51,2	24209
7	1,8	1,0	302 32,2	−22 8,4	− 2,43	− 29,3	24702
8	1,7	1,0	316 45,0	−18 54,8	− 4,13	+ 4,7	25041
9	1,6	1,0	330 22,7	−14 50,2	− 5,47	+ 46,4	25171
10	1,6	1,0	343 30,3	−10 10,3	− 6,53	+ 90,1	25065
11	+1,5	−1,0	356 17,3	− 5 10,4	− 7,48	+130,3	8,24728
12	1,5	1,0	8 55,9	− 0 5,6	− 8,44	+162,3	24197
13	1,4	1,0	21 38,4	+ 4 49,4	− 9,52	+183,3	23530
14	1,4	1,0	34 36,2	+ 9 20,2	−10,69	+192,2	22795
15	1,4	1,0	47 56,8	+13 12,0	−11,84	+189,6	22061
16	1,4	1,0	61 43,5	+16 10,4	−12,71	+177,5	21383
17	1,5	1,0	75 52,7	+18 4,5	−13,07	+158,9	20808
18	1,6	1,0	90 13,6	+18 44,6	−12,77	+137,3	20358
April 2	+2,2	−0,9	281 51,9	−24 47,1	− 0,56	− 53,2	8,23419
3	2,1	0,9	296 45,8	−23 8,8	− 1,92	− 37,0	23841
4	2,0	0,9	311 11,1	−20 20,1	− 3,09	− 10,6	24205
5	1,9	0,9	325 1,7	−16 34,5	− 4,00	+ 23,5	24470
6	1,8	0,9	338 19,6	−12 7,3	− 4,73	+ 62,3	24596
7	1,7	0,9	351 13,0	− 7 13,9	− 5,42	+101,5	24555
8	1,6	0,9	3 53,1	− 2 9,3	− 6,23	+137,2	24333
9	+1,6	−0,9	16 32,6	+ 2 51,5	− 7,26	+165,4	8,23938
10	1,5	0,9	29 23,1	+ 7 34,1	− 8,56	+183,2	23402
11	1,5	0,9	42 34,3	+11 43,8	−10,04	+189,0	22776
12	1,5	0,9	56 11,1	+15 6,0	−11,46	+183,0	22116
13	1,5	0,9	70 12,9	+17 27,6	−12,46	+167,7	21480
14	1,6	0,9	84 31,9	+18 38,1	−12,77	+146,5	20916
15	1,6	0,9	98 55,5	+18 32,5	−12,27	+123,4	20463
16	1,7	0,9	113 9,4	+17 11,7	−11,10	+101,3	20146

1895 Im Meridian von Greenwich		Physische Libration in selenocentr.		Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen		
		Länge	Breite	a	d	in AR.	in Decl.	Parallaxe
						$\alpha_{☾} - \alpha_k$	$\delta_{☾} - \delta_k$	$\log \sin p_k$
		′	′	° ′	° ′	s	″	
Mai	1	+1,6	−0,8	305 34,7	−21 35,5	− 3,96	− 12,4	8,23748
	2	1,5	0,8	319 38,5	−18 11,2	− 4,53	+ 18,3	23904
	3	1,4	0,8	333 7,9	−13 59,0	− 4,84	+ 52,6	23992
	4	1,3	0,8	346 9,0	− 9 14,5	− 5,07	+ 87,7	23997
	5	1,2	0,8	358 52,3	− 4 12,9	− 5,40	+121,0	23900
	6	1,2	0,8	11 29,9	+ 0 51,0	− 5,98	+149,6	23692
	7	1,2	0,8	24 14,0	+ 5 42,7	− 6,91	+171,0	23372
	8	1,2	0,8	37 15,7	+10 7,6	− 8,19	+182,7	22947
	9	+1,2	−0,8	50 41,7	+13 50,8	− 9,69	+183,3	8,22446
	10	1,2	0,8	64 34,2	+16 38,6	−11,07	+173,3	21906
	11	1,2	0,8	78 48,0	+18 18,9	−11,94	+154,9	21367
	12	1,3	0,8	93 12,0	+18 44,5	−12,02	+132,1	20875
	13	1,3	0,8	107 32,0	+17 53,7	−11,29	+108,6	20469
	14	1,4	0,8	121 34,9	+15 50,7	− 9,94	+ 87,0	20179
	15	1,5	0,8	135 13,0	+12 45,1	− 8,22	+ 68,2	20025
	16	1,5	0,8	148 24,4	+ 8 49,0	− 6,36	+ 52,0	20016
	30	+1,0	−0,7	327 54,9	−15 45,5	− 6,06	+ 53,8	8,23908
	31	0,9	0,7	341 4,9	−11 12,1	− 6,11	+ 88,1	23779
Juni	1	0,8	0,7	353 53,3	− 6 15,7	− 6,14	+119,6	23594
	2	0,7	0,7	6 31,0	− 1 11,0	− 6,33	+146,5	23353
	3	0,6	0,7	19 10,6	+ 3 47,5	− 6,82	+167,0	23061
	4	0,5	0,7	32 3,3	+ 8 24,9	− 7,67	+179,8	22722
	5	0,4	0,7	45 18,4	+12 26,7	− 8,85	+183,2	22338
	6	0,4	0,7	58 59,9	+15 38,6	−10,14	+176,8	21919
	7	+0,4	−0,6	73 6,0	+17 47,5	−11,20	+161,6	8,21484
	8	0,4	0,6	87 27,5	+18 44,0	−11,65	+140,1	21055
	9	0,4	0,6	101 50,9	+18 23,9	−11,32	+116,2	20663
	10	0,4	0,6	116 2,2	+16 49,4	−10,28	+ 92,9	20336
	11	0,5	0,6	129 51,0	+14 7,8	− 8,74	+ 72,2	20101
	12	0,5	0,6	143 13,0	+10 30,6	− 6,92	+ 54,5	19982
	13	0,6	0,6	156 10,6	+ 6 11,1	− 5,01	+ 39,4	19997
	14	0,7	0,6	168 50,7	+ 1 23,4	− 3,12	+ 26,1	20154
	15	0,7	0,6	181 23,9	− 3 38,3	− 1,37	+ 13,4	20451

1895 Im Meridian von Greenwich	Physische Libration in selenocentr.		Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen		
					in AR.	in Decl.	Parallaxe
	Länge	Breite	a	d	$\alpha_☾ - \alpha_k$	$\delta_☾ - \delta_k$	log sin p_k
	′	′	° ′	° ′	s	″	
Juni 29	—0,2	—0,6	1 33,4	— 3 12,6	— 7,49	+151,1	8,23472
30	0,3	0,6	14 10,7	+ 1 50,1	— 7,84	+171,9	23050
Juli 1	0,4	0,5	26 56,7	+ 6 37,6	— 8,46	+184,4	22619
2	0,5	0,5	40 1,7	+10 55,5	— 9,36	+188,0	22193
3	0,6	0,5	53 32,1	+14 29,0	—10,43	+182,4	21778
4	0,6	0,5	67 28,6	+17 4,6	—11,39	+168,2	21379
5	0,6	0,5	81 44,9	+18 31,1	—11,91	+147,6	21003
6	—0,6	—0,5	96 9,1	+18 42,0	—11,76	+123,4	8,20659
7	0,5	0,5	110 26,8	+17 36,8	—10,91	+ 98,9	20358
8	0,5	0,5	124 25,4	+15 20,9	— 9,51	+ 76,4	20116
9	0,5	0,5	137 58,2	+12 4,5	— 7,77	+ 56,8	19950
10	0,4	0,5	151 4,9	+ 8 0,1	— 5,86	+ 40,3	19881
11	0,4	0,5	163 50,8	+ 3 22,0	— 3,91	+ 26,2	19921
12	0,3	0,5	176 25,2	— 1 35,8	— 2,02	+ 13,8	20088
13	0,3	0,5	188 59,6	— 6 39,1	— 0,26	+ 2,1	20390
14	0,2	0,5	201 45,8	—11 33,1	+ 1,27	— 10,0	20825
15	0,2	0,5	214 56,9	—16 3,8	+ 2,47	— 22,8	21379
29	—1,4	—0,4	34 50,1	+ 9 17,1	—10,42	+196,8	8,22446
30	1,5	0,4	48 10,0	+13 10,3	—11,50	+191,3	21899
31	1,5	0,4	61 56,2	+16 10,6	—12,45	+176,9	21406
Aug. 1	1,5	0,4	76 5,5	+18 5,9	—12,99	+155,7	20976
2	1,5	0,4	90 28,0	+18 47,6	—12,89	+130,6	20610
3	1,5	0,4	104 49,6	+18 12,6	—12,10	+104,7	20311
4	1,5	0,4	118 56,8	+16 24,2	—10,74	+ 80,4	20078
5	—1,4	—0,4	132 40,3	+13 30,7	— 9,02	+ 59,0	8,19908
6	1,4	0,4	145 57,0	+ 9 43,9	— 7,11	+ 40,6	19813
7	1,3	0,4	158 50,3	+ 5 17,5	— 5,14	+ 25,2	19795
8	1,2	0,4	171 27,9	+ 0 25,8	— 3,19	+ 12,0	19870
9	1,2	0,4	184 0,6	— 4 37,0	— 1,33	— 0,3	20047
10	1,2	0,4	196 40,7	— 9 36,7	+ 0,38	— 10,7	20336
11	1,1	0,4	209 40,1	—14 18,7	+ 1,87	— 21,7	20746
12	1,1	0,4	223 8,7	—18 27,5	+ 2,99	— 33,0	21272
13	1,1	0,4	237 12,5	—21 48,0	+ 3,57	— 44,1	21901
14	1,2	0,4	251 50,6	—24 5,5	+ 3,40	— 53,1	22610

1895 Im Meridian von Greenwich	Physische Libration in selenocentr.		Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen		
					in AR.	in Decl.	Parallaxe
	Länge	Breite	a	d	$\alpha_☾ - \alpha_k$	$\delta_☾ - \delta_k$	$\log \sin p_k$
	′	′	° ′	° ′	s	″	
Aug. 27	−2,2	−0,3	56 27,7	+15 6,1	−13,44	+187,4	8,21857
28	2,3	0,3	70 28,9	+17 29,0	−14,25	+165,5	21272
29	2,3	0,3	84 47,6	+18 40,8	−14,33	+138,7	20783
30	2,2	0,3	99 11,1	+18 36,6	−13,64	+110,9	20393
31	2,2	0,3	113 25,2	+17 16,9	−12,32	+ 84,7	20105
Sept. 1	2,1	0,3	127 18,8	+14 48,1	−10,60	+ 61,4	19911
2	2,1	0,3	140 46,3	+11 21,1	− 8,67	+ 41,5	19801
3	2,0	0,3	153 47,8	+ 7 8,9	− 6,66	+ 21,5	19769
4	−1,9	−0,3	166 30,6	+ 2 25,5	− 4,68	+ 10,1	8,19814
5	1,8	0,3	179 3,9	− 2 34,8	− 2,81	− 2,4	19931
6	1,8	0,3	191 39,5	− 7 37,7	− 1,07	− 13,6	20128
7	1,7	0,2	204 29,8	−12 28,7	+ 0,49	− 24,1	20407
8	1,7	0,2	217 45,7	−16 52,9	+ 1,79	− 34,0	20776
9	1,7	0,2	231 34,8	−20 34,7	+ 2,71	− 43,3	21239
10	1,7	0,2	245 59,2	−23 19,1	+ 3,11	− 51,2	21792
11	1,7	0,2	260 52,4	−24 53,1	+ 2,87	− 55,6	22422
12	1,7	0,2	275 59,7	−25 8,1	+ 1,99	− 53,3	23101
25	−2,6	−0,2	79 7,2	+18 21,8	−15,22	+149,0	8,21333
26	2,5	0,2	93 30,8	+18 48,6	−14,91	+118,9	20784
27	2,5	0,2	107 50,6	+17 58,4	−13,77	+ 90,2	20358
28	2,4	0,2	121 53,3	+15 56,3	−12,08	+ 64,7	20059
29	2,3	0,2	135 31,3	+12 51,1	−10,12	+ 43,1	19883
30	2,3	0,1	148 42,8	+ 8 55,2	− 8,08	+ 25,0	19817
Oct. 1	2,2	0,1	161 32,1	+ 4 22,5	− 6,09	+ 9,6	19845
2	2,1	0,1	174 7,8	− 0 32,8	− 4,22	− 3,7	19956
3	2,0	0,1	186 41,2	− 5 36,7	− 2,51	− 15,5	20137
4	−1,9	−0,1	199 24,3	−10 34,5	− 1,00	− 26,3	8,20378
5	1,9	0,1	212 29,1	−15 11,5	+ 0,25	− 36,1	20675
6	1,8	0,1	226 4,6	−19 12,3	+ 1,18	− 44,8	21024
7	1,8	0,1	240 15,4	−22 21,7	+ 1,69	− 51,4	21426
8	1,8	0,1	254 58,4	−24 25,2	+ 1,73	− 54,8	21882
9	1,8	0,1	270 1,7	−25 12,5	+ 1,32	− 52,9	22385
10	1,8	0,1	285 7,4	−24 39,0	+ 0,62	− 43,6	22924
11	1,9	0,1	309 56,5	−22 47,7	− 0,20	− 25,9	23477

1895 Im Meridian von Greenwich	Physische Libration in selenocentr.		Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen		
	Länge	Breite	a	d	in AR. $\alpha_{☾} - \alpha_k$	in Decl. $\delta_{☾} - \delta_k$	Parallaxe log sin p_k
	′	′	° ′	° ′	s	″	
Oct. 25	−2,3	0,0	116 23,2	+16 54,4	−13,09	+ 70,5	8,20432
26	2,2	0,0	130 11,8	+14 13,1	−11,24	+ 46,5	20116
27	2,1	0,0	143 33,7	+10 35,8	− 9,20	+ 26,8	19945
28	2,0	0,0	156 31,0	+ 6 16,1	− 7,16	+ 10,6	19907
29	1,8	0,0	169 11,0	+ 1 27,9	− 5,24	− 3,0	19990
30	1,7	0,0	181 43,8	− 3 34,6	− 3,49	− 15,1	20174
31	1,6	0,0	194 21,7	− 8 36,7	− 1,98	− 26,0	20438
Nov. 1	1,5	0,0	207 16,8	−13 24,2	− 0,77	− 36,1	20760
2	−1,5	0,0	220 39,6	−17 41,8	+ 0,07	− 45,0	8,21118
3	1,4	0,0	234 36,7	−21 13,8	+ 0,47	− 51,7	21495
4	1,4	0,0	249 8,6	−23 45,3	+ 0,41	− 54,8	21881
5	1,4	0,0	264 6,1	−25 3,9	− 0,05	− 52,2	22267
6	1,4	0,0	279 13,4	−25 2,7	− 0,70	− 42,5	22649
7	1,4	0,0	294 11,0	−23 41,6	− 1,31	− 25,2	23021
8	1,5	0,0	308 42,4	−21 7,8	− 1,75	− 1,1	23378
9	1,5	0,0	322 39,8	−17 33,6	− 2,03	+ 28,5	23711
24	−1,3	+0,1	151 26,6	+ 8 5,4	− 7,65	+ 13,0	8,20009
25	1,2	0,1	164 12,5	+ 3 26,7	− 5,83	− 1,3	19984
26	1,1	0,1	176 46,6	− 1 31,8	− 3,98	− 13,2	20104
27	1,0	0,1	189 20,7	− 6 36,0	− 2,34	− 23,6	20351
28	0,9	0,1	202 7,5	−11 31,4	− 0,99	− 33,3	20703
29	0,8	0,1	215 18,1	−16 3,1	− 0,03	− 42,3	21128
30	0,7	0,1	229 1,3	−19 55,5	+ 0,42	− 49,9	21595
Dec. 1	0,6	0,1	243 20,3	−22 53,2	+ 0,30	− 54,3	22065
2	−0,6	+0,1	258 10,0	−24 42,5	− 0,36	− 53,2	8,22509
3	0,5	0,2	273 16,7	−25 13,7	− 1,34	− 44,0	22901
4	0,6	0,2	288 21,2	−24 23,9	− 2,29	− 26,1	23223
5	0,6	0,2	303 4,9	−22 17,7	− 2,98	− 0,6	23468
6	0,6	0,2	317 16,3	−19 5,7	− 3,31	+ 30,2	23642
7	0,7	0,2	330 52,7	−15 2,3	− 3,41	+ 63,3	23752
8	0,7	0,2	343 58,7	−10 22,9	− 3,46	+ 96,6	23803
9	0,8	0,2	356 44,3	− 5 22,8	− 3,64	+127,9	23796

1895 Im Meridian von Greenwich	Physische Libration in selenocentr.		Selenocentrische AR. und Decl. von Mösting A.		Reduction für Meridianbeobachtungen		
					in AR.	in Decl.	Parallaxe
	Länge	Breite	a	d	$\alpha_{☾} - \alpha_k$	$\delta_{☾} - \delta_k$	$\log \sin p_k$
	′	′	° ′	° ′	s	″	
Dec. 24	0,0	+0,2	184 22,0	− 4 33,7	−2,49	− 20,7	8,20118
25	+0,1	0,2	197 1,8	− 9 34,5	−0,92	− 30,0	20397
26	0,2	0,2	210 1,2	−14 17,7	+0,38	− 38,1	20805
27	0,3	0,3	223 30,1	−18 27,9	+1,22	− 45,7	21318
28	0,4	0,3	237 34,5	−21 49,6	+1,46	− 51,7	21900
29	0,4	0,3	252 13,0	−24 8,1	+1,02	− 53,9	22503
30	0,5	0,3	267 15,5	−25 11,7	−0,04	− 49,0	23073
1896							
Jan. 0	+0,5	+0,3	282 24,0	−24 54,4	−1,40	− 34,3	8,23563
1	0,5	0,3	297 19,1	−23 17,7	−2,66	− 9,4	23931
2	0,4	0,3	311 45,3	−20 30,2	−3,58	+ 23,4	24154
3	0,4	0,3	325 36,3	−16 45,2	−4,13	+ 60,4	24228
4	0,4	0,3	338 54,0	−12 17,9	−4,46	+ 97,5	24171
5	0,3	0,3	351 46,7	− 7 23,9	−4,75	+131,5	24007
6	0,3	0,3	4 25,8	− 2 18,2	−5,18	+159,9	23765
7	0,2	0,3	17 4,1	+ 2 44,4	−5,91	+180,9	23470

Ephemeride zur Beobachtung des Mondkraters Mösting A

mitgetheilt von Herrn Prof. Dr. J. Franz in Königsberg.

Die Ephemeride ist, ebenso wie die für das vorhergehende Jahr, nach der in Abschnitt 5 der Abhandlung »Darlegung der Ephemeridenrechnung von Mösting A« (Astr. Nachr. Nr. 3241) angegebenen Methode berechnet. Jedoch gilt sie für die Culmination in Berlin, während die früheren Ephemeriden für die Culmination in Greenwich gegeben waren.

Zur Anwendung derselben auf Meridianbeobachtungen des Kraters interpolire man $\alpha_{☾} - \alpha_k$, $\delta_{☾} - \delta_k$ und $\log \sin p_k$ unter strenger Berücksichtigung der zweiten Differenzen mit dem Argument »Länge des Beobachtungsortes von Berlin« so, daſs die Länge westlich positiv, östlich negativ genommen wird. Dann befreie man die beobachtete Decl. des Kraters von der Höhenparallaxe, indem man diese in der bekannten Weise mit dem Argument der wahren Kraterdeclination (nicht Monddeclination) unter Benutzung der Horizontaläquatorialparallaxe p_k des Kraters berechnet, welche aus der Horizontaläquatorialparallaxe des Mondes, wie sie der Nautical Almanac angiebt, abgeleitet ist. Bringt man alsdann $\alpha_{☾} - \alpha_k$ und $\delta_{☾} - \delta_k$ an die Beobachtung an, so hat man die AR. und Decl. des Mondes, wie sie vom Mittelpunkt der Erde aus beobachtet wären, für die Beobachtungszeit, d. h. für die Zeit der Culmination des Kraters (nicht des Mondes). Diese Methode ist für jeden Beobachtungsort auf der Erde anwendbar. Ueber die Reduction der Meridianbeobachtungen des Kraters findet man Näheres Astr. Nachr. Nr. 3262.

Für Beobachtungen auſserhalb des Meridians wende man diejenige Reductionsmethode an, welche in dem vorliegenden Jahrbuch im Anhange »Ueber die Einrichtung des Jahrbuchs« Seite (7) angegeben ist.

Nach den Königsberger Beobachtungen Bd. 38 Abh. I sowie nach Astr. Nachr. Nr. 2917 und 3241 ist der selenographische Ort des Kraters Mösting A

$$\lambda = -5^0\,10',32 \qquad \beta = -3^0\,10',40,$$

seine physische Libration

in selenocentrischer Länge $= +2',2 \sin \odot - 0',4 \sin ☾ + 0',3 \sin 2\omega$

in selenocentrischer Breite $= -1',6 \sin (\omega - 5^0,2)$,

wo $\odot$ und ☾ die mittleren Anomalieen von Sonne und Mond und ω der Abstand des Perigaeums vom aufsteigenden Knoten der Mondbahn sind; die constante Neigung des Mondäquators gegen die Ekliptik ist

$$J = 1^0\,31',37.$$

Aus letzterer und aus dem Zusammenfallen des aufsteigenden Knotens des Mondäquators auf der Ekliptik mit dem niedersteigenden Knoten der Mondbahn auf derselben findet sich die folgende, hier anzuwendende

Lage des Mondäquators.

1896	☊'	Δ — ☋	i
	° ′	° ′	° ′
Jan. 1,0	+1 37,01	—1 29,46	22 3,97
21,0	+1 41,06	—1 33,19	22 4,71
Febr. 10,0	+1 45,08	—1 36,89	22 5,46
März 1,0	+1 49,05	—1 40,56	22 6,25
21,0	+1 52,97	—1 44,16	22 7,06
April 10,0	+1 56,86	—1 47,73	22 7,91
30,0	+2 0,68	—1 51,26	22 8,78
Mai 20,0	+2 4,47	—1 54,76	22 9,68
Juni 9,0	+2 8,21	—1 58,18	22 10,61
29,0	+2 11,90	—2 1,57	22 11,57
Juli 19,0	+2 15,53	—2 4,92	22 12,55
Aug. 8,0	+2 19,11	—2 8,21	22 13,56
28,0	+2 22,63	—2 11,45	22 14,60
Sept. 17,0	+2 26,10	—2 14,64	22 15,66
Oct. 7,0	+2 29,51	—2 17,77	22 16,75
27,0	+2 32,86	—2 20,85	22 17,86
Nov. 16,0	+2 36,15	—2 23,87	22 19,00
Dec. 6,0	+2 39,38	—2 26,83	22 20,16
26,0	+2 42,55	—2 29,74	22 21,34

1896.

Möstıng A im Meridian von Berlin.

Tag	Physische Libration in selenocentr.		Reduction für Meridianbeobachtungen							
			in AR.			in Decl.			Parallaxe	
	Länge	Breite	$\alpha_☾ - \alpha_k$	Diff.		$\delta_☾ - \delta_k$	Diff.		lg. sin p_k	Diff.
*)	′	′	s	s	s	″	″	″		
Jan. 23	+1,3	+0,4	+ 1,56			− 41,9			8,20716	
				+0,99			− 5,8			+544
24	1,4	0,4	+ 2,48		−0,59	− 47,7		+ 1,7	21260	
				+0,33			4,1			643
25	1,5	0,4	+ 2,81		0,84	− 51,8		3,7	21903	
				−0,51			− 0,4			703
26	1,5	0,4	+ 2,30		0,56	− 52,2		7,6	22606	
				1,07			+ 7,2			703
27	1,5	0,4	+ 1,23		−0,36	− 45,0		10,4	23309	
				1,43			17,6			641
28	1,5	0,4	− 0,20		+0,01	− 27,4		11,1	23950	
				1,42			28,7			507
29	1,5	0,4	− 1,62		0,24	+ 1,3		8,7	24457	
				−1,18			+37,4			+325
30	+1,5	+0,4	− 2,80		+0,22	+ 38,7		+ 4,3	8,24782	
				0,96			41,7			+115
31	1,4	0,4	− 3,76		+0,16	+ 80,4		− 0,7	24897	
				0,80			41,0			− 95
Febr. 1	1,4	0,4	− 4,56		−0,06	+121,4		5,3	24802	
				0,86			35,7			276
2	1,3	0,4	− 5,42		0,17	+157,1		9,1	24526	
				1,03			26,6			411
3	1,2	0,4	− 6,45		0,29	+183,7		10,4	24115	
				1,32			16,2			496
4	1,2	0,4	− 7,77		−0,35	+199,9		−12,5	23619	
				−1,67			+ 3,7			−531
5	1,1	0,4	− 9,44			+203,6			23088	
22	+2,3	+0,5	+ 3,48			− 51,2			8,21735	
				−0,27			+ 2,2			+720
23	2,3	0,5	+ 3,21		−0,53	− 49,0		+ 7,3	22455	
				0,80			9,5			760
24	2,3	0,5	+ 2,41		0,33	− 39,5		9,8	23215	
				1,13			19,3			737
25	2,3	0,5	+ 1,28		−0,09	− 20,2		10,5	23952	
				1,22			29,8			642
26	2,3	0,5	+ 0,06		+0,02	+ 9,6		8,9	24594	
				1,20			38,7			475
27	2,2	0,5	− 1,14		0,00	+ 48,3		+ 4,8	25069	
				1,30			43,5			+245
28	2,2	0,5	− 2,34		−0,08	+ 91,8		− 0,5	25314	
				−1,28			+43,0			− 5
29	+2,1	+0,5	− 3,62		−0,25	+134,8		− 6,3	8,25309	
				1,53			36,7			251
März 1	2,0	0,6	− 5,15		0,32	+171,5		11,0	25058	
				1,85			25,7			454
2	2,0	0,6	− 7,00		0,31	+197,2		14,3	24604	
				2,16			+11,4			601
3	1,9	0,6	− 9,16		−0,15	+208,6		15,3	24003	
				2,31			− 3,9			669
4	1,9	0,6	−11,47		+0,26	+204,7		14,9	23334	
				2,05			18,8			685
5	1,8	0,6	−13,52		+0,71	+185,9		− 8,8	22649	
				−1,34			−27,6			−651
6	1,8	0,6	−14,86			+158,3			21998	

*) Für Jan. 1 bis 7 findet man die entsprechenden Angaben im Jahrbuch für 1897.

1896.

Mösting A im Meridian von Berlin.

Tag	Physische Libration in selenocentr. Länge	Breite	Reduction für Meridianbeobachtungen in AR. $\alpha_☾ - \alpha_k$	Diff.		in Decl. $\delta_☾ - \delta_k$	Diff.		Parallaxe lg. sin p_k	Diff.
	′	′	s	s	s	″	″	″		
März 22	+2,8	+0,6	+ 3,11	−0,41		− 42,5	+12,9		8,22240	+713
23	2,8	0,6	+ 2,70	0,60	−0,19	− 29,6	20,8	+ 7,9	22953	726
24	2,7	0,6	+ 2,10	0,77	0,17	− 8,8	29,0	8,2	23679	678
25	2,6	0,6	+ 1,33	0,92	0,15	+ 20,2	37,3	8,3	24357	560
26	2,6	0,6	+ 0,41	1,12	0,20	+ 57,5	41,8	+ 4,5	24917	371
27	2,5	0,7	− 0,71	1,47	0,35	+ 99,3	41,4	− 0,4	25288	+135
28	2,4	0,7	− 2,18	−1,95	0,48	+140,7	+35,6	5,8	25423	−125
29	+2,3	+0,7	− 4,13	2,50	−0,55	+176,3	23,4	−12,2	8,25298	373
30	2,3	0,7	− 6,63	2,91	0,41	+199,7	+ 7,0	16,4	24925	569
31	2,2	0,7	− 9,54	2,92	−0,01	+206,7	−10,3	17,3	24356	697
April 1	2,1	0,7	−12,46	2,29	+0,63	+196,4	25,6	15,3	23659	752
2	2,1	0,7	−14,75	−1,16	1,13	+170,8	34,7	9,1	22907	740
3	2,1	0,7	−15,91	+0,12	+1,28	+136,1	−37,2	− 2,5	22167	−673
4	2,0	0,7	−15,79			+ 98,9			21494	
20	+2,7	+0,8	+ 1,99	−0,15		− 12,8	+23,3		8,22770	+615
21	2,6	0,8	+ 1,84	0,28	−0,13	+ 10,5	29,4	+ 6,1	23385	593
22	2,5	0,8	+ 1,56	0,49	0,21	+ 39,9	34,4	5,0	23978	524
23	2,4	0,8	+ 1,07	0,85	0,36	+ 74,3	37,3	+ 2,9	24502	395
24	2,3	0,8	+ 0,22	1,38	0,53	+111,6	36,4	− 0,9	24897	+211
25	2,2	0,8	− 1,16	2,06	0,68	+148,0	30,4	6,0	25108	− 6
26	2,1	0,8	− 3,22	−2,79	0,73	+178,4	+18,4	12,0	25102	−243
27	+2,0	+0,8	− 6,01	3,30	−0,51	+196,8	+ 1,8	−16,6	8,24859	453
28	1,9	0,8	− 9,31	3,18	+0,12	+198,6	−16,4	18,2	24406	614
29	1,9	0,8	−12,49	2,30	0,88	+182,2	30,9	14,5	23792	711
30	1,8	0,8	−14,79	−0,94	1,36	+151,3	38,3	7,4	23081	735
Mai 1	1,8	0,8	−15,73	+0,40	1,34	+113,0	38,5	− 0,2	22346	698
2	1,8	0,8	−15,33	1,42	1,02	+ 74,5	33,9	+ 4,6	21648	610
3	1,8	0,8	−13,91	+1,76	+0,34	+ 40,6	−27,6	+ 6,3	21038	−498
4	1,8	0,8	−12,15			+ 13,0			20540	

1896.

Mösting A im Meridian von Berlin.

Tag		Physische Libration in selenocentr. Länge	Breite	Reduction für Meridianbeobachtungen in AR. $\alpha_☾ - \alpha_k$	Diff.		in Decl. $\delta_☾ - \delta_k$	Diff.		Parallaxe lg. sin p_k	Diff.
		′	′	s	s	s	″	″	″		
Mai	20	+1,9	+0,9	+ 0,85			+ 67,7			8,23712	
					−0,22			+31,7			+376
	21	1,8	0,9	+ 0,63		−0,41	+ 99,4		+ 0,6	24088	
					0,63			32,3			298
	22	1,7	0,9	0,00		0,61	+131,7		− 3,0	24386	
					1,24			29,3			174
	23	1,6	0,9	− 1,24		0,76	+161,0		7,1	24560	
					2,00			22,2			+ 14
	24	1,5	0,9	− 3,24		0,78	+183,2		12,0	24574	
					2,78			+10,2			−159
	25	1,3	0,9	− 6,02		−0,46	+193,4		16,0	24415	
					3,24			− 5,8			337
	26	1,2	0,9	− 9,26		+0,28	+187,6		16,5	24078	
					−2,96			−22,3			−491
	27	+1,2	+0,9	−12,22		+1,01	+165,3		−12,4	8,23587	
					1,95			34,7			598
	28	1,1	0,9	−14,17		1,38	+130,6		− 4,9	22989	
					−0,57			39,6			649
	29	1,1	0,9	−14,74		1,20	+ 91,0		+ 1,7	22340	
					+0,63			37,9			643
	30	1,1	0,9	−14,11		0,78	+ 53,1		5,6	21697	
					1,41			32,3			586
	31	1,1	0,9	−12,70		0,41	+ 20,8		6,7	21111	
					1,82			25,6			489
Juni	1	1,1	0,9	−10,88		+0,13	− 4,8		6,6	20622	
					1,95			19,0			365
	2	1,2	0,9	− 8,93		−0,04	− 23,8		+ 5,4	20257	
					+1,91			−13,6			−225
	3	1,2	0,9	− 7,02			− 37,4			20032	
	18	+0,8	+1,0	− 0,63			+130,3			8,23875	
					−0,71			+27,2			+109
	19	0,7	1,0	− 1,34		−0,55	+157,5		− 6,4	23984	
					1,26			20,8			+ 34
	20	0,6	1,0	− 2,60		0,74	+178,3		8,7	24018	
					2,00			+12,1			− 64
	21	0,5	1,0	− 4,60		0,63	+190,4		12,6	23954	
					2,63			− 0,5			175
	22	0,4	1,0	− 7,23		−0,18	+189,9		14,6	23779	
					2,81			15,1			290
	23	0,3	1,0	−10,04		+0,52	+174,8		13,5	23489	
					2,29			28,6			399
	24	0,2	1,0	−12,33		1,09	+146,2		8,5	23090	
					1,20			37,1			480
	25	0,2	1,0	−13,53		1,15	+109,1		− 2,0	22610	
					−0,05			−39,1			−532
	26	+0,1	+1,0	−13,58		+0,97	+ 70,0		+ 3,3	8,22078	
					+0,92			35,8			541
	27	0,1	1,0	−12,66		0,58	+ 34,2		6,0	21537	
					1,50			29,8			509
	28	0,1	1,0	−11,16		0,27	+ 4,4		6,8	21028	
					1,77			23,0			434
	29	0,1	1,0	− 9,39		+0,07	− 18,6		6,3	20594	
					1,84			16,7			329
	30	0,2	1,0	− 7,55		−0,03	− 35,3		5,4	20265	
					1,81			11,3			204
Juli	1	0,2	1,0	− 5,74		0,13	− 46,6		4,2	20061	
					1,68			7,1			− 63
	2	0,3	1,0	− 4,06		−0,19	− 53,7		+ 3,2	19998	
					+1,49			− 3,9			+ 86
	3	0,3	1,0	− 2,57			− 57,6			20084	

1896.

Mösting A im Meridian von Berlin.

Tag	Physische Libration in selenocentr. Länge	Breite	Reduction für Meridianbeobachtungen in AR. $\alpha_☾ - \alpha_k$	Diff.		in Decl. $\delta_☾ - \delta_k$	Diff.		Parallaxe lg. sin p_k	Diff.
	′	′	s	s	s	″	″	″		
Juli 18	−0,5	+1,1	− 5,01			+194,3			8,23669	
				−2,14			+ 0,8			−224
19	0,6	1,1	− 7,15		−0,26	+195,1		−13,0	23445	
				2,40			−12,2			277
20	0,7	1,1	− 9,55		+0,25	+182,9		12,8	23168	
				2,15			25,0			326
21	0,8	1,1	−11,70		0,80	+157,9		9,1	22842	
				1,35			34,1			373
22	0,8	1,1	−13,05		1,05	+123,8		− 4,0	22469	
				−0,30			38,1			411
23	0,9	1,1	−13,35		0,94	+ 85,7		+ 1,1	22058	
				+0,64			37,0			434
24	0,9	1,1	−12,71		0,64	+ 48,7		4,5	21624	
				+1,28			−32,5			−434
25	−0,9	+1,1	−11,43		+0,34	+ 16,2		+ 6,2	8,21190	
				1,62			26,3			407
26	0,9	1,1	− 9,81		+0,15	− 10,1		6,4	20783	
				1,77			19,9			354
27	0,8	1,1	− 8,04		0,00	− 30,0		5,9	20429	
				1,77			14,0			270
28	0,8	1,1	− 6,27		−0,07	− 44,0		5,0	20159	
				1,70			9,0			165
29	0,8	1,1	− 4,57		0,12	− 53,0		4,1	19994	
				1,58			4,9			− 34
30	0,7	1,1	− 2,99		0,18	− 57,9		2,9	19960	
				1,40			− 2,0			+107
31	0,7	1,1	− 1,59		−0,23	− 59,9		+ 2,3	20067	
				+1,17			+ 0,3			+251
Aug. 1	0,6	1,1	− 0,42			− 59,6			20318	
16	−1,6	+1,2	−10,33			+192,1			8,23194	
				−2,10			−23,9			−433
17	1,6	1,2	−12,43		+0,75	+168,2		− 9,6	22761	
				1,35			33,5			431
18	1,7	1,2	−13,78		1,02	+134,7		− 4,4	22330	
				−0,33			37,9			421
19	1,7	1,2	−14,11		0,94	+ 96,8		+ 0,4	21909	
				+0,61			37,5			404
20	1,8	1,2	−13,50		0,65	+ 59,3		3,8	21505	
				1,26			33,7			383
21	1,8	1,2	−12,24		0,36	+ 25,6		5,7	21122	
				1,62			28,0			355
22	1,7	1,2	−10,62		0,14	− 2,4		5,8	20767	
				+1,76			−22,2			−321
23	−1,7	+1,2	− 8,86		+0,02	− 24,6		+ 6,0	8,20446	
				1,78			16,2			266
24	1,6	1,2	− 7,08		−0,06	− 40,8		5,3	20180	
				1,72			10,9			199
25	1,6	1,2	− 5,36		0,11	− 51,7		4,6	19981	
				1,61			6,3			110
26	1,5	1,2	− 3,75		0,13	− 58,0		3,7	19871	
				1,48			− 2,6			− 3
27	1,4	1,2	− 2,27		0,15	− 60,6		2,8	19868	
				1,33			+ 0,2			+120
28	1,4	1,2	− 0,94		0,21	− 60,4		2,3	19988	
				1,12			2,5			255
29	1,3	1,2	+ 0,18		0,26	− 57,9		2,0	20243	
				0,86			4,5			394
30	1,3	1,2	+ 1,04		−0,28	− 53,4		+ 2,3	20637	
				+0,58			+ 6,8			+522

1896.

Mösting A im Meridian von Berlin.

Tag	Physische Libration in selenocentr.		Reduction für Meridianbeobachtungen							
			in AR.			in Decl.			Parallaxe	
	Länge	Breite	$\alpha_☾ - \alpha_k$	Diff.		$\delta_☾ - \delta_k$	Diff.		lg. sin p_k	Diff.
	′	′	s	s	s	″	″	″		
Sept. 14	−2,2	+1,3	−15,04			+145,0			8,22638	
				−0,52			−39,6			−567
15	2,2	1,3	−15,56		+1,05	+105,4		+0,2	22071	
				+0,53			39,4			517
16	2,3	1,3	−15,03		0,77	+ 66,0		4,0	21554	
				1,30			35,4			451
17	2,2	1,3	−13,73		0,41	+ 30,6		5,9	21103	
				1,71			29,5			384
18	2,2	1,3	−12,02		+0,16	+ 1,1		6,1	20719	
				1,87			23,4			322
19	2,2	1,3	−10,15		−0,02	— 22,3		5,9	20397	
				1,85			17,5			258
20	2,1	1,3	— 8,30		0,08	— 39,8		5,4	20139	
				1,77			12,1			194
21	2,1	1,3	— 6,53		0,13	— 51,9		4,7	19945	
				+1,64			— 7,4			−127
22	−2,0	+1,3	— 4,89		−0,14	— 59,3		+4,1	8,19818	
				1,50			— 3,3			— 50
23	1,9	1,3	— 3,39		0,15	— 62,6		3,3	19768	
				1,35			0,0			+ 38
24	1,8	1,3	— 2,04		0,16	— 62,6		2,8	19806	
				1,19			+ 2,8			133
25	1,7	1,3	— 0,85		0,15	— 59,8		2,4	19939	
				1,04			5,2			246
26	1,7	1,3	+ 0,19		0,17	— 54,6		1,9	20185	
				0,87			7,1			368
27	1,6	1,3	+ 1,06		0,19	— 47,5		2,4	20553	
				0,68			9,5			488
28	1,6	1,3	+ 1,74		−0,18	— 38,0		+2,9	21041	
				+0,50			+12,4			+603
29	1,5	1,3	+ 2,24			— 25,6			21644	
Oct. 13	−2,3	+1,4	−16,42			+ 73,8			8,22084	
				+1,06			−39,2			−621
14	2,3	1,4	−15,36		+0,62	+ 34,6		+6,7	21463	
				1,68			32,5			528
15	2,3	1,4	−13,68		0,25	+ 2,1		7,1	20935	
				1,93			25,4			428
16	2,2	1,4	−11,75		+0,03	— 23,3		6,6	20507	
				1,96			18,8			325
17	2,1	1,4	— 9,79		−0,11	— 42,1		5,9	20182	
				1,85			12,9			227
18	2,1	1,4	— 7,94		0,15	— 55,0		4,9	19955	
				1,70			8,0			141
19	2,0	1,4	— 6,24		0,18	— 63,0		4,3	19814	
				1,52			3,7			— 59
20	1,9	1,4	— 4,72		0,19	— 66,7		3,6	19755	
				+1,33			— 0,1			+ 14
21	−1,8	+1,4	— 3,39		−0,17	— 66,8		+3,2	8,19769	
				1,16			+ 3,1			88
22	1,7	1,4	— 2,23		0,16	— 63,7		2,8	19857	
				1,00			5,9			166
23	1,6	1,4	— 1,23		0,12	— 57,8		2,5	20023	
				0,88			8,4			247
24	1,5	1,4	— 0,35		0,08	— 49,4		2,3	20270	
				0,80			10,7			335
25	1,4	1,4	+ 0,45		0,04	— 38,7		2,3	20605	
				0,76			13,0			426
26	1,4	1,4	+ 1,21		0,06	— 25,7		2,4	21031	
				0,70			15,4			521
27	1,4	1,4	+ 1,91		−0,07	— 10,3		+3,0	21552	
				+0,63			+18,4			+599
28	1,3	1,4	+ 2,54			+ 8,1			22151	

1896.

Mösting A im Meridian von Berlin.

Tag	Physische Libration in selenocentr. Länge	Breite	Reduction für Meridianbeobachtungen: in AR. $\alpha_☾ - \alpha_k$	Diff.		in Decl. $\delta_☾ - \delta_k$	Diff.		Parallaxe lg. sin p_k	Diff.
	′	′	s	s	s	″	″	″		
Nov. 12	−1,8	+1,4	−13,08	+1,93		− 22,9	−21,7		8,20978	−480
13	1,7	1,4	−11,15	1,89	−0,04	− 44,6	14,8	+6,9	20498	353
14	1,6	1,4	− 9,26	1,76	0,13	− 59,4	8,9	5,9	20145	227
15	1,5	1,4	− 7,50	1,57	0,19	− 68,3	4,1	4,8	19918	108
16	1,4	1,4	− 5,93	1,36	0,21	− 72,4	− 0,2	3,9	19810	− 4
17	1,3	1,4	− 4,57	1,14	0,22	− 72,6	+ 3,2	3,4	19806	+ 86
18	1,2	1,4	− 3,43	0,93	0,21	− 69,4	6,4	3,2	19892	161
19	1,1	1,4	− 2,50	+0,77	0,16	− 63,0	+ 9,2	2,8	20053	+225
20	−1,0	+1,4	− 1,73	0,68	−0,09	− 53,8	12,1	+2,9	8,20278	281
21	0,9	1,4	− 1,05	0,66	−0,02	− 41,7	14,8	2,7	20559	332
22	0,8	1,4	− 0,39	0,72	+0,06	− 26,9	17,3	2,5	20891	379
23	0,8	1,4	+ 0,33	0,77	+0,05	− 9,6	19,3	2,0	21270	429
24	0,7	1,4	+ 1,10	0,75	−0,02	+ 9,7	21,3	2,0	21699	474
25	0,7	1,4	+ 1,85	0,65	0,10	+ 31,0	23,4	2,1	22173	515
26	0,8	1,4	+ 2,50	+0,41	−0,24	+ 54,4	+25,2	+1,8	22688	+538
27	0,8	1,4	+ 2,91			+ 79,6			23226	
Dec. 12	−0,8	+1,5	− 8,35	+1,60		− 71,6	− 5,6		8,20221	−238
13	0,6	1,5	− 6,75	1,42	−0,18	− 77,2	− 0,9	+4,7	19983	− 96
14	0,5	1,5	− 5,33	1,21	0,21	− 78,1	+ 3,0	3,9	19887	+ 38
15	0,4	1,5	− 4,12	0,97	0,24	− 75,1	6,1	3,1	19925	151
16	0,3	1,5	− 3,15	0,75	0,22	− 69,0	9,3	3,2	20076	246
17	−0,1	1,5	− 2,40	0,57	0,18	− 59,7	12,3	3,0	20322	316
18	0,0	1,5	− 1,83	0,50	−0,07	− 47,4	15,5	3,2	20638	359
19	0,0	1,5	− 1,33	+0,51	+0,01	− 31,9	+18,7	3,2	20997	+384
20	+0,1	+1,5	− 0,82	0,58	+0,07	− 13,2	21,4	+2,7	8,21381	391
21	0,2	1,5	− 0,24	0,64	0,06	+ 8,2	23,6	2,2	21772	384
22	0,2	1,5	+ 0,40	0,65	+0,01	+ 31,8	24,9	1,3	22156	373
23	0,2	1,5	+ 1,05	0,55	−0,10	+ 56,7	25,7	0,8	22529	361
24	0,2	1,5	+ 1,60	+0,32	0,23	+ 82,4	25,8	+0,1	22890	349
25	0,2	1,5	+ 1,92	−0,05	0,37	+108,2	25,1	−0,7	23239	325
26	0,2	1,5	+ 1,87	−0,59	−0,54	+133,3	+22,7	−2,4	23564	+287

A. W. Schade's Buchdruckerei (L. Schade) in Berlin, Stallschreiberstr. 45/46.